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RESEARCH

Vault Mosaics of the Kukeldash Madrasah, Bukhara, Uzbekistan

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Abstract The vault mosaics at the entrance of Kukeldash Madrasah in Bukhara (built in 1568–69) display three different geometric types, all of them with 8*mm* symmetry. Type 1 is the centrally-positioned Kond-style tiling, a combination of 10-, 12-, and 16-fold rosettes with pentagons and lozenges. Type 2 is a unique curvilinear pattern formed by irregular hexagonal tiles in non-linear hexagonal arrangement. Interpretation of this pattern as a net of intersecting geometric curves is made in the paper and its construction is discussed. Type 3 is represented by colourful non-linear mosaics based on combination of local centers with five and sixfold multiplicity, interpreted as the antithesis of the type 2 mosaic.

Introduction

Kukeldash Madrasah, built in 1568–69 by khan Abdullah II of the Uzbek Shaybanid dynasty (Mayhew et al. 2000), is one of Bukhara's largest theological colleges. It had 150 student cells, two iwans in the courtyard and a monumental entrance. It brought some architectural innovations, such as upstairs loggias. With great ingenuity, the pendentives were organized to form a webbed framing which, filled with brickwork, produced vaults (Gippenreiter et al. 1987). The subject of this present paper are the remarkable mosaics in the vaults of the madrasah entrance, which differ in style, realization and material from the usual Islamic mosaics of the broad region.

I became acquainted with the fascinating ceiling mosaics of the Kukeldash Madrasah in Bukhara in about 1966, from a photographic volume produced by two

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Birkhäuser

trail-blazing Czech authors (Hrbas and Knobloch 1965) who somehow overcame the unwillingness and inertia of both central and provincial Soviet administration and managed to produce a magnificent demonstration of Central Asian Islamic art, by then half-ignored. Finally, in 2015 I had the possibility to admire these mosaics in their full colour and splendour in the entrance corridors of the madrasah, which otherwise is relatively austere compared with other Bukhara monuments.

The mosaics are placed in both halves of the split entrance corridor to the madrasah, in the fields of the ribbed vault of a rather low ceiling (Fig. 1). They show three different mosaic styles/constructions, and even a case in which all three are combined in one field.

Vault Geometry

The rib vault which houses the mosaics can be described as three nested squares: a small square filled by a central mosaic; a larger one, rotated by 45°; and the largest one, with edges approximately three times larger and with corner coffers filled by four more mosaics (Fig. 1). The masonry is accentuated by a combination of masked element contacts (virtual continuation of them across a portion of a pattern) with contacts which are clearly marked by white interspaces between elements. Four and threefold fan-like fields, situated in the intermediate square and beyond the squares, add dynamism to the construction. In one wing of the corridor, settlement of the ceiling introduced cracks in several of these mosaics whereas in the other wing they are in pristine condition.

The eightfold mosaic fields are filled with three types of mosaic. Type 1, the central one (Fig. 2), is closest to a Kond style (Bonner 2014; Makovicky 2016a, among many others), whereas the two mosaics along the corridor axis are a complex net of hexagons (Fig. 3), and the perpendicular ones are two mosaics in which

Fig. 1 Ornamented ceiling of the entrance corridor to the Kukeldash Madrasah in Bukhara, Uzbekistan. The ribbed ceiling (details in text) contains a central, two axial and two lateral mosaics delimited within eightfold star shapes

Fig. 2 The central Kond-style mosaic in the Kukeldash entrance

Fig. 3 The curvilinear pattern of irregularly cut hexagonal tiles from the entrance of the Kukeldash Madrasah

Fig. 4 A colourful lateral mosaic with petal-adorned five and sixfold stars; entrance to the Kukeldash Madrasah

Fig. 5 A composite lateral mosaic in the Kukeldash corridor unifying the principles seen in Figs. 3 and 4

enhanced coloring partly masks their geometry (Figs. 4, 5). The mosaic fields are slightly concave and the question arose whether they can be adequately described by 8*mm* point group of symmetry, with two sets of radially oriented mirror planes intersecting in the eightfold rotation axis. Measurements on the eightfold star field of the type illustrated in Fig. 3, performed on oblique photographs of the ceiling, reveal, however, that the elevation of the vault crest above the level of its periphery is only 7% of the length of the radius of the circle defining the curvature of the vault. The arc spanning the circle circumscribed to the eightfold star field is only 43°. The vault-defining radius is approximately 2.66 times larger than the radius of the field. Therefore, the planar point group is a sufficiently close approximation to the symmetry of individual mosaics.

