COFINE POTENTIAL THEORY

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Let X be a harmonic space with a countable base and a Green function G on $X \times X$. We associate with each point $z \in X$ the potential $g_z = G(\cdot, z)$ whose harmonic support is $\{z\}$. A set A is said to be a *cofine* neighborhood of z if $\hat{R}_{g_z}^{X \setminus A} \neq g_z$.

Let U be a Borel measurable relatively compact finely open subset of X. A point $z \in \partial_{cof} U$ (= the cofine boundary of U) is called *cofinely regular* if

$$\text{cofine} - \lim_{x \to z} \varepsilon_x^{X \setminus U}(f) = f(z)$$

for every $f \in C(\partial U)$.

Problems. (a) Characterize the set of all cofinely regular points.

(b) (The minimum principle.) Let U be a lower bounded lower semicontinuous function on U. Assume that

$$\varepsilon_x^{X \setminus V}(u) \leq u(x)$$

whenever $x \in V \subset \overline{V} \subset U$, and

$$(*) \qquad \text{cofine} - \liminf_{x \to z} u(x) \geq 0$$

for every $z \in \overline{U}^{cof} \setminus U$. Does it follow $u \geq 0$?

(c) The same question as in (b), but (*) is supposed to hold for all cofinely regular points only.

References

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[2] Le JAN, Y. : Quasi-continuous functions associated with a Hunt process, Proc. Amer. Math. Soc. 86 (1982), 133-137.