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Natural inflation with and without modulations in type IIB string theory

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Abstract: We propose a mechanism for the natural inflation with and without modulation in the framework of type IIB string theory on toroidal orientifold or orbifold. We explicitly construct the stabilization potential of complex structure, dilaton and Kähler moduli, where one of the imaginary component of complex structure moduli becomes light which is identified as the inflaton. The inflaton potential is generated by the gaugino-condensation term which receives the one-loop threshold corrections determined by the field value of complex structure moduli and the axion decay constant of inflaton is enhanced by the inverse of one-loop factor. We also find the threshold corrections can also induce the modulations to the original scalar potential for the natural inflation. Depending on these modulations, we can predict several sizes of tensor-to-scalar ratio as well as the other cosmological observables reported by WMAP, Planck and/or BICEP2 collaborations.

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1 Introduction

Cosmic inflation is the most successful scenario which not only explains the current cosmological observations but also solves the fine-tuning problems such as the horizon and flatness problems at the same time.

The inflation scenarios are mostly classified according to the size of the tensor-to-scalar ratio which measures the tensor perturbations of the metric in our universe. One is the small-field inflation scenario which gives the tiny tensor-to-scalar ratio due to the flat potential of the scalar field, called inflaton. The other scenario we consider is the large-field inflation model which gives a sizable and measurable tensor-to-scalar ratio. Recent data reported by BICEP2 collaboration [1] can be explained by dust emission [2, 3] reported by the joint analysis of BICEP2, Keck Array and Planck collaborations. In any case, it is interesting to propose the large-field inflation models which would be tested by future cosmological observations.

When we consider the large-field inflation models, we always encounter the problems how to treat the trans-Planckian field values. For example, in the case of natural inflation [4] known as one of the large-field models, we need the corresponding trans-Planckian axion decay constant of the inflaton which is required by recent Planck data [5, 6]. (See ref. [7] and references therein.)

Especially, in the higher-dimensional theory, there are a lot of axions associated with the internal cycles of the internal manifold and then it would be natural to identify such axions as the inflaton. However, it is in general to be problematic that the scale of axion decay constant is severely constrained by the size of internal manifold and the cut-off scale

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of higher-dimensional theory. To overcome such a problem, there are several approaches to realize trans-Planckian axion decay constant by employing Kim-Niles-Peloso alignment mechanism [8] in the case of multiple axions with sub-Planckian axion decay constant, for more details see refs. [9–19]. In the case of single axion, the trans-Planckian axion decay constant can be realized based on the five-dimensional theory [20, 21] with a small five-dimensional gauge coupling and the weakly-coupled heterotic string theory with certain loop-corrections to the gauge coupling [22].

In this paper, we propose the natural inflation scenario in the framework of type IIB string theory on toroidal orientifold or orbifold and the inflaton is identified as the imaginary part of the complex structure moduli, $\mathrm{Im}\, U^2$. The axion decay constant of inflaton is enhanced to the trans-Planckian field value due to the inverse of one-loop factor in the gauge threshold corrections which have a dependence on the complex structure moduli. The sections are organized as follows. We briefly review the gauge threshold corrections caused by the massive open strings between D-branes in the $\mathcal{N} = 2$ sector of type II string theory in section 2. In section 3, we show the moduli stabilization procedure step by step and identify the lightest mode ($\mathrm{Im}\, U^2$) as the inflaton. First, some linear combinations of dilaton and the complex structure moduli expect for the inflaton sector U^2 can be stabilized by three-form fluxes at the perturbative level. Second, we consider the remaining orthogonal linear combination of dilaton and complex structure moduli and Kähler modulus stabilization by such non-perturbative effects as those employed in the racetrack scenario [23–26] in sections 3.1, 3.2 and as that adopted in the Kachru-Kallosh-Linde-Trivedi (KKLT) scenario [27] in section 3.2. Then the real part of complex structure moduli $\mathrm{Re}\, U^2$ can be also stabilized due to those nonvanishing superpotential terms at the same time. Finally, we extract the effective inflaton potential which is in a type of natural inflation with the trans-Planckian axion decay constant by identifying $\mathrm{Im}\, U^2$ as the inflaton in the large complex structure moduli limit, $\mathrm{Re}\, U^2 > 1$ in section 3.3. On the other hand, in the case of $\mathrm{Re}\, U^2 \simeq 1$, we find the modulations to the original scalar potential for the natural inflation to be discussed in section 3.4. Section 4 is devoted to the conclusion. We show the mass-squared matrices of moduli in appendix A.

2 Moduli-dependent threshold corrections in type II string theory

We briefly review the one-loop stringy threshold corrections to the gauge couplings on D-branes in the framework of type II string theory. (For more details, see refs. [28, 29], and references therein.) The running gauge coupling for scale μ below the string scale M_s is written by

$$\frac{1}{g_a^2(\mu)} = \frac{1}{g_a^2} + b_a \ln\left(\frac{M_s^2}{\mu^2}\right) + \frac{\Delta_a}{16\pi^2}, \tag{2.1}$$

where g_a is the 4D gauge coupling at the string scale M_s, b_a is the beta-function coefficient of the gauge group G_a and Δ_a represents the correction from stringy massive modes at the one-loop level. In type II string theory, in general, the charged open strings between two stacks of D-branes or O-planes contribute to the gauge couplings on D-branes as the threshold corrections Δ_a which are mostly moduli-dependent [30].

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In the case of type IIA string theory on toroidal orientifold or orbifold with O-planes and D6-branes wrapping on a supersymmetric three-cycle (special Lagrangian submanifold) of the internal tori, the gauge threshold corrections are explicitly computed by an exact CFT method via the cylinder and Möbius diagram [28]. (There are similar computations in type IIB string theory and F-theory on the local geometric cycle with fractional D-branes [31, 32].) When we consider the T-dual picture, they correspond to the setup of D3/D7-branes or D5/D9-branes in type IIB orientifold or orbifold which depend on the choice of T-duality. For $\mathcal{N} = 2$ SUSY sector in type IIB string with D3/D7-branes and O3/O7-planes, (which correspond to the stacks of parallel D6- and D6′-branes or O-planes in type IIA string theory), one-loop gauge threshold corrections for the gauge theory living on D7-branes with the gauge group G_a and the wrapping numbers (p_a^k, q_a^k) on three two-tori labeled by $k = 1, 2, 3$ are expressed as

$$\Delta_a = -\sum_c b_{ac}^{N=2} \left[\ln |\eta(i\,U^k)|^4 + \ln \left(\mathrm{Re}\, U^k \frac{|p_a^k + i\, q_a^k \mathrm{Re}\, T^k|^2}{\mathrm{Re}\, T^k} \right) - \kappa \right], \tag{2.2}$$

where T^k and U^k are Kähler and complex structure moduli, respectively, κ is the IR regularization constant and η is the Dedekind eta-function. The beta-function coefficients $b_{ac}^{N=2}$ represent contributions from the charged massive modes in open strings stretched between the a-stack of D7-branes and the other c-stack of D-branes, and the summation over c implicitly extracts the all contributions from the other stacks of D-branes. Note that the imaginary part of T^k is given by the Neveu-Schwarz field.

