STEREO AND MOTION

Vertical and Horizontal Disparities from Phase

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We apply the notion that phase differences can be used to interpret disparity between a pair of stereoscopic images. Indeed, phase relationships can also be used to obtain probabilistic measures both edges and corners, as well as the directional instantaneous frequency of an image field. The method of phase differences is shown to be equivalent to a Newton-Raphson root finding iteration through the resolutions of band-pass filtering. The method does, however, suffer from stability problems, and in particular stationary phase and aliasing. The stability problems associated with this technique are implicitly derived from the mechanism used to interpet disparity, which in general requires an assumption of linear phase and the local instantaneous frequency. We present two techniques. Firstly, we use the centre frequency of the applied band-pass filter to interpret disparity. This interpretation, however, suffers heavily from phase error and requires considerable damping prior to convergence. Secondly, we use the derivative of phase to obtain the instantaneous frequency from an image, which is then used to improve the disparity estimate. These ideas are extended into 2-D where it is possible to extract both vertical and horizontal disparities.

1 Introduction

Horizontal disparities provide important depth cues for passive range finding systems. Unfortunately, the computation of disparities has proven particularly difficult because of the *correspondence* problem. Indeed, there have been several attempts to formulate solutions to the problem which may categorised into feature and area based correspondence solutions [2]. Both methods, however, suffer from similar forms of the uncertainty principle [6]. With edge based stereoscopic algorithms, the presence of edge information alone yields considerable ambiguity, since edges themselves are difficult to distinguish, and can also occur spuriously within an image owing to the presence of noise, and the complexity of natural image data. In terms of the uncertainty principle we know *where* but not *what.* In contrast, area based correspondence algorithms usually apply normalised cross-correlation techniques to obtain measures of similarity between image functions. These techniques suffer from poor definition of window sizes that are used during the cross-correlation proceedure. This may also be considered as providing information equivalent to *where* but not *what.* We form the premise that at the lowest level of vision processing, *what* refers to both the local spectral properties (instantaneous frequency) and orientation of our image data and *where* is obtained through the resolutions of band-pass filtering. Marr [13], was also aware of the uncertainty principle. He proposed a coarse to fine edge based matching algorithm using the Laplacian of the Gaussian kernel whose zero-crossings can be interpreted

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as edges. The approach presented here, is an extension of Marr's work but instead of using one operator, we apply quadrature filter pairs and track phase differences. Fundamental to Marr's work, lies the notion of eye vergence. His work was criticised because of the nature of his vergence mechanism [2]. We propose an explicit method to drive optical vergence. Indeed, we find that many of the criticisms directed towards Marr's work can be explained by our theory, which we also extend to include mechanisms for computing both vertical and horizontal disparities.

2 The Gabor representation

The problem that Gabor addressed, was the simultaneous representation of a signal in both space and frequency, which has received extensive application within the field of image processing (e.g [18]). The measure of duration that Gabor used to formulate his Uncertainty hypothesis was based upon a minimisation of the second moment of an arbitrary signal in both space and frequency by an expansion of *elementary* signals:

$$\Psi(x,\omega_g) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{4}}} \exp[-\frac{(x-x_o)^2}{4\sigma^2}]\; cis(\omega_g x + \phi) \tag{1}$$

2.0.1 Phase from Gabor functions

Consider a pair of images, one of which has experienced a phase shift owing to horizontal disparities, we obtain the phase difference between image pairs $I_r(x)$ and $I_l(x)$ at x_o[9] by solving:

$$I_s = \mathcal{R}e[\Psi(x_o,\omega_g) * I_l(x_o)] \quad I_a = \mathcal{I}m[\Psi(x_o,\omega_g) * I_l(x_o)] \tag{2}$$

$$I_{s\phi} = \mathcal{R}e[\Psi(x_o,\omega_g) * I_r(x_o)] \quad I_{a\phi} = \mathcal{I}m[\Psi(x_o,\omega_g) * I_r(x_o)] \tag{3}$$

Let $I_l(X) = \cos\omega_o x$ and $I_r(x) = \cos\omega_o x + \phi$ represent our 1-D image function, then expanding equations (2) and (3), phase may be represented in terms of the rotation matrix: $R_{\theta\zeta}$ where $\zeta = \tanh(8\pi^2\sigma^2 u_o u_g)$ i.e:

$$\begin{bmatrix} I_{s\phi} \\ I_{a\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & -\frac{\sin\phi}{\zeta} \\ \sin\phi\;\zeta & \cos\phi \end{bmatrix} \begin{bmatrix} I_s \\ I_a \end{bmatrix} \tag{4}$$

