Measuring the Natural Output Level by DSGE Models: An Empirical Investigation for Switzerland[a]

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> The "natural rate of unemployment", in other words, is the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actual characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labor availability, the costs of mobility and so on.
>
> Milton Friedman (1968)

1. Introduction

Modern microfounded macroeconomic models have incorporated into a dynamic stochastic general equilibrium framework imperfect product and factor markets along with nominal wage and price rigidities. As the departures from perfect competition and the Walrasian general equilibrium system is made explicit, these models allow for rigorous, although model-based, definitions of the level of *potential output* and the level of *natural level of output*. The former being the level of output that would prevail under perfect competition and no price or wage rigidities; whereas the latter being the level of output that would prevail under imperfect competition, but with flexibel prices and wages. Thus, the two concepts are quite different and should therefore be clearly distinguished.[1] It is

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1 Here we follow the terminology of Justiniano and Primiceri (2008). In order to avoid confusion the level of potential output might also be labeled as the level of efficient output.

also clear that the latter concept is closer to Friedman's original definition.[2] This conceptual issue is not only of theoretical importance, but can also be relevant empirically as pointed out by McCallum (2001, p. 261).

In this paper we take a clear stand and follow the lead of McCallum and Nelson (1999) and McCallum (2001) to estimate a model-based natural level of output. For this purpose, we specify a fully fledged DSGE small open economy model. This model was taken from DelNegro and Schorfheide (2008) and Lubik and Schorfheide (2007) and is itself a simplified version of the small open economy model of Galí and Monacelli (2007). The model consists of an open economy IS curve, a New Keynesian Phillips curve and a Taylor rule. In contrast to DelNegro and Schorfheide (2008) and Lubik and Schorfheide (2007), we allow for habit persistence and indexation. In this model, the natural level of output is defined as the output for which the deviation of marginal costs from its steady state level is zero. This allows us to define the output gap as percentage difference between actual and natural level of output. This is the entity we are actually interested in.

The model is estimated for Swiss data using the Bayesian econometric techniques as exposed by An and Schorfheide (2007) and Fernández-Villaverde (2009). The aim is to update an a priori distribution over the parameters using sample observations to arrive at the posterior distribution. This involves several steps. First, for given parameters we have to solve a forward looking rational expectations model. Second, we apply the Kalman filter to compute the likelihood of the data. Third, we approximate the posterior distribution by a Monte Carlo Markov Chain (MCMC). An output of the Kalman filter is an estimate of the state vector at point t given the information up to and including period t. These so-called filtered values are then taken as our estimates for the output gap. Finally, we investigate the properties of our estimate and compare it to other measures. We also estimate different variants of the model to investigate the robustness of our results.

2 Compare the definition of the natural output with the definition of the natural rate of unemployment by Friedman (1968). See also the discussion in McCallum (2001).

2. Review of the Literature

There is a large empirical literature on the estimation of the output gap as well as the potential output because of its utmost importance for the assessment and conduct of monetary policy. This literature usually assumes that potential output evolves smoothly over time letting the output gap fluctuate with the business cycle. This ad hoc *identifying assumption* underlies most of the statistical filtering procedures, like HP-filter, band-pass filter, unobserved components, et cetera.[3] While the application of these procedures is straightforward, it proved to be difficult to come up with reliable estimates in real time due to end-of-sample problems. Therefore, some researchers have questioned the usefulness of these procedures for the conduct of monetary policy (see among others Orphanides and van Norden, 2002; or Watson, 2007). Others, like Hall (2005), have actually rejected the notion of a smoothly evolving potential output surrounded by business cycle fluctuations. This problem cannot be overcome by moving to multivariate filters.

