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Correction

Correction to: On Semi-finite Hexagons of Order $(2,t)$ Containing a Subhexagon

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Correction to: Ann. Comb. 20 (2016) 433–452
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In Section 4.2 of [1], we showed that there does not exist any infinite near hexagon $\mathcal{N}$ of order $(2,t)$ that contains an isometrically embedded subgeometry $\mathcal{H}$ isomorphic to $H(2)$. The proofs of Lemmas 4.6 and 4.7 in [1] have been spoiled by the same error: points of $\mathcal{N}$ at distance 1 from $\mathcal{H}$ are not necessarily collinear with a unique point of $\mathcal{H}$ (see Page 446, Line -8 and Page 447, Line 2). This is true in case $\mathcal{N}$ is a generalised hexagon, but not if $\mathcal{N}$ is a general near hexagon. Luckily, these errors can be corrected.

The following proof should replace the proof of Lemma 4.6 in [1].

Lemma 1. *There are only finitely many points of type B_1 in $\mathcal{N}$.*

Proof. Let $\mathcal{B}$ denote the set of those points of $\mathcal{N}$ that have type B_i for some $i \in \{2,3,4,5\}$. Then $\mathcal{B}$ is finite by [1, Lemma 4.5]. Let $\mathcal{A}$ denote the set of those points of $\mathcal{N}$ that have type A, i.e., the points of $\mathcal{H}$. Then the set $\mathcal{A} \cup \mathcal{B}$ is also finite. Let x be a point of type B_1 in $\mathcal{N}$. Then by [1, Lemma 4.2], x is at distance 1 from $\mathcal{H}$, and since $\mathcal{O}_{f_x}$ is a singleton, there exists a unique point $\pi(x)$ in $\mathcal{H}$ collinear with x. If x is only collinear with points of type A, B_1 or C, then by the same reasoning as in the proof of [1, Theorem 4.4], we get a contradiction. So, x is collinear with at least one point of $\mathcal{B}$, and we have already seen that it is collinear with at least one point of $\mathcal{A}$. Thus x is the common neighbour of two points at distance 2 in the finite set $\mathcal{A} \cup \mathcal{B}$. Since each such pair of points at distance 2 in the near polygon $\mathcal{N}$ has finitely many

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(at most five) common neighbours, we see that the set of points of type B_1 must be finite; in fact, the cardinality of this set is bounded by five times the number of unordered pairs of points at distance 2 in $\mathcal{A} \cup \mathcal{B}$. □

The following proof should replace the proof of Lemma 4.7 in [1].

Lemma 2. *There are only finitely many points of type C in $\mathcal{N}$.*

Proof. Let x be a point of type C in $\mathcal{N}$. Then the set of points of $\mathcal{H}$ at distance 2 from x is a 1-ovoid of $\mathcal{H}$ and hence it has cardinality 21. Let S_x be the set of common neighbours between x and the elements of $\mathcal{O}_{f_x}$ (the 1-ovoid of $\mathcal{H}$ induced by x). By [1, Lemma 4.2], each element y of S_x has type B_i for some $i \in \{1, 2, \ldots, 5\}$ and hence by [1, Table 3] y is collinear with at most nine points of $\mathcal{H}$. Therefore, $|S_x| \geq \frac{21}{9}$, and we get two points of the set $\Gamma_1(\mathcal{H})$ at distance 2 from each other having x as a common neighbour. By [1, Lemma 4.5] and Lemma 1, the set $\Gamma_1(\mathcal{H})$ is finite. A similar reasoning as in the proof of Lemma 1 then shows that there are only finitely many points of type C in $\mathcal{N}$. □

The rest of the discussion in Section 4.2 of [1] can remain as it is. In the proof of Lemma 4.3, there is however a typo. The condition $\mathrm{d}(x, y_1) = \mathrm{d}(x, y_2) = \mathrm{d}(x, y_3)$ should be replaced with $\mathrm{d}(y, x_1) = \mathrm{d}(y, x_2) = \mathrm{d}(y, x_3)$.

Reference

[1] Bishnoi, A., De Bruyn, B.: On semi-finite hexagons of order $(2, t)$ containing a subhexagon. Ann. Comb. 20(3), 433–452 (2016)

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