As pointed out in ref. [33], only holomorphic threshold corrections contribute to the gauge kinetic function on D7-branes, which is extracted from the first term on the right-handed side of eq. (2.2),

$$f_a^{1-\mathrm{loop}} = -\frac{1}{4\pi^2} \sum_c b_{ac}^{N=2} \ln \left(\eta(i\,U^k) \right). \tag{2.3}$$

Especially, in the large complex moduli limit, the logarithmic factor in eq. (2.3) behaves as

$$\ln \eta(i\,U^k) \to -\frac{\pi}{12} U^k, \tag{2.4}$$

due to the asymptotic form of the Dedekind eta-function. In this limit, the gauge kinetic function on D7-branes receives the following threshold correction,

$$f_a \simeq \sum_i \frac{T^i}{4\pi} + \sum_j \frac{b^j}{48\pi} U^j, \tag{2.5}$$

where the summations of Kähler and complex structure moduli are only performed over the cycle wrapped by the D7-branes and b^j represents the contribution from the massive open-string modes. Here we consider the D7-branes , otherwise the dilaton dependence also appears in the gauge kinetic function depending on the two-form fluxes, because such fluxes are irrelevant in our scenario of moduli stabilization and inflation. The case with a more general form of gauge kinetic function in terms of the Dedekind eta-function in eq. (2.2) are discussed later.

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In the following, we propose the moduli stabilization and inflation scenario in the framework of type IIB string theory on toroidal orientifold or orbifold such as T^2/Z_2 or $T^2/(Z_2 \times Z_2)$ with D-branes.

3 Natural inflation in type IIB string theory on toroidal orientifold or orbifold

In this section, we propose the natural inflation in the framework of type IIB string theory on toroidal orientifold or orbifold such as T^2/Z_2 or $T^2/(Z_2 \times Z_2)$ with D-branes. The inflaton is considered as the axion paired with one of the complex structure moduli into $\mathcal{N} = 1$ SUSY multiplet and the axion decay constant is enhanced to trans-Planckian value due to the inverse of a loop-factor accompanying the one-loop corrections to the gauge kinetic function which makes it possible to realize a successful natural inflation, as shown later.

As pointed out in ref. [34], in the type IIB string theory (unlike the heterotic string theory) on Calabi-Yau (CY) three-fold, three-form fluxes induce the superpotential $W_{\rm flux}$ which depends on the dilaton S and complex structure moduli U^k as

$$W_{\rm flux} = \int_{\rm CY} G_3 \wedge \Omega, \tag{3.1}$$

where Ω is the holomorphic three-form of the CY manifold and $G_3 = F_3 - i\,S H_3$ is the three-form flux determined by Ramond-Ramond (RR) three-form flux F_3 and Neveu-Schwarz (NS) three-form flux H_3. Such flux-induced superpotential can stabilize the dilaton and all complex structure moduli at the perturbative level [34].

In order to show the essential idea of our scenario, as mentioned above, we consider the type IIB string on the simple toroidal orientifold or orbifold such as T^2/Z_2 or $T^2/(Z_2 \times Z_2)$ whose moduli are characterized by dilaton S, three complex structure moduli U^1, U^2, U^3 and single overall Kähler modulus T.[1] In order to obtain the desired inflation potential, we follow a similar step to the KKLT scenario [27] for stabilizing all the moduli other than $\mathrm{Im}\, U^2$ which is identified as the inflaton field.

3.1 Moduli stabilization with three-form fluxes

First, let us focus on the stabilization of the dilaton and complex structure moduli by employing the three-form flux. We consider the following Kähler potential and superpotential of S, U^1, U^2 and U^3 in the framework of 4D $\mathcal{N} = 1$ supergravity,

$$\begin{aligned} K =& -\ln(S+\bar{S}) - \sum_{i=1}^{3} \ln(U^i + \bar{U}^i), \\ W_{\rm flux} =& w_1 + i w_2\,(U^1 - U^2) + i w_3\, U^3 + i w_4\, S + w_5 U^3\,(U^1 - U^2) + w_6 S\, U^3 + w_7 S\,(U^1 - U^2) \\ &+ i w_8 S U^3 (U^1 - U^2), \end{aligned} \tag{3.2}$$

[1] It is straightforward to extend our stabilization mechanism to the case with three Kähler moduli.

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in the Planck unit, $M_{\rm Pl} = 1$, where all the dimensionful quantities are measured by the reduced Plank mass[2] $M_{\rm Pl} = 2.4 \times 10^{18}$GeV and then the coefficients w_m $(m = 1, 2, \ldots, 8)$ are integers determined by the RR- and NS-fluxes.[3]

To brighten the outlook for analyzing the stabilization of dilaton S and complex structure moduli U^1, U^2 and U^3, we redefine one of the latter as

$$U^4 = U^1 - U^2. \tag{3.3}$$

In the field base S, U^2, U^3 and U^4, the superpotential and Kähler potential are rewritten as

$$\begin{aligned} K &= -\ln(S+\bar{S}) - \ln(U^2+\bar{U}^2) - \ln(U^3+\bar{U}^3) - \ln(U^4+\bar{U}^4+U^2+\bar{U}^2), \\ W_{\rm flux} &= w_1 + iw_2\, U^4 + iw_3\, U^3 + iw_4\, S + w_5 U^3\, U^4 + w_6 S\, U^3 + w_7 S\, U^4 + iw_8 S U^3 U^4. \end{aligned} \tag{3.4}$$

Then the vacuum expectation values of dilaton and complex structure moduli are determined by the supersymmetric condition,

$$D_I W = 0, \tag{3.5}$$

where $D_I W = W_I + K_I W$, with $W_I = \partial W/\partial \Phi^I$ and $K_I = \partial K/\partial \Phi^I$, is the covariant derivative with respect to the moduli fields Φ^I, $\Phi^I = S, U^2, U^3$ and U^4.