Type 1: The Centrally-Positioned Kond-Style Tiling

The central mosaic is four-colored, with small ubiquitous pentagons and their central dot rendered in green–red and black–white combinations (Fig. 2). It is an eightfold radial composition which consists of three concentric areas: a central one with the 16-fold star, intermediate one with tenfold rosettes, and the peripheral one with incomplete 12-fold rosettes. Tenfold rosettes represent radially a white–black–white combination, and they have a central green fivefold star (Fig. 2). The central 16-fold star is white–black–white with a central double-ring, whereas marginal, incomplete 12-fold stars are black–white with a central black-and-red circle. Corners of the limiting (slightly broken) octagon field of the mosaic contain black–white star fragments, probably eightfold stars. What remains entirely suppressed by the tile coloring are the three different rhombs (lozenges), respectively placed between 12–12, 10–10 and 10–16 rosettes (accentuated in Fig. 6).

These lozenges are marked by V-shaped internal divisions which correspond to those in the Kond (or Maragha-) tiling lozenges, here accentuated by green fill and red frame in two cases, and by red fill and green frame in the central ring of lozenges (Fig. 2). The lozenges interconnecting two tenfold rosettes are slim, even slimmer than those in the decagonal tiling at Maragha (Makovicky 2008), whereas those interconnecting other rosette pairs have the smaller inner angles considerably enlarged, in line with observations on Kond tilings by Makovicky (2016b). The

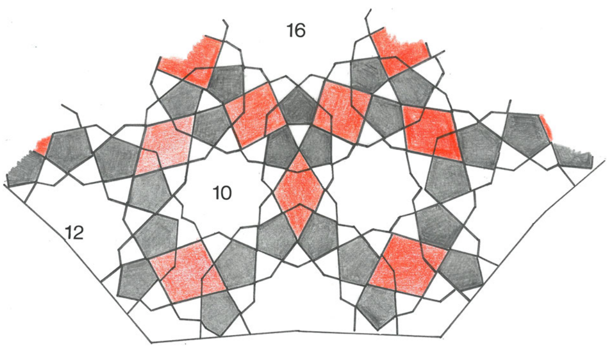

Fig. 6 A portion of the central mosaic redrawn from the photograph. The Kond-style marked lozenges (red), and small pentagons (grey) have been selectively coloured whereas the 10-, 12-, and 16-fold rosettes are left blank together with their outer rays. One lozenge plus four pentagons side-attached to it form a unit which is treated in some detail in the text

'four pentagons-and-a lozenge' configuration, defined in Kond tilings from Iran and Uzbekistan by Makovicky (2016b), dominates the pattern when redrawn and appropriately coloured in Fig. 6. Triples of such configurations meet via shared pentagons to form the central wreath; these configurations comprise two angles of 116.5° and once (centrally) of 127°. These values and the details of the intersecting configurations are reminiscent of the hexagonal pattern from Divrigi (Makovicky 2016b, Fig. 23) but with appropriate distortions to fit a different angular situation. As for the stars involved, we noticed that two times tenfold plus once 16-fold adds to 36, which divided by 3 is 12 as in the hexagonal pattern: does this make the formation of this pattern easier?

