INTRODUCTION

Spaces of functions have been used since the late 19th century to form a framework in which convergence of sequences of functions could be studied. Since then several natural topologies have been frequently used to study function spaces. The purpose of this book is to bring together the techniques used in studying the topological properties of such function spaces and to organize and present the theory in a general setting. In particular, a study is made of C(X,R), the space of all continuous functions from a topological space X into a topological space R.

For almost any natural topology imposed on C(X,R), the topological properties of X and R interact with the topological properties of C(X,R). One of the things which is emphasized is the study of these interactions, especially the deduction of the topological properties of C(X,R) from those of X and R. The two major classes of topologies on C(X,R) which are studied are the set-open topologies and the uniform topologies. Each chapter has a number of exercises, not only about these two classes of topologies, but about other kinds of function space topologies found in the literature. Chapters I, II and III contain basic properties and techniques, as well as classical theory. Chapters IV and V have the characterizations of many topological properties of function spaces. Those in Chapter IV are given in the more general setting of cardinal functions.

The range space throughout this book is denoted by R, and whenever the properties of R are not important for the discussion, C(X,R) is abbreviated as C(X). in order to eliminate pathologies and ensure that C(X,R) is large enough, all spaces are assumed to be completely regular Hausdorff spaces, and R is assumed to contain a nontrivial path. The symbol ω denotes the first infinite ordinal number (which is the set of all natural numbers), and $\mathbb{R}$ is used to indicate the space of real numbers with the usual topology.