Notice that the non-linearity in phase incorporates a shearing of the transformation matrix, with a phase difference found from:

$$\phi = \tan^{-1}(\frac{\frac{I_{a\phi}}{\zeta}}{I_{s\phi}}) - \tan^{-1}(\frac{\frac{I_a}{\zeta}}{I_s}) \tag{5}$$

and the disparity estimate given by:

$$D_{est} \approx \frac{\phi}{2\pi u_g} = \frac{u_o}{u_g} d \tag{6}$$

Naturally, this interpretation of disparity contains an implicit error based on the differences between the centre frequency of the Gabor function and the image signal [11]. We can ensure that the Gabor function approximates a linear phase filter by assuming $u_o = u_g$ and $8\pi^2 u_o u_g \sigma^2 \geq 8.0$ Consider the argument from $\Psi(x) * I(x)$ which may be defined as the instantaneous phase of a signal expressed as:

$$F(x) = \tan^{-1}(\frac{I_a}{I_s}) = \tan^{-1}(\frac{\tan(2\pi u_o x + \phi)}{\zeta}) \tag{7}$$

Since the right hand side of the above equation is independent of the centre frequency from the Gabor filter. It can easily be shown that:

$$\frac{d[F(x)]}{dx} = \frac{2\pi u_o}{\frac{\sin^2(2\pi u_o x)}{\zeta} + \zeta\cos^2(2\pi u_o x)} \tag{8}$$

Providing $\zeta = 1.0$ then the derivative of the phase response may be considered as an estimate of the instantaneous frequency of a signal. This may be used to improve the estimation of phase differences with little extra cost in computation, and without regard to the energy response.

3 The method of phase differences in 1-D

The method of phase differences has been applied to stereoscopic computation by several authors (Wilson and Knuttson,1987;Larcombe,1984; and Langley and Atherton,1988). Miller [15] , also implemented the same principle in hardware by the application of a Phase-Locked Loop. The approach is based upon the transformation of a real intensity signal into its analytic form by the addition of its quadrature counterpart. We therefore form the complex 1-D image function ($\mathbf{z}(x) = \Psi(x,\omega_o) * I(x)$):

$$\mathbf{z}(x) = E(x)\exp[j\phi(x)]$$

which at any one instance may be represented more succinctly in terms of the rotation matrix ($\mathbf{R}_\phi$) as:

$$\mathbf{z}_l = \mathbf{R}_\phi \mathbf{z}_r + \mathbf{n} \tag{9}$$

and $\mathbf{n}$ represents additive quadrature noise which receives contributions from sensor noise, and discontinuities in the disparity field. General closed-form solutions to functions of the type presented in equation (9) requires the probability density function (PDF) of the phase error. Unfortunately such a PDF is not yet known, but may be approximated by the Tichonov density [5], which is derived from the first order phase-locked loop whose input is the sum of Gaussian noise and a sinusoid. While it is possible to make quantitative statements regarding sensor noise, it unlikely that discontinuities in the disparity field may be equally represented. We must therefore, resort to smoothing to reduce the effects of noise under these conditions. We define the phase difference between two image functions at a spatial position (x) as the roots of:

$$\phi_l(x) - \phi_r(x_o) = 0 \tag{10}$$

Suppose that x is an exact root of the problem we require to solve, then expanding the above equation as a Taylor series with $x = x_o + d$ as an exact solution we have:

$$\phi_l(x_o + d) - \phi_r(x_o) = 0$$

$$= \phi_l(x_o) - \phi_r(x_o) + d\phi_l'(x_o) + \ldots\ldots$$

let us then make d_1 an approximation to d in which case we have:

$$d_1 = -\frac{-[\phi_l(x_o) - \phi_r(x_o)]}{\phi_l'(x_o)}$$

from which we iterate to find the root of our initial equation by:

$$x_n = x_{n-1} - \frac{\phi_l(x_{n-1}) - \phi_r(x_o)}{\phi_l'(x_{n-1})} \tag{11}$$

which is in fact Newton-Raphson convergence problem with d_n as our current disparity estimate. We can immediately state that convergence from this method can only occur providing the new estimation of the root to x_n lies between the previous estimate x_{n-1} and the exact solution x. In addition, we can also state that the **sgn** of the derivative of phase must also correspond in each image pair, otherwise the method does not converge into a stable solution. We propose to reduce the phase error by iteration in the same manner as a Phase-Locked Loop and additionally resolve the aliasing problem. By the aliasing problem we refer to the difficulty in obtaining large disparity estimates from filter pairs tuned to high spatial frequencies.