Here the angular 'test' of pentagon rotation in the '*four pentagons-and-a lozenge*' configuration, defined in (Makovicky 2016b) gives rather erratic results, primarily because the pentagons were already fashioned as distorted. The only exceptions are the radially oriented four-pentagons-and-a lozenge configurations which interconnect the 16- and 12-fold rosettes, and are squeezed between two tenfold rosettes. This local situation corresponds to that in some 2D-periodic Kond patterns (e.g., Bonner 2017, Fig. 252) but in our case these configurations are not 5 star rays apart, counting around one tenfold rosette, but 6 rays when counting counterclockwise, i.e., they diverge from one another. We should remember that all elements of the mosaic are freely hand-cut and hand-fitted loose tiles, so that a regularity observed in interlaced rib patterns cannot be expected here. The 'butterflies' or 'bow-ties' known from other Kond tilings (Makovicky 2016a) are missing at Kukeldash.

This central mosaic is an attempt to construct a Kond tiling which combines a central 16-fold rosette with 10-, 12- and eightfold rosettes in a single circular pattern. It contains, although only in principle and imperfectly, a built-in similarity operation combined with eightfold rotation. Any problems which could be detected are solved, or masked (e.g., in creating the eightfold pattern, 8 times unit intervals of a tenfold rosette do not give 360°) by approximate hand-cutting and fitting of tiles. Judging from the details of their form, the interconnecting white tiles appear to have been adapted in situ until they fitted the relevant interstices (Fig. 2). The local distortion of pattern symmetry is widespread, but it does not detract from the attractiveness of the visually prominent design.

Type 2: Curvilinear Mosaics

Two mosaics of this category are composed of hand-shaped hexagons in 6.3.6.3 arrangement. They are of great interest because of their curvilinearity (Fig. 3). The central rosette is eightfold, with white petals pointing toward the arms of the field-limiting eightfold star. The surrounding white-centered black hexagons are elongate and petal-like (a rather common shape in that position, see Bonner 2017, Fig. 322C); the rest of hexagons are irregular because of hand-cutting. Only the radial rows towards the recesses of the star-like limit are linear, all the other traceable interconnections of adjacent hexagonal discs are curved, creating a non-linear hexagonal pattern with discs and intervals decreasing towards boundaries (Fig. 3).

There are three concave lines plus two rudimentary outermost lines in every 135° circular segment. Instead of the irregular outline of individual hexagons, we concentrate on their center markings (small ceramic circles in Fig. 3), which are closer to the imaginary connection lines. The same system of lines repeats eight times, every 45°, creating composite nets which can also be interpreted as a system of nested concave ('hyperbolic') octagons. The nets are curved nets, with lines being most curved close to the centre of the 135° cut-out and straightening out towards edge (Fig. 7). Some curves can be drawn through well-aligned tile centers (Fig. 7), while other curves suffer from the scattering of these points around them, causing uncertainty in interpretation. In this situation, the question of which type of curve was used can be studied only by comparison of the fitted curves with graphs and shapes of mathematically defined curves.

The obvious candidates are *inward oriented circle segments* and *close packing of small graded circles*. There are eight orientations of the set of circle segments in Kukeldash, 45° apart. Three adjacent sets form a quadrant of the curvilinear net (Fig. 7). The circles are *purposely* non-concentric, resulting in size-graded hexagons. Looking for alternative descriptions, hyperbolae limited by a rectangular pair of asymptotes are far from the observed shape but an affinely modified conjugate hyperbola drawn in a 'widely open' 135° system of asymptotes, in accordance with Pedoe (1988) and Wikipedia (2017), also fits the observed

