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Erratum: Refining the boundaries of the classical de Sitter landscape

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Summary and consequences

In the trace of the Einstein equation along internal parallel flat directions, namely equations (4.14) and (4.15), a few terms have been missed. As a consequence the corrected equations will have additional terms which depend on specific components of fluxes, such as $|H^{(2)}|^2 + 2|H^{(3)}|^2$ which are the squares of the components $H_{a_{||}b_{||}c_{\perp}}$ and $H_{a_{||}b_{||}c_{||}}$. These terms are then absent in the final $\tilde{\mathcal{R}}_4$ expression (4.21) and (4.29). The only change that impacts our conclusion, (4.36), is that the curvature terms $2\mathcal{R}_{||} + 2\mathcal{R}_{||}^{\perp}$ should be replaced by

$$2\mathcal{R}_{||} + 2\mathcal{R}_{||}^{\perp} - |H^{(2)}|^2 - 2|H^{(3)}|^2 \,. \tag{1}$$

As in (4.36), this combination gets bounded by two inequalities, in order to get classical de Sitter solutions for parallel $p = 4, 5, 6$ sources. While this change modifies the final expression, it has little impact on the physics result: we obtain tight constraints on a combination of fields for de Sitter solutions to exist with parallel $p = 4, 5, 6$ sources. The no-go theorems for parallel $p = 3, 7, 8$ sources are not affected at all.

The combination (1) is better motivated than the curvature terms alone, as it now appears to be T-duality invariant, on geometric backgrounds. This statement can be made more precise by considering group manifolds, where the $f^a{}_{bc}$, building the curvature terms,

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are constant, and some are set to zero by the orientifold projection. In addition, the H-flux is odd under an orientifold involution, imposing $H^{(3)} = 0$ for a constant flux; avoiding the Freed-Witten anomaly also sets $H^{(3)}$ to zero. The (opposite sign of the) combination (1) then reduces to

$$\begin{aligned}&\delta^{ab} f^{d_{||}}{}_{c_{||}a_{||}} f^{c_{||}}{}_{d_{||}b_{||}} + \frac{1}{2}\delta^{ch}\delta^{dj}\delta_{ab} f^{a_{||}}{}_{c_{||}j_{||}} f^{b_{||}}{}_{h_{||}d_{||}} + \delta^{ab} f^{d_\perp}{}_{c_\perp a_{||}} f^{c_\perp}{}_{d_\perp b_{||}} \\ &+ \delta^{ab}\delta^{dg}\delta_{ch} f^{h_\perp}{}_{g_\perp a_{||}} f^{c_\perp}{}_{d_\perp b_{||}} + \frac{1}{2}\delta^{ad}\delta^{be}\delta^{cf} H_{a_{||}b_{||}c_\perp} H_{d_{||}e_{||}f_\perp} ,\end{aligned} \tag{2}$$

and the first and third terms vanish on nilmanifolds. The H-flux component schematically transforms under T-duality into one or the other structure constant, depending on the T-duality direction

$$H_{a_{||}b_{||}c_\perp} \to f^{c_{||}}{}_{a_{||}b_{||}} \text{ or } -f^{a_\perp}{}_{c_\perp b_{||}} , \tag{3}$$

showing the T-duality invariance of the combination (1) in that setting.

A practical consequence for the paper is that several occurrences of "curvature terms" should be replaced by the above "field combination": it is the case for equations (1.2), (1.3), (4.30), and the text of the Outlook. The discussed consequences of the results are unchanged: to start with, the remark on the solutions T-dual to one with an O_3, at the end of section 4.2, remains valid. The requirement of having $f^{a_{||}}{}_{b_\perp c_\perp} \neq 0$ for a de Sitter solution still holds, from the constraints on the new combination, implying the no-go theorem for $p = 8$ (footnote 6) and the impossibility to embed a specific monodromy inflation mechanism, as mentioned in the Outlook.