4 A mechanism for eye vergence

For our purposes, eye vergence holds some interesting properties. By verging the eyes, we bring features of small spatial extent into nearer correspondence, with the reduced possibility of aliasing and disparity error. Initiating a vergence also increases the effective disparity range that each filter can detect, and thereby reducing the need for filters tuned to the very lowest spatial frequency elements. Vergence is simply obtained from coarse to fine resolutions by:

- Obtain the mean disparity estimates from the responses by convolving an Image function with quadrature Gabor filters and weighting the disparities by their associated energy responses summed from both image pairs.
- Induce a vergence mechanism based upon the mean disparity and progress to the next resolution of filtering.

Our measured mean disparirity is therefore taken from:

$$D_{mean} = \frac{\sum_{x=-N}^{x=N} P_x d_x}{\sum_{x=-N}^{x=N} P_x} \tag{12}$$

Where $P_x = \sqrt{E_l(x)E_r(x)}$ represents the product of energy from the left and right image image pairs respectively, and we convolve from -N to N pixels at a given resolution from the optical centre. A useful indication for the choice of N is the spatial standard deviation of the applied filter, at the central point of the image. For compuational purposes, we may rather our image pairs were fused at some mean disparity. To this end, we suggest increasing the size of N to incorporate the whole image array for all resolutions of filters. Thus we would anticipate arriving at the mean least squares estimate of disparity between the two image functions.

5 Reducing the effect from quadrature noise

The mechanism of subtracting phase, is unfortunately highly unstable, and particularly sensitive to aliasing problems. Wilson and Knuttson [19], suggest that phase can be damped by a modification of the Willsky [17] error measure on a circle:

$$\eta = \frac{1}{2}[1 + \cos(\phi_d)]$$

This term, however, is not sufficient with a 1-D filter, particularly when iterating phase diffences as a phase-locked loop. Fundamental to stability lies the notion of subsequently smoothing the disparity estimates at a given resolution. We know from Papouillis [16] that if we have a signal:

$$g(x) = g_f(x) + g_n(x)$$

where $g_n(x)$ is the output from a linear system, whoes input n(x) is white noise, then for a constant $g_f(x)$ signal, the minimum mean-square estimation error of g_f smoothed with a window $w(x)$, is obtained if $w(x)$ is the truncated parabola given by:

$$w(x) \;=\; \frac{3}{4X}[1-(\frac{x-x_o}{X})^2]p_X(x-x_o)$$

where $p_X(x-x_o)$ is a pulse of width 2X. We argue that this is indeed a suitable filter to apply to the disparity estimates because shear and compression/expansion differences between stereo images significantly alter the local spectral properties, which can be difficult to interpret by this method alone. We are therefore only relying on this method to obtain approximate correspondence. Higher orders of disparity differences requires the examination of both orientation and curvature differences between image pairs [8] with the former reducing to differences in the first two moments taken from a band of filters applied in a circle. We must also consider the presence of negative frequency which is an unusual form of aliasing [10] (e.g fig. 1). Its occurence may be reduced by either applying a first order quadrature pair (e.g the derivative of the Gabor function) or by removing the mean d.c level from the image data. In addition, subtracting the mean intensity also reduces the large energy bias often observed at the extrema of image data, and reduces the instability of phase owing to edge effects. We therefore propose to add a recursive weighting to our phase locking iteration: We form a measure Q based upon:

$$Q(x) \;=\; \frac{1}{4}[1 \;+\; \cos(\phi_d)]^2 \tag{13}$$

As the square of the Willsky error measure. We choose the higher order to reduce the effects from noise. At a given scale of the iteration, we define our disparity as:

$$\bar{D}_{k+1}(x) \;=\; \frac{Q_k(x)\bar{D}_k(x) \;+\; Q_{k+1}(x)(D_{k+1}(x)+\bar{D}_k(x))}{Q_{k+1}(x)+Q_k(x)} \tag{14}$$

