## §2. Semigroups

For any $u \in Q$ we put

$$\mathrm{denom}(u) = \{0 \neq n \in Z\colon nu \in Z\}$$

and for any $u' \subset Q$ we put

$$\mathrm{denom}(u') = \{0 \neq n \in Z\colon nu \in Z \text{ for all } u \in u'\}.$$

We note that $Q$ is an additive abelian semigroup and $Z$ is an (additive) subsemigroup of $Q$. In fact $Q$ is a nonnegative ordered additive abelian semigroup where by a nonnegative ordered additive abelian semigroup we mean the nonnegative part of an ordered additive abelian group, i.e., the set of all nonnegative elements of an ordered additive abelian group. Likewise $Z$ is a nonnegative ordered additive abelian semigroup. Moreover $Q$ is divisible, but $Z$ is not, in the following sense.

An additive abelian semigroup $G$ is said to be divisible if for every $v \in G$ and $0 \neq n \in Z$ there exists $v^* \in G$ such that $v = nv^*$. Note that if $\bar{G}$ is a divisible ordered additive abelian group then for every $w \in \bar{G}$ and $u \in Q$ there exists a unique $w^* \in \bar{G}$ such that for every $n \in \mathrm{denom}(u)$ we have $(nu)w = nw^*$; we define: $uw = w^*$; we observe that this notation is consistent with regarding $\bar{G}$ as a module over the ring of rational numbers; we also note that if $G$ is the nonnegative part of $\bar{G}$ then for every $w \in G$ and $u \in Q$ we now have $uw \in G$.