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Correction to: Wellposedness and Convergence of Solutions to a Class of Forced Non-diffusive Equations with Applications

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In this erratum we revise the hypothesis and statement of [1, Theorem 2.3] to prove local in time existence of analytic, rather than Gevrey class solutions. As a consequence, we also revise the results in [1, Theorem 2.5] for the convergence of analytic solutions as ν goes to zero.

To begin with, we first recall that the abstract active scalar equations which are given by

$$\begin{cases} \partial_t \theta^\nu + u^\nu \cdot \nabla \theta^\nu = S, \\ u^\nu_j = \partial_{x_i} T^\nu_{ij}[\theta^\nu], \theta^\nu(x,0) = \theta_0(x) \end{cases} \tag{1.1}$$

where $\mathbb{T}^d \times (0,\infty) = [0,2\pi]^d \times (0,\infty)$ with $d \geq 2$. We assume that

$$\int_{\mathbb{T}^d} \theta^\nu(x,t)dx = \int_{\mathbb{T}^d} S(x) = 0 \text{ for all } t \geq 0.$$

The symbols $\{T^\nu_{ij}\}_{\nu\geq 0}$ refer to a sequence of operators which satisfy:

A1 $\partial_i \partial_j T^\nu_{ij} f = 0$ for any smooth functions f for all $\nu \geq 0$.

A2 $T^\nu_{ij} : L^\infty(\mathbb{T}^d) \to BMO(\mathbb{T}^d)$ are bounded for all $\nu \geq 0$.

A3 For each $\nu > 0$, there exists a constant $C_\nu > 0$ such that for all $1 \leq i, j \leq d$,

$$|\widehat{T}^\nu_{ij}(k)| \leq C_\nu |k|^{-3}, \forall k \in \mathbb{Z}^d.$$

A4 For each $1 \leq i, j \leq d$,

$$\lim_{\nu\to 0} \sum_{k\in\mathbb{Z}^d} |\widehat{T^\nu_{ij}}(k) - \widehat{T^0_{ij}}(k)|^2 |\widehat{g}(k)|^2 = 0$$

for all $g \in L^2$.

Moreover, we further assume that $\{T^\nu_{ij}\}_{\nu\geq 0}$ satisfy either one of the following assumptions:

A5$_1$ There exists a constant $C_0 > 0$ independent of ν, such that for all $1 \leq i, j \leq d$,

$$\sup_{\nu\in(0,1]} \sup_{\{k\in\mathbb{Z}^d\}} |\widehat{T}^\nu_{ij}(k)| \leq C_0;$$

$$\sup_{\{k\in\mathbb{Z}^d\}} |\widehat{T}^0_{ij}(k)| \leq C_0. \tag{1.2}$$

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A5_2 There exists a constant $C_0 > 0$ independent of ν, such that for all $1 \leq i, j \leq d$,

$$\sup_{\nu\in(0,1]} \sup_{\{k\in\mathbb{Z}^d\}} |k_i \widehat{T}^\nu_{ij}(k)| \leq C_0;$$

$$\sup_{\{k\in\mathbb{Z}^d\}} |k_i \widehat{T}^0_{ij}(k)| \leq C_0. \tag{1.3}$$

We now give the revised statement for Theorem 2.3 given in [1]:

Theorem 2.3. (Analytic local wellposedness in the case $\nu = 0$). *Fix $r > \frac{d}{2} + \frac{3}{2}$ and $K_0 > 0$. Let $\theta^0(\cdot, 0) = \theta_0$ and S be analytic functions with radius of convergence $\tau_0 > 0$ and satisfy*

$$\|\Lambda^r e^{\tau_0\Lambda}\theta^0(\cdot, 0)\|_{L^2} \leq K_0, \qquad \|\Lambda^r e^{\tau_0\Lambda} S\|_{L^2} \leq K_0. \tag{2.4}$$

For $\nu = 0$, under the assumptions A1–A2 and A5_1, there exists $\bar{T}, \bar{\tau} > 0$ and a unique analytic solution θ^0 to (1.1) defined on $\mathbb{T}^d \times [0, \bar{T}]$ with radius of convergence at least $\bar{\tau}$. In particular, there exists a constant $C = C(K_0) > 0$ such that for all $t \in [0, \bar{T}]$,

$$\|\Lambda^r e^{\bar{\tau}\Lambda}\theta^0(\cdot, t)\|_{L^2} \leq C. \tag{2.5}$$

Moreover, if the assumption A3 holds as well, then we have

$$\|\Lambda^r e^{\bar{\tau}\Lambda}\theta^\nu(\cdot, t)\|_{L^2} \leq C, \, \forall \nu > 0, \tag{2.6}$$

where θ^ν are analytic solutions to (1.1) for $\nu > 0$ as described in [1, Theorem 2.2].

Proof of Theorem 2.3. The results follow by the similar argument given by Friedlander and Vicol [2] for the unforced system with $S \equiv 0$ in (1.1). □

We also revise the statement of [1, Theorem 2.5] for the convergence of analytic solutions to (1.1).

Theorem 2.5. (Convergence of solutions as $\nu \to 0$). *Depending on the assumptions A5_1 and A5_2, we have the following cases:*

- *Assume that the hypotheses and notations of Theorem 2.3 are in force. Under the assumptions A3–A4, if θ^ν and θ^0 are analytic solutions to (1.1) for $\nu > 0$ and $\nu = 0$ respectively with initial datum θ_0 on $\mathbb{T}^d \times [0, \bar{T}]$ with radius of convergence at least $\bar{\tau}$ as described in Theorem 2.3, then there exists $T < \bar{T}$ and $\tau = \tau(t) < \bar{\tau}$ such that, for $t \in [0, T]$, we have*

$$\lim_{\nu\to 0} \|(\Lambda^r e^{\tau\Lambda}\theta^\nu - \Lambda^r e^{\tau\Lambda}\theta^0)(\cdot, t)\|_{L^2} = 0. \tag{2.7}$$

- *Assume that the hypotheses and notations of* [1, Theorem 2.4] *are in force. Under the assumptions A3–A4, for $d \geq 2$ and $s > \frac{d}{2} + 1$ and $t \in [0, T]$, we have*

$$\lim_{\nu\to 0} \|(\theta^\nu - \theta^0)(\cdot, t)\|_{H^{s-1}} = 0. \tag{2.8}$$

Remark 2.6. The proof for the convergence result (2.7) follows by the same argument given in [1, pp. 16–18] by taking $s = 1$.

Remark 2.7. The applications to the magnetostrophic equations given in [1, Sect. 6] now hold under the revised statements of Theorems 2.3 and 2.5.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interests.

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