The above stabilization condition (3.5) can be satisfied by

$$W_S = W_{U^3} = W_{U^4} = W = 0. \tag{3.6}$$

For simplicity and concreteness, we further restrict the RR- and NS-fluxes to those satisfying

$$w_1 = w_2\, w_6, \;\; w_3 = -w_5\, w_6, \;\; w_4 = -w_6\, w_7, \;\; w_8 = 1, \tag{3.7}$$

with which the expectation value of S, U^3 and U^4 are given by

$$\mathrm{Re}\, U^3 \mathrm{Re}\, S = -(w_2 + w_5\, w_7), \;\; \mathrm{Re} U^4 = 0, \;\; \mathrm{Im}\, U^3 = w_7, \;\; \mathrm{Im}\, U^4 = w_6, \;\; \mathrm{Im}\, S = w_5, \tag{3.8}$$

at the minimum given by eq. (3.6). Thus U^4 and the linear combination of S and U^3 can be stabilized at the supersymmetric Minkowski minimum and their mass-squared matrices are found as

$$\begin{aligned} m_S^2 &= \\ &= \begin{pmatrix} K^{U^3\bar{U}^3}|W_{U^3U^4}|^2 + K^{S\bar{S}}|W_{SU^4}|^2 & 0 & 0 \\ 0 & K^{U^4\bar{U}^4}|W_{U^3U^4}|^2 & K^{U^4\bar{U}^4} W_{U^3U^4} \bar{W}_{\bar{S}\bar{U}^4} \\ 0 & K^{U^4\bar{U}^4} \bar{W}_{\bar{U}^3\bar{U}^4} W_{SU^4} & K^{U^4\bar{U}^4}|W_{SU^4}|^2 \end{pmatrix}, \end{aligned} \tag{3.9}$$

in the field basis (U^4, U^3, S), which has rank 2 with some appropriate choices of the integers w_m.[4] Note that since there is a Kähler mixing between U^2 and U^4 as can be seen from

[2] Here and hereafter, we adopt the Planck unit.

[3] We choose the certain ansatz of three-form flux that yields the superpotential terms in eq. (3.2) through eq. (3.1) in order to realize the moduli inflation as discussed later.

[4] In ref. [35], it is shown that when one leaves one modulus massless at the supersymmetry breaking minimum, there appears another massless moduli.

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eq. (3.4), we have to canonically normalize them as summarized in the appendix A. The Kähler mixing between U^2 and U^4 are the essential ingredients to realize the supersymmetric Minkowski minimum in the physical domain of the moduli space.

We remark that, with the above choice of RR- and NS-fluxes, the tadpole cancellation condition may not occur among themselves, however, our moduli stabilization and natural inflation scenario would not depend on the detail structure of tadpole condition.

3.2 Light moduli stabilization with non-perturbative effects

Next, we discuss the stabilization of remnant complex structure modulus U^2, the linear combination of S and U^3, Kähler modulus T below the mass scale of the stabilized complex structure moduli U^4. As mentioned above, for simplicity, we focus on the case with a single overall Kähler modulus T and then its Kähler potential is expressed as

$$K = -3\ln(T+\bar{T}), \tag{3.10}$$

in the large volume limit. As a source of stabilizing the Kähler modulus T, dilaton S and $\mathrm{Re}\,U^2$, we assume the non-perturbative effects such as the gaugino condensation on D7-branes and D3-brane,

$$W_{\rm non} = A(U)e^{-\frac{8\pi^2 f_1}{N_1}} - B(U)\,e^{-\frac{8\pi^2 f_2}{N_2}} + C(U)e^{-\frac{8\pi^2 f_3}{N_3}} - D(U)\,e^{-\frac{8\pi^2 f_4}{N_4}}, \tag{3.11}$$

where f_1 and f_2 denote the gauge kinetic functions of pure $\mathrm{SU}(N_1)\times\mathrm{SU}(N_2)$ gauge theories on D7-branes,

$$f_1 = f_2 = \frac{T}{4\pi} + \frac{b^2U^2}{48\pi}, \tag{3.12}$$

where we assume that both of them receive the same threshold corrections depending on the complex structure modulus U^2 determined by the size of b^2 given by eq. (2.5). f_3 and f_4 denote the gauge kinetic functions of pure $\mathrm{SU}(N_3)\times\mathrm{SU}(N_4)$ gauge theories on D3-branes at the orbifold fixed points,

$$f_3 = f_4 = \frac{S}{4\pi}. \tag{3.13}$$

$A(U)$, $B(U)$, $C(U)$ and $D(U)$ are functions of only the heavy complex structure moduli stabilized by the flux-induced superpotential (3.1). Thus we can treat these functions $A(U)$ and $B(U)$ as constants, neglecting the fluctuations of these heavy moduli around the stabilized value.

In the same way as employed in the section 3.1, we redefine the Kähler modulus as

$$\tilde{T} = T + \frac{b^2}{12}U^2. \tag{3.14}$$

Then the stabilization of the Kähler modulus $\tilde{T}$ and dilaton S can be achieved by two gaugino-condensation terms in the same way as the racetrack scenario [23–26], i.e.,

$$\begin{aligned} D_{\tilde{T}}W_{\rm non} &= (W_{\rm non})_{\tilde{T}} + K_{\tilde{T}}W_{\rm non} = 0, \\ D_S W_{\rm non} &= (W_{\rm non})_S + K_S W_{\rm non} = 0, \end{aligned} \tag{3.15}$$

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which leads to the following value of the Kähler moduli and dilaton at the racetrack minimum,

$$\langle \tilde{T} \rangle \simeq \frac{N_1 N_2}{2\pi(N_2 - N_1)} \ln \frac{N_2\, A}{N_1\, B}, \quad \langle S \rangle \simeq \frac{N_3 N_4}{2\pi(N_4 - N_3)} \ln \frac{N_4\, C}{N_3\, D}, \tag{3.16}$$

where the explicit values of parameters are explored by evaluating the cosmological observables. The above racetrack minimum can be realized due to the following relation, $\langle W_{\rm non} \rangle \ll \langle (W_{\rm non})_{\tilde{T}} \rangle, \langle (W_{\rm non})_S \rangle$ which is satisfied in our parameter regions as shown later. As mentioned in section 3.1, the linear combination of S and U^3 is already stabilized by the flux induced superpotential. Here the remaining orthogonal combination can be stabilized by this racetrack superpotential for S. (The stabilization point of $\mathrm{Im}\, S$ is the same as that given by eq. (3.8) when we choose $w_5 = 0$ in the superpotential (3.4).)