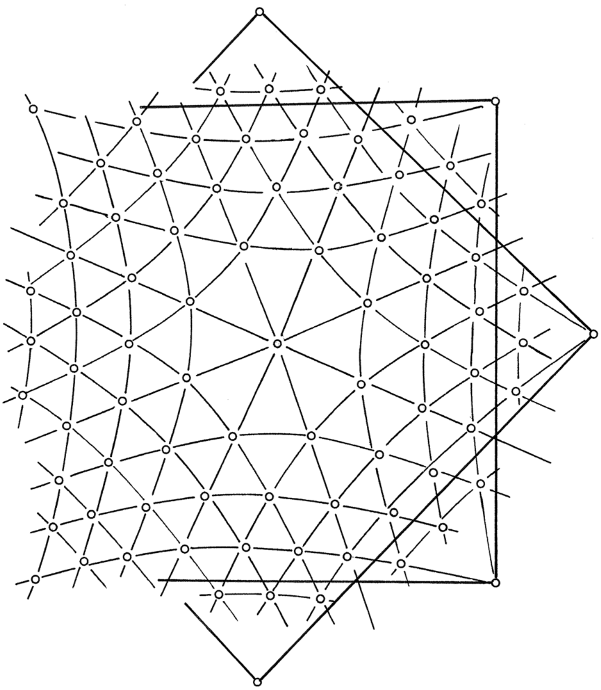

Fig. 7 Curvilinear net based on the marked centers of irregular hexagonal tiles. Small circles reflect true positions of tile centers. Geometry of field boundaries is idealized

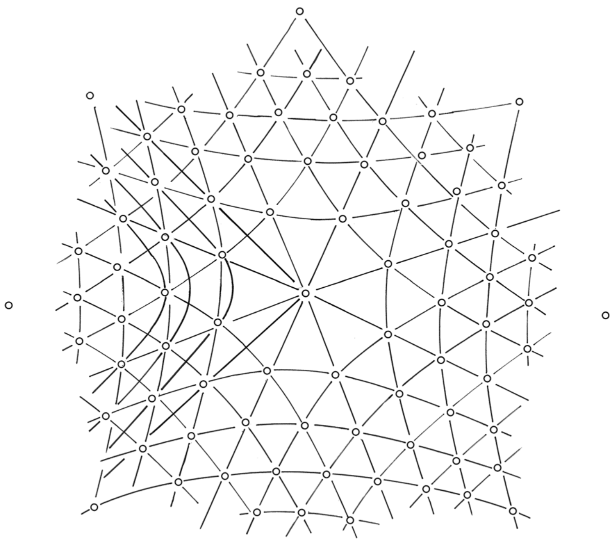

Fig. 8 Secondary hyperbolic curves superposed on the curvilinear net

Kukeldash curve. The net of intersecting curves allows a system of secondary hyperbolae (Fig. 8) running through every second intersection. The least expectable type of curves, a four-branch hypocycloid (Bronshtein 1973; Brette 1976) gives a good fit as well.

Small circles with a different radius for each curve satisfactorily fill the Kukeldash curves when radius and exact position are selected striving for six contacts for each circle. An array of circles in essentially ubiquitous sixfold-contact fitting configuration, with one eightfold configuration (which extends its symmetry elements over the array) positioned in its center, forces the circle size to decrease towards the edges of the field. It fits the curvilinear Kukeldash pattern, although Fig. 9 shows that the circle centers rarely coincide with the centers of pattern hexagons. Every deviation from regularity leaves a clear imprint in the array of filling circles. The studied array, with the central point which has symmetry higher than the rest of the array, has trend opposite to the circle arrays which have the center as a point of lowest symmetry, as illustrated by Stephenson (2003).

Whichever interpretation is accepted, the irregularities observed in the original mosaic, and illustrated in real terms in Fig. 7, result in permanent uncertainty in the selection of the best description and interpretation for the curved pattern. The concept of non-concentric circular segments with different radii has the advantage of simplicity.

How was this difficult geometry transferred onto the target star-shaped space? Work with a ruler and long string (for segments with large circle radii) must have been hindered by the boundaries and complex surface of the ceiling. Direct projection from a drawing on the floor appears impossible because of the very detailed texture of the pattern, and a prick-transfer from a paper drawing clashes with the complex surface and ribs of the ceiling, besides a problem of large paper

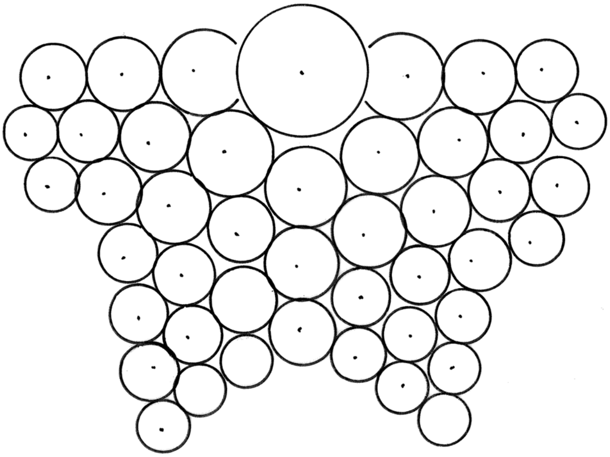

Fig. 9 Array of contact-fitted circles substituting for the hexagonal tiles of the curvilinear pattern from Fig. 3