Corrected equations

For a p-dimensional source, any internal flux F_q was decomposed in (4.11) as $F_q = \sum_{n=0}^{p-3} F_q^{(n)}$, where the components of $F_q^{(n)}$ have n internal parallel flat indices, and $F_q^{(0)} = F_q|_\perp$. As a consequence, one has

$$\begin{aligned}|F_q|^2 &= \sum_{n=0}^{p-3} |F_q^{(n)}|^2 , \text{ where } |F_q|^2 = \frac{1}{q!} F_{q\ a_1\ldots a_q} F_q^{a_1\ldots a_q} , \\ |F_q^{(n)}|^2 &= \frac{1}{n!(q-n)!} F_{q\ a_{1||}\ldots a_{n||}a_{n+1\perp}\ldots a_{q\perp}} F_q^{a_{1||}\ldots a_{n||}a_{n+1\perp}\ldots a_{q\perp}} ,\end{aligned} \tag{4}$$

the indices being lifted by the flat internal metric. We now consider the trace of the Einstein equation along the internal parallel directions. An internal flux F_q appears in it as follows

$$\begin{aligned}\delta^{ab}\frac{1}{(q-1)!} F_{q\ a_{1||}a_2\ldots a_q} F_{q\ b_{1||}}{}^{a_2\ldots a_q} &= \sum_{n\geq 1}^{p-3} \delta^{ab}\frac{1}{(n-1)!(q-n)!} F^{(n)}_{q\ a_{1||}a_{2||}\ldots a_{n||}a_{n+1\perp}\ldots a_{q\perp}} \\ &\quad \times F^{(n)}_{q\ b_{1||}}{}^{a_{2||}\ldots a_{n||}a_{n+1\perp}\ldots a_{q\perp}} \\ &= \sum_{n\geq 0}^{p-3} n |F_q^{(n)}|^2 = |F_q|^2 - |F_q|_\perp|^2 + \sum_{n\geq 2}^{p-3} (n-1)|F_q^{(n)}|^2 .\end{aligned} \tag{5}$$

The last sum is absent of (4.14) and (4.15). These two equations are corrected towards

$$\begin{aligned}
\mathcal{R}_{6||} + 2(\nabla\partial\phi)_{6||} &= \frac{p-3}{4}\left(\mathcal{R}_4 + 2(\nabla\partial\phi)_4 + 2e^{2\phi}|F_6|^2\right) \\
&+ \frac{1}{2}\left(|H|^2 - |H|_\perp|^2 + e^{2\phi}(|F_2|^2 - |F_2|_\perp|^2 + |F_4|^2 - |F_4|_\perp|^2\right) \\
&+ \frac{1}{2}\sum_{n\geq 2}^{p-3}(n-1)\left(|H^{(n)}|^2 + e^{2\phi}(|F_2^{(n)}|^2 + |F_4^{(n)}|^2)\right) \\
\mathcal{R}_{6||} + 2(\nabla\partial\phi)_{6||} &= \frac{p-3}{4}\left(\mathcal{R}_4 + 2(\nabla\partial\phi)_4 + e^{2\phi}|F_5|^2\right) \qquad (6) \\
&+ \frac{1}{2}\left(|H|^2 - |H|_\perp|^2 + e^{2\phi}(|F_1|^2 - |F_1|_\perp|^2 + |F_3|^2 - |F_3|_\perp|^2\right) \\
&+ \frac{1}{4}e^{2\phi}\left(|F_5|^2 - |F_5|_\perp|^2 - |*_6 F_5|^2 + |(*_6 F_5)|_\perp|^2\right) \\
&+ \frac{1}{2}\sum_{n\geq 2}^{p-3}(n-1)\left(|H^{(n)}|^2 + e^{2\phi}\left(|F_3^{(n)}|^2 + \frac{1}{2}|F_5^{(n)}|^2\right)\right),
\end{aligned}$$