Where the measure k refers to the k_{th} resolution of bandpass filtering, and $\bar{D}_{k+1}(x)$ refers to the measurement of disparity from the recent update measured by $D_k(x)$. Thus we are updating our measurements based on the *goodness of fit*, at successive resolutions. To preserve a recursive nature, we re-arrange and modify the above expression to obtain:

$$\bar{D}_{k+1}(x) \;=\; \bar{D}_k \;+\; \frac{Q_{k+1}(x)}{Q_{k+1}(x)+Q_k(x)}D_{k+1}(x) \tag{15}$$

Where we redefine Q_{k+1} to function recursively as:

$$Q_{k+1} \;=\; \frac{Q_k}{2} \;+\; \frac{\bar{Q}_{k+1}}{2}$$

Which we propose holds the properties that we require. The combination of both damping at a given resolution of filtering, and between scales can overdamp the phase locking proceedure. To compensate for this, we propose incorporating the vergence mechanism.

5.0.2 Disparity interpretation from local spectral analysis

An examination of equation (11) indicates that the local derivative of phase (instantaneous frequency) from either image function can be used to interpet disparity. To resolve this ambiguity, we suggest taking a weighted mean (f_{av}) from both image pairs to interpret disparity:

$$f_{av}(x) \;=\; \frac{E_l^2(x)f_l(x) \;+\; E_r^2(x)f_l(x)}{E_l^2(x) \;+\; E_r^2(x)} \tag{16}$$

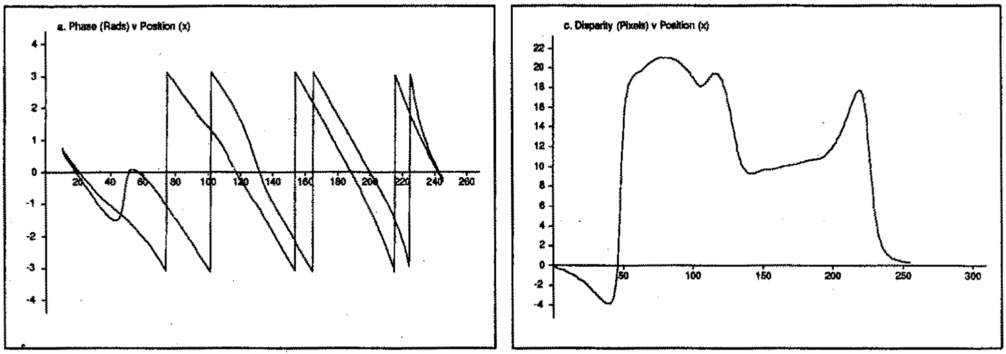

Figure 1: *Phase response from convolution with filters tuned to 1/64 cpp of raster line 110 from figure 2. Actual disparity varied from 25 pixels (lamp) to 10 pixels (background). (a) Left and Right image response superimposed. (b) Disparity interpreted from centre frequency of Gabor function.*

Where f_l and f_r correspond to the instantaneous frequency of both left and right images. This interpretation does suffer from the problems associated with stationary phase since we place a pole into our equation for disparity interpretation. At present, we threshold the data based upon the upper and lower cut-off frequency characteristics of our filter pairs. In addition, we also impose a further threshold restricting the difference between image spectral properties of 1.25 octaves, which we obtain from the disparity gradient hypothesis. In this case, smoothing also acts as a crude interpolator within regions that are not-analytic by this method.

6 The Stereoscopic Aperture Problem

Extending the work presented here into the 2-D case is interesting. In particular, we apply orientationally selective filters using the Compact Pyramid [4] code to reduce our computational load. Unfortunately, extending the method of phase differences into 2-D poses a dilemma. While in principle, it is possible to obtain unrotated phase differences which are resolved both horizontally and vertically [11], this implicitly also introduces a similar form of the motion aperture problem, which in this case is a direct consequence of orientationally selective filtering. Similarly, we can only resolve this ambiguity in image regions which are not restricted to singly directional signals. As is well established [1], corners provide the best image regions for resolving the aperture problem. Fortunately[12], there are methods for determining corner and edge confidence measures using directional filtering. We merely require the linear Fourier transform taken from the circle of energy responses from our orientationally selective filters. We have by Parseval's theorem a probabilistic measure for both "cornerness" ($P_c - P_e$) and "edgeness" (P_e) from:

$$P_e = \frac{[\sum_{i=0}^{N} E_i^2 \cos 2\theta_i]^2 + [\sum_{i=0}^{N} E_i^2 \sin 2\theta_i]^2}{[\sum_{i=0}^{N} E_i^2]^2} \quad ; \quad 0 \le P_e \le 1 \tag{17}$$