size. Instead of laboriously drawing the same pattern again and again onto four distinct star-shaped fields, master mosaicist might have used a template consisting of four radially arranged and properly marked thin laths, to which the marginal, star-like periphery-defining laths were constructed. Subsequently, the curvilinear net composed of highly elastic, 'bamboo-like', properly curved rods was fixed to it, in agreement with an underlying drawing prepared on the floor. Perpendicular projection of all intersections in this pattern, performed with a help of an ink-rod, transferred the pattern but left the template undamaged in spite of repeated use. The observed approximations may have arisen from technical problems of the transfer and, especially, from fitting the smaller and smaller discs at the ends of curved lines which were squeezed into the star-like eightfold frame.

Type 3: Lateral Mosaics

The third type of mosaic in the Kukeldash corridors are those composed of colourful tiles without white interspaces, a relatively rare case in historic Islamic art. One exemplar is entirely without white tiles, except for selected petals of eight and tenfold rosettes, and those petals of fivefold stars which are oriented towards the centre of the eightfold-star mosaic (Fig. 4). Another exemplar starts with a central disc of the Type 2 mosaic (above) and converts into the colourful type after the first wreath of irregular sixfold coordinations, the centres of which were picked out as green and red hexagons centering white-pointed stars (Fig. 5). The next ring of sixfold petals are centred by a green star, followed centrifugally by an imperfect red star situated in the lobe of the mosaic limit. Importantly, the space between two adjacent rays of this type contains fivefold red stars (Fig. 4), the surrounding petals of which were altered into irregular fivefold configuration, instead of a sixfold one present in that position in the 'Type 2' mosaic. The presence of fivefold stars results

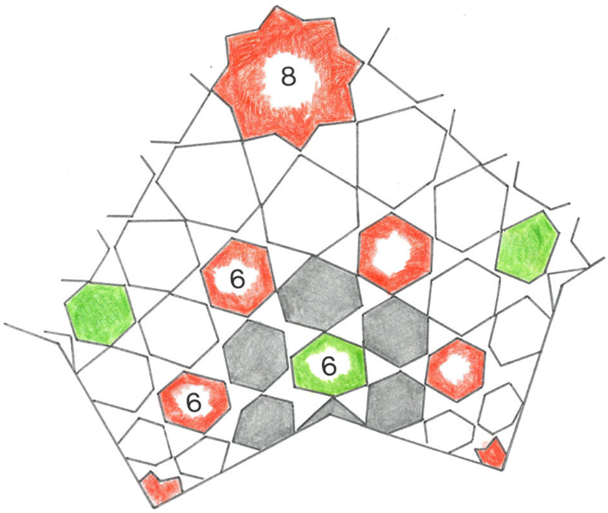

Fig. 10 A portion of the curvilinear pattern with the central eightfold tile (top) and a portion of its 'petal' ring. Selected irregular sixfold tiles in the surrounding pattern are accentuated by colouring and the vertex number in order to contrast them against corresponding configurations in Figs. 11 and 12

