GENERALIZATIONS OF PRODUCT ISOMORPHISMS

S. M. Ulam, University of Colorado, Boulder, CO 80304

RESUME

Given two subsets A, B of the direct product $E^n = E \times E \times \dots E$ of an abstract set E, the product isomorphism of $A \approx B$ was defined iff there exists a one-to-one transformation f(p) of E onto itself such that the transformation: $(p_1, p_2 \dots p_n)$ goes into $(f(p_1), f(p_2), \dots f(p_n))$ carries A onto B.
For (n = 2 this notion coincides with the isomorphism of graphs represented by sets of pairs: A, B.) Combinatorial and set theoretical properties of product isomorphisms were defined and studied in my master thesis in 1932.

In the present paper this notion is generalized in two different ways: We call 2 subsets A, B <u>quasi-isomorphic</u> if there exist one-to-one transformations $f_1, f_2 \dots f_n$ of E onto itself, perhaps different from each other, such that the transformation $(p_1 \dots p_n) \to (f_1(p_1) \dots f_n(p_n))$, carries A onto B.

The second generalization is that of "<u>equivalence by decomposition</u>" (finite or countable). One calls two sets A, B equivalent by decomposition if $A = \bigcup A_i$, $A_i \cap A_j = 0$ for $i \neq j$, $B = \bigcup B_i$, $B_i \cap B_j$, $i \neq j$ and $A_i \approx B_i$ for all i (perhaps by different f's)
Some elementary set theoretical properties and enumeration results and problems are discussed. A fuller account of this work will appear elsewhere.