In the following, let us discuss the stabilization of the remnant complex structure modulus U^2. Since the Kähler modulus is stabilized at the minimum $\langle W \rangle \neq 0$, the real part of U^2 is stabilized by the Kähler potential,

$$K = -\ln(U^2 + \bar{U}^2) - \ln(U^4 + \bar{U}^4 + U^2 + \bar{U}^2) - 3 \ln \left(\tilde{T} + \bar{\tilde{T}} - \frac{b^2}{12}(U^2 + \bar{U}^2) \right), \tag{3.17}$$

under the following condition:

$$K_{U^2} = -\frac{1}{U^2 + \bar{U}^2} - \frac{1}{U^4 + \bar{U}^4 + U^2 + \bar{U}^2} + \frac{b^2}{4} \frac{1}{\tilde{T} + \bar{\tilde{T}} - \frac{b^2}{12}(U^2 + \bar{U}^2)} = 0, \tag{3.18}$$

which determines the expectation value of $\mathrm{Re}\, U^2$ as

$$\mathrm{Re}\, U^2 = \frac{24 \langle \mathrm{Re}\, \tilde{T} \rangle}{5 b^2}, \tag{3.19}$$

satisfying the extremal condition $V_{U^2} = \partial V / \partial U^2 = 0$ and we employed $\langle \mathrm{Re}\, U^4 \rangle = 0$. We again remark that U^4 and the linear combination of U^3 and S are fixed at a high-scale by the condition $D_{U^3} W = D_{U^4} W = D_S W = 0$. Therefore, if the gaugino condensation scale is much smaller than the mass scale of U^4 and the linear combination of U^3 and S, their deviations from the minimum given by eq. (3.8) are sufficiently small, and we can replace the heavy moduli U^4, the linear combination of U^3 and S by their expectation values (3.8) in evaluating the stabilization of light moduli $\tilde{T}$, S and $\mathrm{Re}\, U^2$.

To confirm the stabilization of $\mathrm{Re}\, U^2$, we have to check that the rank of the full mass-squared matrices for U^2, U^3, U^4, S and $\tilde{T}$. The explicit form of them and the canonical normalization of all moduli are summarized in appendix A. From the mass matrices shown in eq. (A.4), we find the squared mass of $\mathrm{Re}\, U^2$ is positive, if the mass scales of U^4 and the linear combination of U^3 and S are much heavier than the gaugino condensation scale determined by the superpotential (3.11), that is consistent with the above argument.

In the above analysis, the vacuum energy is negative at the minimum $D_I W = 0$ for $I = U^2, U^3, U^4, S$ and $\tilde{T}$. Therefore we assume the existence of some uplifting sector with which the total scalar potential V is in KKLT-type [27],

$$V = V_F + V_{\rm up}, \tag{3.20}$$

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where V_F is written by the usual $\mathcal{N}=1$ supergravity formula,

$$V_F = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right), \tag{3.21}$$

with $K^{I\bar{J}}$ is the inverse of the Kähler metric $K_{I\bar{J}} = \partial^2 K / \partial\Phi^I \partial\bar{\Phi}^{\bar{J}}$ for $\Phi^I = S, \tilde{T}, U^2, U^3$ and U^4. The uplifting potential $V_{\rm up}$ may come from anti-D3-branes [27] and/or nonvanishing F-terms in some dynamical SUSY breaking sector [36–39], etc. In the next section, we show the inflaton potential by identifying the light axion $\mathrm{Im}\, U^2$ as the inflaton.

Finally, we comment on the stabilization of the Kähler modulus T. In the above analysis, the Kähler modulus is stabilized in the same way as the racetrack scenario [23–26]. In the case of the KKLT scenario [27] which is achieved, instead of eq. (3.11), by a single gaugino-condensation term and a tiny constant value of the nonvanishing flux-induced superpotential, we can also derive the similar inflaton potential with a trans-Planckian axion decay constant as seen in the next section. This is because our inflaton potential does not depend on the stabilization of T. However, in the latter case, we need to tune the RR- and NS-flux to obtain the tiny expectation value of superpotential $\langle W \rangle \simeq 10^{-2}$ in order to realize the large volume limit required to ensure the form of Kähler potential shown in eq. (3.10). When the three-form fluxes are turned on, we may have to consider a more general geometry than CY (locally) warped due to the energies of these fluxes as well as some sources for the tadpole cancellation [34]. Thus we further assume that the possible backreactions from the three-form fluxes are negligible in the relevant sector to our scenario of moduli stabilization and inflation.

So far, we focus on the single overall Kähler modulus T. The other Kähler moduli T_i $i = 1, 2, \cdots$, are also stabilized by the non-perturbative effects such as the gaugino condensation on D7-branes irrelevant to the cycle associated with the modulus T,

$$W = \sum_i A_i(U) e^{-\frac{8\pi^2 f_1^{(i)}}{M_1^{(i)}}} - B_i(U)\, e^{-\frac{8\pi^2 f_2^{(i)}}{M_2^{(i)}}}, \tag{3.22}$$

at the racetrack minimum, where $A_i(U)$, $B_i(U)$ are the functions of only the heavy complex structure moduli by the flux-induced superpotential (3.1). $f_1^{(i)}$ and $f_2^{(i)}$ denote the gauge kinetic functions of $\mathrm{SU}(M_1^{(i)}) \times \mathrm{SU}(M_2^{(i)})$ gauge theories, e.g., $f_1^{(i)} = a_1 T_i$ and $f_2^{(i)} = a_2 T_i$ with a_1, a_2 are constants. It is then assumed these gaugino condenstion scales are much heavier than the those for the modulus T.

3.3 Natural inflation without modulations

Now we are ready to write down the inflaton potential. The effective scalar potential for $\mathrm{Im}\, U^2$ is generated from another $\mathrm{SU}(L)$ gaugino-condensation term,

$$W \supset E(\langle U \rangle) e^{-\frac{2\pi}{L}\langle T \rangle - \frac{b\pi}{6L}\langle \mathrm{Re}\, U^2 \rangle - i\frac{b\pi}{6L}\mathrm{Im}\, U^2}, \tag{3.23}$$

where we assume the gauge coupling on $\mathrm{SU}(L)$ gauge theory receives the threshold corrections which have U^2-dependence. The factor $E(\langle U \rangle)$ denotes possible threshold corrections from the heavy complex structure moduli, U^3 and U^4, as in the previous step. We assume

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again that all the other moduli are strictly fixed at their minimum given by eqs. (3.8) and (3.16) obtaining heavy masses and the fluctuations around their vacuum expectation values are neglected in the effective potential for $\mathrm{Im}\, U^2$. Such a situation can be realized if the rank of the SU(L), SU(N_i) ($i = 1, 2, 3, 4$) gauge theories are chosen as $L < N_i$ with $i = 1, 2, 3, 4$. In this case, $\mathrm{Im}\, U^2$ is lighter enough than all the other moduli those receive much heavier masses from the high-scale gaugino-condensation terms (3.11) and the flux-induced superpotential (3.1), respectively.

