Weighted Norm Inequalities for the Restriction of Fourier Transform to S^{n-1}

LOU HONGWEI

Ninbo University

In [1,3,5], E. M. Stein, C. Fefferman, A. Zygmund, and P. Tomas considered the restriction of Fouier transform to the unit sphere S^{n-1}. They proved that the a-priori inequality

$$(1) \qquad (\int_{S^{n-1}} |\hat{f}(\theta)|^q d\theta)^{1/q} \leq C\|f\|_p \quad f \in S(\mathbb{R}^n)$$

holds if and only if $1 \leq p < 4/3$, $q \leq p'/3$ when $n = 2$. When $n \geq 3$, they proved that (1) holds if $1 \leq p \leq \frac{2(n+1)}{n+3}$ and $q \leq \frac{n-1}{n+1}p'$.

Now we consider the weighted norm inequalities for the restriction of Fourier transform to S^{n-1}:

$$(2) \qquad (\int_{S^{n-1}} |\hat{f}(\theta)|^q d\theta)^{1/q} \leq C(\int_{\mathbb{R}^n} |f(x)|^p \varphi(|x|) dx)^{1/p} \quad f \in S$$

where $1 \leq q < \infty$, $1 \leq p \leq 2$ and $\varphi(r) \geq 0$ is a measurable function on $[0, \infty)$, we prove that:

Theorem 1: Let $p = 2$, $1 \leq q \leq 2$, $\mathcal{N} = \{0, 1, 2, 3, \cdots .\}$, if:

$$(3) \qquad \sup_{k \in \mathcal{N}} \int_0^\infty \frac{r}{\varphi(r)} J^2_{k+\frac{n-2}{2}}(2\pi r) dr < \infty$$

then (2) holds. When $n = 2$ or $q = 2$, (2) holds if and only if (3) holds.

Theorem 2: Let $\varphi(r) = r^a$, $1 \leq q \leq 2 \leq p < \infty$, then (2) holds if and only if $\frac{n+1}{2}p - n < a < n(p-1)$.

On the other hand, we have:

Theorem 3: Let $g(\theta) \in L^2(S^{n-1})$, $q \leq \frac{2n}{n-1}$, if $\|(gd\theta)\check{}(x)\|_q < \infty$ then $g(\theta) = 0$ for almost every $\theta \in S^{n-1}$, where we define $(gd\theta)\check{}(x) = \int_{S^{n-1}} g(\theta) e^{-2\pi i x \cdot \theta} d\theta$.

Acknowledgment: I am grateful to my advisers, Prof. Wan Si-Lei and Prof. Shi Xian-Liang, for their many helpful suggestions and discussions.

References

1. C. Fefferman, Acta Math. 124(1970), 9-36.

2. E. M. Stein & G. Weiss, Introduction to Fourier analysis in Euclidean spaces, Princeton University Press. Princeton, NJ.1975.

3. P. Tomas, Bull. Am. Math. Soc. 81(1975), 477-478.

4. G. N. Watson, Theory of Bessel functions, Cambridge University Press, Cambridge, 1944.

5. A. Zygmund, Studia Math. 50(1974), 189-201.