where in IIB, the one-form fluxes, F_1 and $*_6F_5$, do not contribute to the new terms because the sum starts with $n \geq 2$. For the same reason, these new terms only contribute for $p \geq 5$. A general rewriting of these two equations, correcting equation (4.16), is then given by

$$\begin{aligned}
2\mathcal{R}_{6||} + 4(\nabla\partial\phi)_{6||} - \frac{p-3}{2}\left(\mathcal{R}_4 + 2(\nabla\partial\phi)_4\right) &= |H|^2 - |H|_\perp|^2 + e^{2\phi}\left(|F_{k-2}|^2 - |F_{k-2}|_\perp|^2\right) \\
+ e^{2\phi}\Bigg(|F_k|^2 - |F_k|_\perp|^2 + |F_{k+2}|^2 + (9-p)|F_{k+4}|^2 &+ 5|F_{k+6}|^2 + \frac{1}{2}(|(*_6F_5)|_\perp|^2 - |F_5|_\perp|^2)\Bigg) \\
+ \sum_{n\geq 2}^{p-3}(n-1)\left(|H^{(n)}|^2 + e^{2\phi}\left(|F_k^{(n)}|^2 + |F_{k+2}^{(n)}|^2 + \frac{p-6}{2}|F_{k+4}^{(n)}|^2 + \frac{p-7}{4}|F_5^{(n)}|^2\right)\right), &\qquad (7)
\end{aligned}$$

where the F_5 terms should only be considered in IIB. Equation (4.17) gets corrected by adding the same new line, while the final formula (4.21) becomes

$$\begin{aligned}
2e^{-2A}\tilde{\mathcal{R}}_4 = &- \left|*_\perp H|_\perp + \varepsilon_p e^{\phi} F_{k-2}|_\perp\right|^2 - 2e^{2\phi}\left|g_s^{-1}\tilde{*}_\perp \mathrm{d}e^{-4A} - \varepsilon_p F_k^{(0)}\right|^2 \\
&- \sum_{a_{||}}\left|*_\perp(\mathrm{d}e^{a_{||}})|_\perp - \varepsilon_p e^{\phi}(\iota_{\partial_{a_{||}}} F_k^{(1)})\right|^2 - 2\mathcal{R}_{||} - 2\mathcal{R}_{||}^{\perp} \\
&- 2e^{-2A}\left(\mathrm{d}\left(e^{8A}\tilde{*}_\perp \mathrm{d}e^{-4A} - e^{8A}\varepsilon_p g_s F_k^{(0)}\right)\right)_{\tilde{\perp}} \qquad (8) \\
- e^{2\phi}&\left(|F_k|^2 - |F_k^{(0)}|^2 - |F_k^{(1)}|^2 + 2|F_{k+2}|^2 + (p-5)|F_{k+4}|^2 + \frac{1}{2}(|F_5|_\perp|^2 - |(*_6F_5)|_\perp|^2)\right) \\
+ \sum_{n\geq 2}^{p-3}&(n-1)\left(|H^{(n)}|^2 + e^{2\phi}(|F_k^{(n)}|^2 + |F_{k+2}^{(n)}|^2 + \frac{p-6}{2}|F_{k+4}^{(n)}|^2 + \frac{p-7}{4}|F_5^{(n)}|^2)\right).
\end{aligned}$$