After all, the effective scalar potential for $\mathrm{Im}\, U^2$ is generated from V_F in eq. (3.21), which is written as

$$V_{\rm eff} = \Lambda_1 \left(1 - \cos\left(\lambda_1 \phi\right)\right), \tag{3.24}$$

where $\Lambda_1 \simeq 6 e^{\langle K \rangle} \langle W_{\rm non} \rangle E(\langle U \rangle) e^{-\frac{2\pi}{L}\langle T \rangle - \frac{b\pi}{6L}\mathrm{Re}\langle U^2 \rangle}$, $\lambda_1 = b\pi/6d\,L$ and $\phi = d\,\mathrm{Im}\, U^2$ is the canonically normalized axion field. The normalization factor $d \simeq 1/\langle \mathrm{Re}\, U^2 \rangle$ is determined by the canonical normalization of relevant complex structure moduli which is explicitly shown in appendix A. Even though U^2, U^4 and $\tilde{T}$ have a kinetic-mixing from the structure of Kähler potential (3.17) as mentioned in section 3.2, the effects from the mixing between ϕ and $\mathrm{Im}\, U^4$, $\mathrm{Im}\, \tilde{T}$ is negligible on the inflation mechanism discussed in the following. This is because $\mathrm{Im}\, U^4$ and $\mathrm{Im}\, \tilde{T}$ are heavier enough than $\mathrm{Im}\, U^2$ and already decoupled from the inflaton dynamics.

When we identify the inflaton as ϕ, the axion potential is considered as the type of natural inflation. As seen in the scalar potential (3.24), the axion decay constant is enhanced by the inverse of one-loop factor and is determined by the ratio b/L and the vacuum expectation value, $\langle \mathrm{Re}\, U^2 \rangle$. Since the trans-Planckian axion decay constant is required in order to explain the cosmological observations reported by Planck data [6], the ratio b/L and $\langle \mathrm{Re}\, U^2 \rangle$ have to be properly chosen. Note that the beta-function coefficient b in $\mathcal{N} = 2$ sector is not related with the sector of SU(L) gauge theory.

To evaluate the cosmological observables, we define the slow-roll parameters for the inflaton ϕ,

$$\begin{aligned}
\epsilon &= \frac{1}{2}\left(\frac{\partial_\phi V_{\rm eff}}{V_{\rm eff}}\right)^2 = \frac{(\lambda_1)^2}{2}\frac{1-\cos^2(\lambda_1\,\phi)}{(1-\cos(\lambda_1\,\phi))^2}, \\
\eta &= \frac{\partial_\phi \partial_\phi V_{\rm eff}}{V_{\rm eff}} = (\lambda_1)^2 \frac{\cos(\lambda_1\,\phi)}{1-\cos(\lambda_1\,\phi)}, \\
\xi &= \frac{\partial_\phi V_{\rm eff}\partial_\phi\partial_\phi\partial_\phi V_{\rm eff}}{V_{\rm eff}^2} = -(\lambda_1)^4 \frac{1-\cos^2(\lambda_1\,\phi)}{(1-\cos(\lambda_1\,\phi))^2},
\end{aligned} \tag{3.25}$$

and then the cosmological observables such as the power spectrum of the scalar density perturbation P_ζ, the spectral index n_s, its running $dn_s/d\ln k$ and the tensor-to-scalar ratio r are written as

$$P_\zeta = \frac{V}{24\pi^2\,\epsilon}, \quad n_s = 1 + 2\eta - 6\epsilon, \quad r = 16\epsilon, \quad \frac{d\,n_s}{d\,\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi, \tag{3.26}$$

by employing the slow-roll approximation at the leading order. The e-folding number is also evaluated as

$$N_e = \int_{\phi_{\rm end}}^{\phi} \frac{V_{\rm eff}}{\partial_\phi V_{\rm eff}} d\phi, \tag{3.27}$$

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where $\phi_{\rm end}$ denotes the field value at the end of inflation with which the slow-roll condition is violated, $\max\{\epsilon, \eta, \xi\} = 1$.

In order to explain the power spectrum of the scalar density perturbation, $P_\zeta \simeq 2.2 \times 10^{-9}$ reported by Planck [6], we set the parameters in the superpotential given by eqs. (3.4), (3.16), (3.23) and the Kähler potential given by eq. (3.17),

$$w_5 = 0,\ \ w_6 = 1,\ \ w_2 = -8,\ \ w_7 = -3,\ \ N_1 = N_3 = 12,\ \ N_2 = N_4 = 20,\ \ L = 10,\ \ b = 1,$$
$$b^2 = 12,\ \ A = -8,\ \ B = -3,\ \ C = 9,\ \ D = 3,\ \ E = \frac{1}{12}, \tag{3.28}$$

and the other parameters in eq. (3.4) are fixed such that the conditions given by eq. (3.7) are satisfied, those lead to the vacuum expectation values of moduli,

$$\langle {\rm Re}\, U^1 \rangle \simeq \langle {\rm Re}\, U^2 \rangle \simeq 2.8,\ \ \langle {\rm Re}\, U^3 \rangle \simeq 1,\ \ \langle {\rm Im}\, U^1 \rangle \simeq 1,\ \ \langle {\rm Im}\, U^2 \rangle \simeq 0,\ \ \langle {\rm Im}\, U^3 \rangle \simeq -3,$$
$$\langle {\rm Re}\, S \rangle \simeq 7.7,\ \ \langle {\rm Im}\, S \rangle \simeq 0,\ \ \langle \hat{T} \rangle \simeq 7.1. \tag{3.29}$$

With the above set of parameters, the above cosmological observables and the e-folding number are evaluated as

$$n_s \simeq 0.963,\ \ r \simeq 0.06,\ \ dn_s/d\ln k \simeq -8 \times 10^{-4},\ \ N_e \simeq 61, \tag{3.30}$$

which are consistent with WMAP, Planck data [6],

$$n_s = 0.9655 \pm 0.0062, \tag{3.31}$$

at the pivot scale $k_* = 0.05{\rm Mpc}^{-1}$ and the upper limit of r [6],

$$r < 0.11, \tag{3.32}$$

when we properly choose the initial condition for inflaton ϕ.

We remark that, in our model, the axion decay constant is enhanced by the inverse of loop factor through the stringy threshold corrections which are characterized by the Dedekind eta-function and the beta-function coefficients b induced by the massive open-string modes between D-branes in eq. (3.12). Thus, we can realize several values of axion decay constant depending on the brane configurations, which means that the tensor-to-scalar ratio can be of $\mathcal{O}(0.01 - 0.1)$ in our framework.

3.4 Natural inflation with modulations

The previous section shows the usual natural inflation with trans-Planckian decay constant, which is valid only when the Dedekind eta-function can be approximated by the leading term as shown in eq. (2.4) in the large field limit of complex structure moduli.

In this section, we estimate the deviations from the large complex-structure limit, by introducing the next leading term in the Dedekind function,

$$\eta(i\,U^2) \to e^{-\frac{\pi}{12}U^2}\Big[1 - e^{-2\pi U^2} - \mathcal{O}(e^{-4\pi U^2})\Big], \tag{3.33}$$

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which induces the following correction to the inflaton potential given by eq. (3.24),

$$V_{\rm inf} = V_{\rm eff} + V_{\rm mod}, \tag{3.34}$$

where

$$V_{\rm mod} = \Lambda_2 \cos\left(\lambda_2 \phi\right), \tag{3.35}$$

with $\Lambda_2 = \Lambda_1 \frac{2b}{L} e^{-\left(2\pi + \frac{b\,\pi}{6L}\right)\langle {\rm Re}\, U^2\rangle}$, $\lambda_2 = (2\pi + b\,\pi/6L)/d$. Note that the correction $V_{\rm mod}$ would in general yield the modulations [40–43] to the leading inflaton potential $V_{\rm eff}$ in the case of $\langle {\rm Re}\, U^2\rangle \simeq 1$, though it is not the case in the above analysis with the numerical values of parameters (3.28) resulting eqs. (3.29)–(3.32).

