On a paper of J.G. Sinai on dynamical systems.

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It was given an information lecture on Sinai's paper "Classical dynamical systems with countable Lebesgue spectrum. II", Iswestya 30, 15 - 69 (1966). The main concepts were explained in detail, the hints for proofs were poor. Furthermore things were illustrated at the example of a continuous automorphism of a compact connected Lie group G. Already here the main ideas can be shown and no technical troubles come in. From the results of J.G.Sinai follows: If the induced linear mapping $A = (dT)_e$ on the tangent space E at the unit element has no eigenvalues of module 1, T is a K-automorphism and has therefore all nice mixing properties with respect to Haar measure.

For simplicity let us assume that eigenvectors $v_1, \ldots, v_n$ with eigenvalues $\lambda_1, \ldots, \lambda_n$ $(|\lambda_i| < 1,\ i \leqslant k,\ |\lambda_i| > 1,\ i > k)$ span the tangent space. We introduce a metric on E by defining $v_1, \ldots, v_n$ orthogonal and of equal length and transport this all over G by left translations. This gives a left invariant Riemannian metric on G. The length of the v's is chosen so that the total volume of G equals 1. Let V be the linear subspace, spanned by $v_1, \ldots, v_k$, W spanned by $v_{k+1}, \ldots, v_n$. By the exponential mapping we project the parallels to V resp. W down to G and obtain two measurable foliations, expanding resp. contracting transversal fields for T and absolutely continuous one with respect to the other. Thus all conditions needed in Sinai's theorems are fulfilled.

It is clear that T represents an "U"-cascade in the sense of D.W. Anosov (Soviet Math. Doklady 4 N° 4 (1963), 1153 - 1156).