$$P_c = \frac{[\sum_{i=0}^{N} E_i^2 \cos 4\theta_i]^2 + [\sum_{i=0}^{N} E_i^2 \sin 4\theta_i]^2}{[\sum_{i=0}^{N} E_i^2]^2} \quad ; \quad 0 \le P_c \le 1 \tag{18}$$

which may also be modified to obtain the orientation of edges and corners. Edge orientation from directional filtering is attributed to Knuttson et al [7]. Equations (17) and (18) normalise the energy responses from the band of filters for both corner and edge detection. However, to distinguish between both corners and edges, we must also apply:

$$C_{orner} = P_c - P_e \quad (19)$$

Because the linear Fourier transform from the circle of edge energies.also gives significant energy contributions to the corner measure. Fortunately, the converse is not the case. The calculation of disparity may be obtained by solving a weighted least-squares fit from a band of filter's energy responses, and phase differences applied in a circle at the same pixel location, i.e;

$$\mathbf{P}\ \mathbf{\Phi D} = \mathbf{P}\ \mathbf{d} \quad (20)$$

Where P is the MxM diagonal matrix of the energy responses from the i^{th} filter in a circle whose leading elements are formed from $P_{ii} = \sqrt{E_{il}E_{ir}}$. $\Phi = [\cos\phi_i, \sin\phi_i]$ is the Mx2 matrix of directional orientation, and $\mathbf{d}$ represents the vector of measured disparity at all M orientations. E_{il} and E_{ir} represent the power response of the i^{th} filter in both image domains, and: $\mathbf{D} = [D_x, D_y]^T$ represents the unknown disparity estimate with both horizontal and vertical components. The above equation may be solved by numerical methods related to over determined sets of equations. We apply equation (20) to obtain the horizontal component of disparity. In the presence of dominant edge signals, there will be an interpretive error in horizontal disparity estimation that varies as a cosine of edge orientation. In this case, tracking horizontal disparities on the assumption of the epipolar constraint does not retain useful vertical disparities. This is the consequence of the aperture problem. It is possible to extract both vertical and horizontal disparities with great confidence at image regions exhibiting measures of cornerness. We turn to the Neurophysiologists for some assistance to this problem. Maske et al [14], studied the response patterns from orientationally selective cells in the striate cortex of the cat. They were interested in investigating the claims by Bishop and Pettigrew [3], that only cells with preferred stimulus orientation near to the vertical can make significant horizontal disparity interpretations. They showed that cells that were sufficiently end-stopped can make precise horizontal disparity discriminations, independent of the optimal stimulus orientation. They also showed, that end-free cells were only *effective* for the measurement of horizontal disparity providing their preferred orientations were near to the vertical. Alternatively, the end-stopped cells that were sensitive to disparity measurement showed no such preferences for orientation in their ability to interpret disparity information. Interestingly, a receptive field profile similar to the end-stopped cell can be produced from the linear sum of two orthogonal but orientationally selective Gabor functions [12], which is therefore ideally suited to respond in image regions from which the stereoscopic aperture problem can be solved. Thus it might appear, that the human visual system has also evolved to deal with this issue. There are many schemes that can be proposed to model these observations. We present the following:

$$D_h = P_e D_{hx} + (1 - P_e) D_x \quad (21)$$

Where D_h is the measure for horizontal disparity at a given scale and position in the image function. D_{hx}, D_x represent the horizontal estimates for disparity in the presence and absence of edge information respectively. We obtain D_{hx} from a separate calculation forming:

$$D_{hi} = \frac{d_i}{\cos(\theta_i)} \ ; \ \theta_i \neq \frac{n\pi}{2} \ , \ n = 1, 3...$$

Where $\cos\theta_i$ represents the orientation (frequency domain) of the ith filter from which disparity is interpreted. We would then anticipate estimating the final disparity estimate in a least squares sense from:

$$D_{hx} = \frac{\sum_{i=-\frac{\pi}{4}}^{\frac{\pi}{4}} P_{ii} Dhi}{\sum_{i=-\frac{\pi}{4}}^{\frac{\pi}{4}} P_{ii}} \quad (22)$$

We choose this form, because we have observed that edge information gives rise to a particularly large energy response. Since the aperture problem is a consequence of directional image energy, we suggest it is sufficient to detect the presence and absence of edge information only. This has the advantage of not requiring a description to any arbitrary response pattern (textures) that may occur from the circle of filters. The vertical component of disparity is then similarly obtained from:

$$D_v = (1 - P_e)D_y \qquad (23)$$

where the subscript 'v' denotes a vertical component. Thus vertical components of disparity are only evaluated in the absence of edges.