In the following analysis, we also take care of the vacuum expectation value of ${\rm Re}\, U^2$ and the ratio b/L in order to avoid the tachyonic scalar potential around the origin, $\phi = 0$. Actually we can avoid the nonvanishing field value of the axion ϕ at the minimum, which would lead to the strong CP problem if it couples to the QCD sector, that is, the physical $\bar{\theta}$ term is severely constrained by the non-observation of electric dipole moment of the neutron [44, 45]. The axion mass squared at the origin is described by

$$\partial_\phi^2 V_{\rm inf}\big|_{\phi=0} = (\lambda_1)^2 \Lambda_1 - (\lambda_2)^2 \Lambda_2, \tag{3.36}$$

and its positivity is ensured by the following condition:

$$(\lambda_1)^2 \Lambda_1 - (\lambda_2)^2 \Lambda_2 > 0 \;\leftrightarrow\; \left(\frac{\pi}{6}\right)^2 \frac{b}{L} > 2\left(2\pi + \frac{\pi\, b}{6L}\right)^2 e^{-2\pi\langle {\rm Re}\, U^2\rangle}. \tag{3.37}$$

For general cases, it is interesting to discuss the contributions from the additional scalar potential $V_{\rm mod}$ to the inflaton dynamics. By the inclusion of $V_{\rm mod}$, the slow-roll parameters of the inflaton potential $V_{\rm inf}$ for the inflaton ϕ are written as

$$\begin{aligned}
\epsilon &= \frac{\left(\lambda_1 \Lambda_1 \sin\left(\lambda_1\,\phi\right) - \lambda_2 \Lambda_2 \sin\left(\lambda_2\,\phi\right)\right)^2}{2\, V_{\rm inf}^2}, \\
\eta &= \frac{(\lambda_1)^2 \Lambda_1 \cos\left(\lambda_1\,\phi\right) - (\lambda_2)^2 \Lambda_2 \cos\left(\lambda_2\,\phi\right)}{V_{\rm inf}}, \\
\xi^2 &= -\frac{\lambda_1 \Lambda_1 \sin\left(\lambda_1\,\phi\right) - \lambda_2 \Lambda_2 \sin\left(\lambda_2\,\phi\right)}{V_{\rm inf}} \times \frac{(\lambda_1)^3 \Lambda_1 \sin\left(\lambda_1\,\phi\right) - (\lambda_2)^3 \Lambda_2 \sin\left(\lambda_2\,\phi\right)}{V_{\rm inf}},
\end{aligned} \tag{3.38}$$

while the spectral index n_s including the higher-order corrections is found as

$$n_s = 1 + 2\eta - 6\epsilon + 2\left[-\left(\frac{5}{3} + 12C\right)\epsilon^2 + (8C-1)\epsilon\,\eta + \frac{1}{3}\eta^2 - \left(C - \frac{1}{3}\right)\xi^2\right] + \cdots, \tag{3.39}$$

where $C = -2 + \ln 2 + \gamma$ with $\gamma \simeq 0.577$ is the Euler-Mascheroni constant and the ellipsis stands for more higher corrections which are given by the fourth derivative with respect to the inflaton. (See ref. [46] and references therein.) As discussed later, in models which have $\xi^2 = \mathcal{O}(0.01)$, the higher-order terms contribute to the numerical value of n_s in eq. (3.39), while the higher-order corrections to P_ζ do not give sizable effects. Note that our inflaton effective potential is controlled by $\langle {\rm Re}\, U^2\rangle$ and b/L in the superpotential given by eq. (3.4). In the following analysis, we assume certain numerical values of parameters different from

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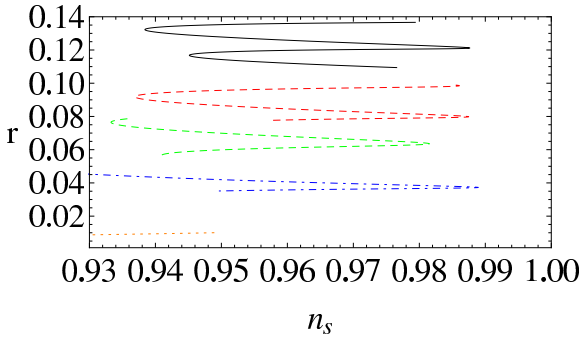

Figure 1. Predictions of (n_s, r) in the range of e-folding number, $50 \leq N_e \leq 60$. For the universal value of $\langle \mathrm{Re}\, U^2 \rangle = 1$, black-solid, red-dashed, green-dashed, blue-dotdashed and orange-dotted lines correspond to the fixed ratios $b/L = 1/10, 1/5, 1/4, 1/3, 1/2$, respectively.

eq. (3.28), those realize the particular value $\langle \mathrm{Re}\, U^2 \rangle \simeq 1$, with which $V_{\rm mod}$ does affect the inflaton dynamics.

Then we numerically evaluate the cosmological observables r, n_s, $dn_s/d\ln k$ by putting several values of b/L. The scalar density perturbation P_ζ can be obtained as 2.2×10^{-9} also in this case by suitably choosing the gaugino-condensation terms in eq. (3.16) which stabilize the Kähler modulus at the racetrack minimum. Figure 1 shows the prediction of the spectral index n_s and the tensor-to-scalar ratio r in the range of e-folding number, $50 \leq N_e \leq 60$. Several oscillating curves are drawn by varying N_e with the corresponding fixed values of the ratio b/L in figure 1. This is because the slow-roll parameters oscillate due to the inclusion of the deviations from the large complex-structure limit as can be seen in figure 2 which shows the behavior of the slow-roll parameters by setting $b/L = 1/5$ $(1/10)$ and $\langle \mathrm{Re}\, U^2 \rangle = 1.2$ (2.4) in the left (right) panel. Although, in the both left and right panels in figure 2, the leading scalar potential $V_{\rm eff}$ has the same structure, the next-leading scalar potential $V_{\rm mod}$ gives sizable corrections in the left rather than the right panel. The scalar potential with and without such modulations is shown in the figure 3. As mentioned above, the detectability of such modulations is governed by the expectation value of $\mathrm{Re}\, U^2$ and then the next-to-next leading scalar potential which comes from the expansion of the Dedekind functions would be important in the case of $\langle \mathrm{Re}\, U^2 \rangle < 1$. We summarize our predictions for the cosmological observables in table 1.

