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Quadrature Domains

Errata

I. In the definition of piecewise smooth arcs on page 8, we have merely assumed that it can be expressed as the union of a finite number of smooth arcs each of which is the image of the closed interval $[0, 1]$ under a function of class C^1.

However, to prove Lemma 2.4, which discusses the growth rate of the normal derivative of the Green function at a corner, we need the continuity of the derivative on each of smooth arcs as stated on page 14. Hence our smooth arcs should satisfy a stronger condition so that the normal derivative of the Green function is continuous on each of them.

It is enough to replace "under a function of class C^1" in the definition of a smooth arc with "under a function of class C^2". A weaker condition is "under a Lyapunov-Dini smooth function". A function γ of class C^1 on $[0, 1]$ is called a Lyapunov-Dini smooth function if the modulus of continuity

$$\omega(t) = \sup\{|\gamma'(t_2) - \gamma'(t_1)| : t_j \in [0, 1],\ |t_2 - t_1| \leq t\}$$

of the derivative γ' of γ satisfies

$$\int_0^1 \omega(t)\frac{dt}{t} < +\infty.$$

II. We have mainly discussed quadrature domains of positive measures as stated in Introduction. However, we have treated quadrature domains of real measures and quadrature domains of complex measures without their explicit definitions. For examples, Proposition 8.1 and its corollary are for real measures and Proposition 9.4 is for complex measures. The definitions are the same as in Introduction. The definition of a quadrature domain of a complex measure ν for class AL^1 is the following: A nonempty domain Ω is called a quadrature domain of ν for class AL^1 if

(1) ν is concentrated in Ω, namely, $\nu|\Omega^c = 0$;

(2) $\int_\Omega |f| d|\nu| < +\infty$ and $\int_\Omega f d\nu = \int_\Omega f dm$ for every $f \in AL^1(\Omega)$.

III. When we use the argument as in the proof of Theorem 3.5, for example, in the proofs of Lemma 3.6, Theorem 3.7 and Proposition 3.10, we treat not only a finite positive measure ν stated as in Theorem 3.4, but also a measure of form $\nu+\xi$ for some finite positive measure ξ. In the proof of Theorem 3.5, we depart from $\overline{W} \subset \Omega$ and arrive $\overline{W^{(n)}} \subset \Omega$. We modify the measure ν to $\nu^{(n)}$ in this process. We apply our argument to a measure $\nu+\xi$ for a finite positive measure ξ on Ω such that every $s \in SL^1(\Omega)$ has an integral on Ω and modify $\nu+\xi$ to $\nu^{(n)}+\xi$.