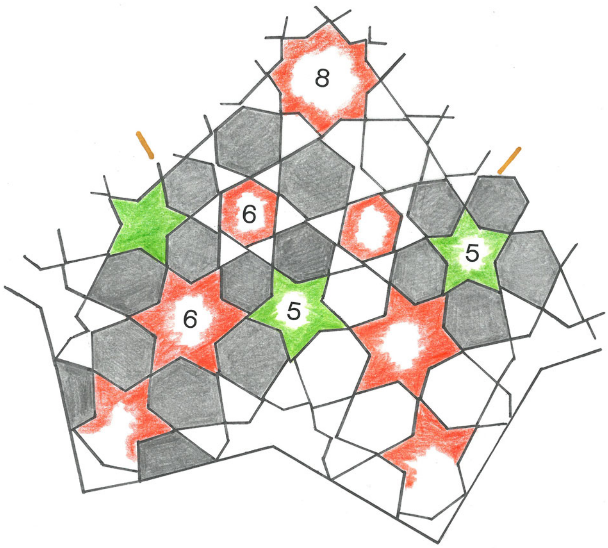

Fig. 11 The lateral mosaic from Fig. 4, with hexagon-and-star combination throughout. In spite of the changes of some tile shapes, the same principle as in Fig. 12 holds for this arrangement

from the uniform size of surrounding tiles, as is shown by contrasting Fig. 10 with Fig. 12.

In both of the just described colourful Type 3 mosaics the miniaturization of tiles towards the edges is reversed on a circular boundary and the size of petals increases

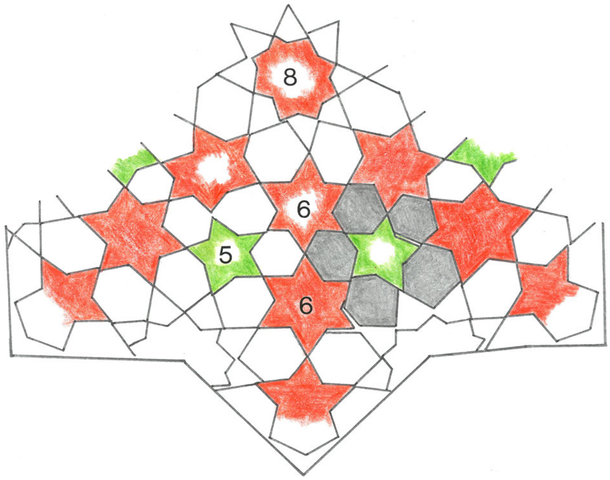

Fig. 12 A portion of the lateral mosaic in Fig. 5 with the central parts composed of hexagons and the outer parts combining stars and hexagons. Comparing it with Fig. 10 we see that reduction of local symmetry from sixfold (red) to fivefold (green) opened the possibility to expand the elements of the outer portion against those in Fig. 10. Conversion of element reduction into expansion starts from the semicircular line which is indicated by brown ticks on both sides of the figure

towards outermost tenfold stars, which are in the recesses of the delimiting line. In the fully colourful Type 3 (Fig. 3), already the innermost sixfold groups are star centred and colourful. The principal change from the Type 2 mosaics happens just beyond the level of centres of second sixfold petal configurations from the centre (see brown markings in Fig. 12): the petal configurations can expand again because in the intervening space the sixfold configuration was altered to a fivefold one, creating a space which was filled by enlarging the surrounding petals. This cannot happen in the exclusively sixfold packing scheme of Type 2 mosaic (compare Figs. 10, 11 and 12).

Conclusion

The Kukeldash mosaics differ from the majority of regional mosaics in technique (free-hand fitted tiles with approximate shapes), geometric character (curvilinearity; curvilinear nets; similarity operations applied to tiles), and material. Freedom of tile shapes, positioning, and tile-fitting into fanciful combinations of local symmetries/multiplicities makes them exceptional in their geographic area. Most outstanding is a hexagon-based mosaic which, except for the eightfold 8*mm* point-group symmetry, is composed of *sixfold packing configurations embedded in a curvilinear grid.* These mosaics were (presumably) created shortly before A.D. 1570 and they show the mannerism of mosaicists of the time, manifested by creating non-linear geometries used in their designs.

Acknowledgements All images are by the author.

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Emil Makovicky was born in Bratislava, Slovakia, in 1940. He obtained his M.Sc and RNDr. degrees at the University of Bratislava and Ph.D. from McGill University, Montreal, Canada (in 1970). After a stay at Yale University he obtained a stable position in mineralogy at the University of Copenhagen, Denmark, in 1972, where he became professor in 1995, now as emeritus. His contributions to symmetry in art started in 1977 and include applications of diverse symmetry groups and quasiperiodicity by traditional artists and artisans