Our results suggest that we can realize several values of the tensor-to-scalar ratio and spectral index independently to each other when we consider the particular value of complex-structure modulus, $\langle \mathrm{Re}\, U^2 \rangle \simeq 1$. This nature is different from the original natural inflation model [4] and is also seen in the multi-natural inflation scenario [10, 11]. However, up to now, we do not know which amount of gravitational waves are observed reported by BICEP2 collaborations [1]. We expect that future cosmological observations select more precisely certain values of cosmological observables.

In summary, in our framework of type IIB string theory on toroidal orientifold or orbifold, the deviation from the natural inflation depends on the expectation value of the real part of complex structure modulus, $\langle \mathrm{Re}\, U^2 \rangle$. In the large field limit of complex structure

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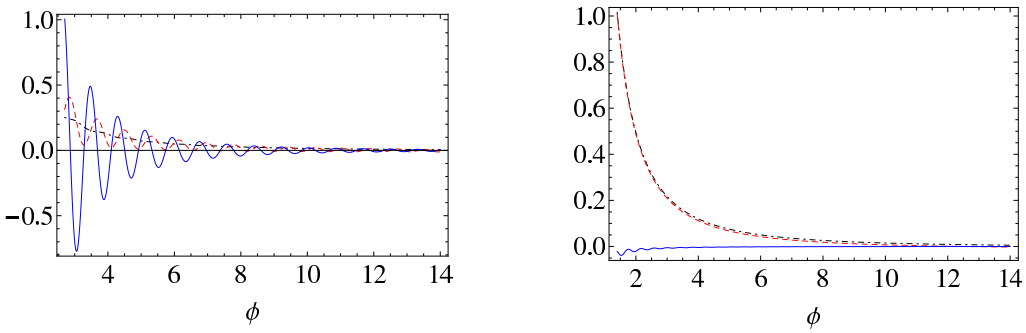

Figure 2. The behavior of the slow-roll parameters, ϵ, η and ξ^2, which correspond to black-dotdashed, red-dashed and blue-solid curves, respectively. In the left (right) panel, we set $b/L = 1/5$ $(1/10)$ and $\langle \mathrm{Re}\, U^2 \rangle = 1.2$ (2.4).

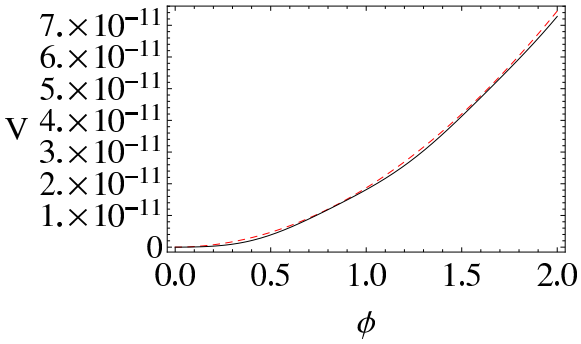

Figure 3. The scalar potential V versus the inflaton value ϕ. Along with figure 2, the black-solid curve corresponds to the scalar potential (3.34) with modulations for the parameter $b/L = 1/5$ and $\langle \mathrm{Re}\, U^2 \rangle = 1.2$. On the other hand, the red-dotted curve corresponds to the leading scalar potential (3.24) without modulations for the same parameters.

b/L	$\langle \mathrm{Re}\, U^2 \rangle$	N_e	n_s	r	$dn_s/d\ln k$
1/10	1.3	50	0.96	0.14	−0.0008
1/10	1.3	57	0.96	0.12	−0.012
1/5	1.2	55	0.96	0.08	−0.002
1/5	1.2	60	0.96	0.08	−0.001
1/4	1.2	53	0.96	0.07	−0.002
1/4	1.2	58	0.96	0.06	−0.001
1/3	1.1	54	0.96	0.04	−0.002
1/3	1.1	60	0.96	0.04	−0.001
1/2	1.1	50	0.95	0.01	−0.0003

Table 1. The input values of b/L, $\mathrm{Re}\, U^2$ and the output values of the e-folding number N_e, spectral index n_s, tensor-to-scalar ratio r and the running of spectral index $dn_s/d\ln k$.

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moduli, our inflaton potential is considered as the original natural inflation scenario [4]. When we construct the standard model sector on Dp-branes ($p > 3$), the matter fields in the standard model generically couple to the complex structure moduli. Such couplings affect the inflaton dynamics after the end of inflation which are related to the reheating processes. Thus, it is interesting to study toward such a direction in a future work.

4 Conclusion

We proposed a mechanism for the natural inflation with and without modulations in the framework of type IIB string theory on toroidal orientifold or orbifold. The essential ingredient to obtain the trans-Planckian decay constant which is required in the natural inflation is the holomorphic gauge threshold corrections to the gauge kinetic function. Such threshold corrections are exactly computed in type II string theory on toroidal orientifold or orbifold by employing the CFT method (see refs. [28, 29], and references therein) which suggests the gauge threshold corrections have moduli dependences. Note that when one of the moduli is identified as the inflaton, the moduli-dependent threshold corrections are important not only to discuss about the gauge coupling unification, but also to enhance the axion decay constant of the inflaton by the inverse of one-loop factor accompanying the correction.

In our model, the inflaton is considered as $\mathrm{Im}\, U^2$ which is the imaginary part of the complex structure modulus and the inflaton potential is extracted from the gaugino-condensation term whose gauge coupling receives the complex structure moduli-dependent terms characterized by the Dedekind function. We presented that in the large complex-structure limit, $\langle \mathrm{Re}\, U^2 \rangle > 1$, the Dedekind function is approximated as the single exponential term and then the inflaton potential is close to that of the natural inflation which is consistent with cosmological observations such as WMAP, Planck [5, 6] and the joint analysis of BICEP2, Keck Array and Planck [3]. On the other hand, in the regime with $\langle \mathrm{Re}\, U^2 \rangle \simeq 1$, we have to take account of the explicit Dedekind function, which leads to the modulations to the original natural inflation [4]. The modulations give a sizable modification to the predictions [40–43] of the original natural inflation in the same way as the multi-natural inflation scenario [10, 11]. The natural inflation with modulations predicts the different predictions unlike the original natural inflation without modulations. In fact, we can achieve the small and large tensor-to-scalar ratio without changing the value of spectral index so much. Thus such natural inflation with modulations can be tested in the near future experiments.

In both inflation scenarios, we stabilize the complex structure moduli except for the inflaton sector by employing three-form fluxes in the usual manner. The dilaton S, overall Kähler modulus T and $\mathrm{Re}\, U^2$ are stabilized at the racetrack (KKLT) minimum by double (single) gaugino-condensation terms (term) above the inflation scale. In general, although it seems to be difficult to obtain the mass difference between $\mathrm{Re}\, U^2$ and $\mathrm{Im}\, U^2$, it can be achieved by the Kähler mixing between U^2 and the other moduli in our model.