page	line	
4	12	$m(\nu) = \\|\nu\\| \longrightarrow m(\Omega) = \\|\nu\\|$
5	2, 7	The double integral $\iint (1/\|\zeta - z\|) d\|\nu\|(\zeta) dm(z)$ is taken over $(\operatorname{supp}\nu)^2$.
7	13	$f \in AL^1(R_\alpha) \longrightarrow f \in A(R_\alpha)$
7	$\uparrow$8	$(\log r)r > 1$ if $r > 1 \longrightarrow (\log r)r > 0$ if $r > 1$
11	$\uparrow$4	$d(G_j, \partial O_{j-1} \cup \partial O_{j+1}) \longrightarrow d(G_j, \partial O_{j-2} \cup \partial O_{j+1})$
17	2	Put a period at the end of line.
18	4	are $\longrightarrow$ arc
21	6	with repsect to $\longrightarrow$ with respect to
23	$\uparrow$3	$q \in \partial W' \longrightarrow q' \in \partial W'$
29	12	$W_n \subset W_{n+1}, n=1,2,\cdots, \bigcup W_n = W$ and $\int_{W_1} \nu dm > m(W_1)$.
	$\longrightarrow$	$W_n \subset W_{n+1} \subset \tilde{W}_n \cup W, n = 1,2,\cdots, \bigcup W_n \supset W$, $m(\bigcup W_n \setminus W) = 0$ and $\int_{W_1} \nu dm > m(W_1)$,
29	13	Let $\tilde{W}_n$ be $\longrightarrow$ where $\tilde{W}_n$ denotes
31	7	that $\nu_1(z) + \nu_2(z) \geq 1$ a.e. $\longrightarrow$ that $\nu_2(z) \geq 0$ a.e. on $\mathbb{C}$, $\nu_1(z) + \nu_2(z) \geq 1$ a.e.
35	13	these lemma $\longrightarrow$ these lemmas
37	$\uparrow$9	$E_1 = \overline{R_0} \cap W \longrightarrow E_1 = \overline{R_0} \cap \partial W$
37	$\uparrow$2–1	$\epsilon = \min\{d(\overline{R_0}, \partial R)/10\sqrt{2}, \inf_{q \in \overline{R_0} \cap \partial W} \lambda_S(q)\}. \longrightarrow \epsilon = \inf_{q \in \overline{R_0} \cap \partial W} \lambda_S(q)$.
38	$\uparrow$3–2	$\epsilon = d(\overline{R_0}, \partial R)/10\sqrt{2}, \longrightarrow \epsilon = \min\{d(\overline{R_0}, \partial R)/10\sqrt{2}, \inf_{q \in \overline{R_0} \cap \partial W} \lambda_S(q)\}$,
39	3–4	$\epsilon = d(\overline{R_1}, \partial R)/10\sqrt{2} \longrightarrow \epsilon = \min\{d(\overline{R_1}, \partial R)/10\sqrt{2}, \inf_{q \in \overline{R_1} \cap \partial W'} \lambda'_S(q)\}$
42	$\uparrow$9	$\beta_\Omega^{(n)}(\overline{\Delta(r;p)}) = 0 \longrightarrow \beta_\Omega^{(n)}(\overline{\Delta(R;p)}) = 0$
42	$\uparrow$8	for some $r > 0 \longrightarrow$ for some $R > r > 0$.
48	$\uparrow$2	if $v(r) = 0$. $\longrightarrow$ if $v(r) = 0$, and A and B are nonnegative constants.
65	1	The integral $\int g(\zeta; z, \Omega) dm(\zeta)$ is taken over Ω.
68	$\uparrow$9	angle $V_1 < 2\Pi \longrightarrow$ angle $V_1 < 3\pi/2$
72	13	$\operatorname{disc}\{\Omega(t)\} = [\bigcup_{t \geq 0} \Omega(t) \setminus \Omega(0)] \setminus \bigcup_{t \geq 0} \partial\Omega(t)$
	$\longrightarrow$	$\operatorname{disc}\{\Omega(t)\} = \left(\bigcup_{t \geq 0} \Omega(t) \setminus \Omega(0)\right) \setminus \bigcup_{t \geq 0} \partial\Omega(t)$
79	1	$\operatorname{stag}\{[W(t)]\} \longrightarrow \operatorname{stag}\{[\tilde{W}(t)]\}$
79	$\uparrow$10	$Q(\{\chi_\Omega m + \nu(t) - \nu(0)\}, F) \longrightarrow Q(\{\chi_{\Omega(0)} m + \nu(t) - \nu(0)\}, F)$
80	$\uparrow$9	$Q(0) \in Q(\nu(0), HL^1) \longrightarrow \Omega(0) \in Q(\nu(0), HL^1)$

84	↑1	contradict $\longrightarrow$ contradicts
95	10	Lemma 6.8, $\longrightarrow$ Lemma 7.1,
105	8	$Q(\nu, AL') \longrightarrow Q(\nu, AL^1)$
105	↑7–6	$\lvert a_1\rvert \leq \sqrt{2/\pi}(a_0/3)^{3/2} \longrightarrow a_1 = \pi b_1^2 \overline{b_2}$
106	↑10	$\overline{u_j}(x_1, x_2) = \longrightarrow \overline{u_j}(x_1, x_2, t) =$
106	↑9	$(1/d)\int_0^d u_j(x_1, x_2, x_3)dx_3 \longrightarrow (1/d)\int_0^d u_j(x_1, x_2, x_3, t)dx_3$
108	3	For every $z \in C_1(t)$ with angle $V_1 < \pi$, $\longrightarrow$ For every $z \in C_j(t)$,
108	5	$U \cap \partial\Omega(s)$ is connected $\longrightarrow$ $U \cap \partial\Omega(s)$ consists of j connected components
109	↑8	every harmonic function on $\overline{\Omega(\tau)}$ $\longrightarrow$ every harmonic function h on $\overline{\Omega(\tau)}$
110	8	$\mathrm{stat}\{\Omega(t)\} \subset \{z \in C_1(0) \lvert \mathrm{angle} V_1 \leq \pi/2\} \subset C_1(0)$ $\longrightarrow$ $\mathrm{stat}\{\Omega(t)\} \subset \{z \in C_1(0) \lvert \mathrm{angle} V_1 \leq \pi/2\} \cup \{z \in C_2(0) \lvert \mathrm{angle} V_j \leq \pi/2, j = 1, 2\}$
110	↑8	onto $\longrightarrow$ into
115	5	minimum open set in $Q(\nu, SL^1)$ $\longrightarrow$ minimum open set Ω in $Q(\nu, SL^1)$
116	↑12	$[G] \longrightarrow G^c$
116	↑4	$\hat{\chi}_{\tilde{G}(t_1)}(\zeta) \longrightarrow \hat{\chi}_{[\tilde{G}(t_1)]}(\zeta)$
117	4	$\varphi'(\zeta_0) > 0 \longrightarrow \varphi'(\zeta) > 0$
122	5	Let G be connected component $\longrightarrow$ Let G be a connected component
122	12	$\theta(x, G) \longrightarrow \theta(x_i, G)$
124	2, 3	$-(-1)^{n+1}i\} \longrightarrow -(-1)^{n+1}(-1+i)\}$