We now detail the last two lines of (8): they are equal to

$p = 3:\ 0$

$p = 4:\ -2e^{2\phi}|F_6|^2$

$$
\begin{aligned}
&p=5:\ |H^{(2)}|^2-e^{2\phi}\left(2|F_5|^2-\frac{1}{2}|(*_6F_5)|_\perp|^2-\frac{1}{2}|F_5^{(2)}|^2\right)\\
&p=6:\ |H^{(2)}|^2+2|H^{(3)}|^2-e^{2\phi}\left(2|F_4|^2-|F_4^{(2)}|^2-2|F_4^{(3)}|^2+|F_6|^2\right)\\
&p=7:\ |H^{(2)}|^2+2|H^{(3)}|^2-e^{2\phi}\Bigg(2|F_3|^2-|F_3^{(2)}|^2-2|F_3^{(3)}|^2+2|F_5|^2-\frac{1}{2}|(*_6F_5)|_\perp|^2\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad-\frac{1}{2}\sum_{n\geq 2}^{4}(n-1)|F_5^{(n)}|^2\Bigg)\\
&p=8:\ |H^{(2)}|^2+2|H^{(3)}|^2-e^{2\phi}\left(2|F_2|^2-|F_2^{(2)}|^2+3|F_4|^2-\sum_{n\geq 2}^{4}(n-1)|F_4^{(n)}|^2\right). \qquad (9)
\end{aligned}
$$

We used (4), that leads to the cancelation of all F_k terms. That equation, together with $|F_5|^2=|*_6 F_5|^2\geq|(*_6F_5)|_\perp|^2$, allows us to prove that the Ramond-Ramond contributions to these lines are always negative (semi-)definite. We rewrite the final equation (8) as

$$
\begin{aligned}
2e^{-2A}\tilde{\mathcal{R}}_4 = &-\left|*_\perp H|_\perp+\varepsilon_p e^{\phi}F_{k-2}|_\perp\right|^2-2e^{2\phi}\left|g_s^{-1}\tilde{*}_\perp \mathrm{d}e^{-4A}-\varepsilon_p F_k^{(0)}\right|^2\\
&-\sum_{a_{||}}\left|*_\perp(\mathrm{d}e^{a_{||}})|_\perp-\varepsilon_p e^{\phi}(\iota_{\partial_{a_{||}}}F_k^{(1)})\right|^2-2\mathcal{R}_{||}-2\mathcal{R}_{||}^{\perp}+|H^{(2)}|^2+2|H^{(3)}|^2\\
&-2e^{-2A}\left(\mathrm{d}\left(e^{8A}\tilde{*}_\perp \mathrm{d}e^{-4A}-e^{8A}\varepsilon_p g_s F_k^{(0)}\right)\right)_{\tilde{\perp}} \qquad (10)\\
&-e^{2\phi}\Bigg(2|F_{k+2}|^2+(p-5)|F_{k+4}|^2+\frac{1}{2}(|F_5|_\perp|^2-|(*_6F_5)|_\perp|^2)\\
&\qquad-\sum_{n\geq 2}^{p-3}(n-1)\left(|F_{k+2}^{(n)}|^2+\frac{p-6}{2}|F_{k+4}^{(n)}|^2+\frac{p-7}{4}|F_5^{(n)}|^2\right)\Bigg),
\end{aligned}
$$

where the last two lines are a negative (semi-)definite contribution. The new combination (1) now appears. The integral version of this expression, (4.29), is similarly corrected. Turning to the no-go theorems, equations (4.33), (4.34) and (4.35) still hold in view of (7), the corrected version of (4.16). They can however be refined with the new H-flux terms, towards

$$
2\mathcal{R}_{6||}+4(\nabla\partial\phi)_{6||}-\frac{p-3}{2}\left(\mathcal{R}_4+2(\nabla\partial\phi)_4\right)-|H^{(2)}|^2-2|H^{(3)}|^2\geq 0\,, \qquad (11)
$$

for (4.33). We deduce the following version of the main result, correcting (4.36)

> There is no de Sitter vacuum for $p=4,5$, or 6, if the inequalities
> $$
> -\int_{\widetilde{\mathcal{M}}}\widetilde{\mathrm{vol}}_6\, e^{2A}\sum_{a_{||}}|(\mathrm{d}e^{a_{||}})|_\perp|^2<\int_{\widetilde{\mathcal{M}}}\widetilde{\mathrm{vol}}_6\, e^{2A}\left(2\mathcal{R}_{||}+2\mathcal{R}_{||}^{\perp}-|H^{(2)}|^2-2|H^{(3)}|^2\right)<0 \qquad (12)
> $$
> are *not* satisfied.