We have not discussed the reheating process. When we construct the standard model sector on Dp-branes ($p > 3$), the matter fields in the standard model generically couple to the complex structure modulus (inflaton). Since such couplings affect the inflaton dynamics after the end of inflation, it is interesting to study in such a direction for the future work.

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It is also interesting to extend our set-up to more general Calabi-Yau manifold. However, the one-loop threshold corrections are unknown on D-branes which wrap the internal cycles of Calabi-Yau manifold. Thus it is beyond our scope.

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A The canonical normalization and mass-squared matrices

In this appendix, we show the canonical normalization and the mass-squared matrices of all moduli given by the Kähler potential (3.4), (3.10) and the superpotential (3.4), (3.11).

The Kähler metric generated by the Kähler potential (3.4), (3.10) is given by

$$K_{I\bar{J}} = \begin{pmatrix} K_{U^4\bar{U}^4} & K_{U^4\bar{U}^2} & 0 & 0 & 0 \\ K_{U^2\bar{U}^4} & K_{U^2\bar{U}^2} & K_{U^2\bar{\tilde{T}}} & 0 & 0 \\ 0 & K_{\tilde{T}\bar{U}^2} & K_{\tilde{T}\bar{\tilde{T}}} & 0 & 0 \\ 0 & 0 & 0 & K_{U^3\bar{U}^3} & 0 \\ 0 & 0 & 0 & 0 & K_{S\bar{S}} \end{pmatrix}, \tag{A.1}$$

where

$$\begin{aligned} K_{U^2\bar{U}^2} &= \frac{1}{(U^2+\bar{U}^2)^2} + \frac{1}{(U^4+\bar{U}^4+U^2+\bar{U}^2)^2} + \frac{3c_2^2}{(\tilde{T}+\bar{\tilde{T}}-c_2(U^2+\bar{U}^2))^2} = \frac{10}{3}\frac{1}{(U^2+\bar{U}^2)^2}, \\ K_{U^2\bar{U}^4} &= K_{U^4\bar{U}^2} = K_{U^4\bar{U}^4} = \frac{1}{(U^2+\bar{U}^2)^2}, \quad K_{U^2\bar{\tilde{T}}} = K_{\tilde{T}\bar{U}^2} = -\frac{4}{3c_2}\frac{1}{(U^2+\bar{U}^2)^2}, \\ K_{U^3\bar{U}^3} &= \frac{1}{(U^3+\bar{U}^3)^2}, \quad K_{S\bar{S}} = \frac{1}{(S+\bar{S})^2}, \quad K_{T\bar{T}} = \frac{3}{(\tilde{T}+\bar{\tilde{T}}-c_2(U^2+\bar{U}^2))^2} = \frac{4}{3c_2^2}\frac{1}{(U^2+\bar{U}^2)^2}, \end{aligned} \tag{A.2}$$

where $c_2 = b^2/12$. Here we employ the stabilization condition given by eq. (3.18) as discussed in section 3.2. Then the eigenvalues $(K_{\rm eig})_I$, and the matrix $U_{I\bar{J}}$ diagonalizing the above Kähler metric $K_{I\bar{J}}$ for $I, J = U^4, U^2, \tilde{T}, U^3, S$ are numerically estimated in the case of $c_2 = 1$,

$$\begin{aligned} &(K_{\rm eig})_{U^4} \simeq \frac{4.3}{(U^2+\bar{U}^2)^2}, \quad (K_{\rm eig})_{U^2} \simeq \frac{1.1}{(U^2+\bar{U}^2)^2}, \quad (K_{\rm eig})_{\tilde{T}} = \frac{0.27}{(U^2+\bar{U}^2)^2}, \\ &(K_{\rm eig})_{U^3} = K_{U^3\bar{U}^3}, \quad (K_{\rm eig})_S = K_{S\bar{S}}, \end{aligned}$$

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$$U_{I\bar{J}} = \begin{pmatrix} -0.67 & -2.19 & 1 & 0 & 0 \\ 1 & 0.14 & 1 & 0 & 0 \\ -1.1 & 0.8 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{A.3}$$

Next, we show the mass-squared matrix obtained from the scalar potential which is consisted of the Kähler potential (3.4), (3.10) and the superpotential (3.4), (3.11),

$$m^2_{I\bar{J}} = (K_{\rm eig})_{I\bar{K}} (U^{-1})_{\bar{K}L} V_{L\bar{M}} U_{\bar{M}N} (K_{\rm eig})_{N\bar{J}}, \tag{A.4}$$

where

$$(K_{\rm eig})_{I\bar{K}} = \delta_{I\bar{K}} / \sqrt{(K_{\rm eig})_I}, \tag{A.5}$$

and

$$V_{L\bar{M}} \simeq \begin{pmatrix} V_{U^4\bar{U}^4} & 0 & 0 & 0 & 0 \\ 0 & V_{U^2\bar{U}^2} & V_{U^2\bar{\tilde{T}}} & 0 & 0 \\ 0 & V_{\tilde{T}\bar{U}^2} & V_{\tilde{T}\bar{\tilde{U}}} & 0 & V_{\tilde{T}\bar{S}} \\ 0 & 0 & 0 & V_{U^3\bar{U}^3} & V_{U^3\bar{S}} \\ 0 & 0 & V_{S\bar{\tilde{T}}} & V_{S\bar{U}^3} & V_{S\bar{S}} \end{pmatrix}. \tag{A.6}$$

Note that here the mass-squared matrix is evaluated in the canonically normalized field basis $(\Phi^2, \Phi^4, \Phi^3, \Phi^S, \Phi^T)$ with

$$\begin{aligned} &\bar{\Phi}^2 = \sqrt{2(K_{\rm eig})_{U^2}} (U^{-1})_{U^2\bar{U}^J} \bar{U}^{\bar{J}}, \quad \bar{\Phi}^4 = \sqrt{2(K_{\rm eig})_{U^4}} (U^{-1})_{U^4\bar{U}^J} \bar{U}^{\bar{J}}, \quad \bar{\Phi}^T = \sqrt{2(K_{\rm eig})_{\tilde{T}}} (U^{-1})_{\tilde{T}\bar{U}^J} \bar{\tilde{T}} \\ &\bar{\Phi}^3 = \sqrt{2(K_{\rm eig})_{U^3}} \bar{U}^{\bar{3}}, \qquad \bar{\Phi}^S = \sqrt{2(K_{\rm eig})_S} \bar{S}, \end{aligned} \tag{A.7}$$

and the mass-squared matrix m^2_{IJ}, $m^2_{\bar{I}\bar{J}}$ can be obtained in the same way.

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