7 Results

We present results from our algorithms to the "room" image for both 1-D techniques, and the extension into 2-D. It is apparent, that the process is capable of obtaining very large disparity estimates with little difficulty. However, the algorithm is severely dependent on the spectral properties contained within the image pairs. In particular with a 1-D filter, it is difficult to obtain phase locking from features of high spatial frequency elements but large disparity differences. We observe that coarse features can easily be identified in disparity space with very large disparity differences. We also present a depth image (intensity proportional to disparity) based on the interpretation of disparity from the local derivative of phase and the threshold constraints that we have also proposed (fig. 4). Our final depth image was obtained from orientationally selective 2-D filters (fig. 5), using the least squares measure to interpret horizontal disparity only.

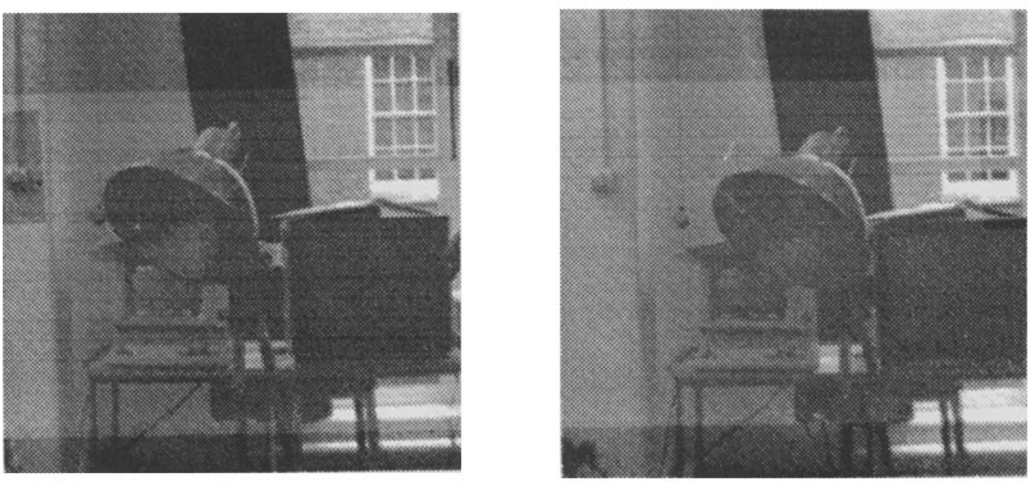

Figure 2: *Images taken from a room with a 512x512 CCD camera. Data was compressed to 256x256 using the compact Pyramid. (a) Left image (b) Right image. Disparity differences ranged from 25 (central lamp) to 4 pixels (outside window).*

Figure 3: *Intensity depth image produced using the centre frequency of the Gabor function to interpret disparity.*

Figure 4: *Intensity depth image produced using the derivative of phase as a measure of instantaneous frequency to interpret disparity.*

Figure 5: *Intensity depth image produced using centre frequency of the Gabor function to interpret disparity with a least squares estimate of disparity taken from 8 orientationally selective filters.*

8 Discussion and conclusion

From our results, it is clear that the method of phase differences is particularly suited for obtaining stereoscopic correspondence from large image features, or constant disparities accross an image field. Under these conditions, accurate results can be obtained for very large disparities, limited only by aliasing. To reduce this problem, we have proposed a vergence mechanism. In the presence of noise, the results cannot be exact since noise affects the response of a phase sensitive process. Indeed, even with noiseless image data, the method of phase differences has difficulty should the local spectral properties between image pairs differ significantly as a result of the transformation between image pairs. Features with small spatial extent, but large disparity differences also provide difficulty with this method. These problems may be reduced by the vergence mechanism, which brings features into approximate correspondence, and also by applying 2-D filters with orientation preferences. By the nature of applying orientationally selective filters, we are able to obtain estimates for vertical disparity. However, this implicitly also introduces a stereoscopic aperture problem. One way in which this problem may be disambiguated is by the detection of corner features. Fortunately, this is also possible with the same filters that we use to interpret disparity. In the case of single edges, however, the aperture problem is particularly undesirable for edges that approach a horizontal orientation. We have therefore proposed a scheme which obtains a horizontal disparity in the presence of edge information alone, while vertical disparity is restricted to image regions absent of single edges.

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