An alternative to ad hoc filtering procedures has recently been provided by Justiniano and Primiceri (2008), Andrés et al. (2005), Neiss and Nelson (2005), and Edge et al. (2008). They estimate and analyze well-specified DSGE models using Kalman filtering techniques. These techniques make a clear distinction between the model equations given as state transition equations and the measurements available to the econometrician. Although the states (model variables) may be unobserved, like the natural output level in our case, the Kalman filter provides an optimal estimate of the states given the observations. The papers cited above apply this technique to US data using closed economy models. Here we make, for the first time, an attempt to use a small open economy DSGE model.

Although the estimation of open economy DSGE models ranks high on the international research agenda (see Lubik and Schorfheide, 2007; DelNegro and Schorfheide, 2008; or Justiniano and Preston, 2009, among others), there are, to our knowledge, only few applications to Switzerland. Notable exceptions are Beltran and Draper (2008) or Bäurle and Menz (2008). Thus, besides the estimation of the output gap, there is an independent interest with regard to the assessment of small open economy DSGE models for Switzerland.

Most of the literature concerning Switzerland followed the "trend approach" as in Assenmacher-Wesche and Gerlach (2008). Nevertheless, there are attempts to use more structural procedures. Bignasca and Rossi (2007) put forward a

3 See Mills (2003) for an introductory survey.

multivariate extension of the Hodrick-Prescott filter which allows to incorporate structural information like a backward looking Phillips curve and Okun's Law.[4] However, this procedure remains ad hoc and does not use a consistent and fully articulated model of the Swiss economy. In addition, there is also a production function approach in place (see Stalder, 2002, Section 2). Although this approach is economically based, the estimates correspond to the concept of potential output and not to Friedman's natural rate.

3. The Model

Our model falls within the class of small open economy models put forward by Galí and Monacelli (2007). Models in this framework can be written in the canonical form of New Keynesian models as presented and analyzed in detail by Woodford (2003). In its simple form, openness only influences the slope coefficients but plays otherwise no independent role. More elaborate models with a more prominent role for openness have been discussed, among others, by Monacelli (2005), Justiniano and Preston (2009) or Beltran and Draper (2008).

As our goal is mostly empirical, we implement the model proposed by Del Negro and Schorfheide (2002) which is itself a simplified version of Galí and Monacelli (2007).[5] The building blocks are a two-stage production sector with a perfectly competitive final goods sector and a monopolistic intermediate goods sector combined with Calvo-price setting behavior. With respect to the open economy, we assume that the law of one price for the foreign good holds and that the asset markets are complete enough to allow for perfect risk sharing with the rest of the world. The model actually consists of three behavioral equations: a dynamic IS-equation, a new Keynesian Phillips curve (NKPC) and a monetary policy rule (MP-rule). However, we add external habit persistence in consumption and price indexation to improve the fit of the model. These features are added in an ad hoc manner as in Ireland (2004) and are not grounded in terms of microeconomic foundations.[6]

We give a brief exposition of the equations of the model. Denote by y_t the percentage deviation of actual output from trend output and by y_t^n the corresponding

4 See Gerlach and Smets (1999) for a similar approach at the European level.

5 See also Lubik and Schorfheide (2007) and Bäurle and Menz (2008).

6 A rationale of these features based on microeconomic arguments can be found, for example, in Justiniano and Preston (2009).

natural level of output. Thus the output gap is defined as $x_t = y_t - y_t^n$. The trend output is driven by a world wide technological process $\{Z_t\}$ with the property that $\ln Z_t \sim I(1)$ with

$$\begin{aligned} z_t &= \Delta \ln Z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \\ \varepsilon_{z,t} &\sim \mathrm{WN}(0, \sigma_z^2). \end{aligned} \tag{1}$$

In the following, all quantitative variables are deflated by Z_t and measured as percentage deviations from the steady state.

The dynamic small open economy IS-equation with exogenous habit formation is written with respect to the output gap:

$$\begin{aligned} x_t - h x_{t-1} = {} & (1-h)\mathbb{E}_t x_{t+1} \\ & -(1-h)(\tau+\lambda)[R_t - \mathbb{E}_t(\pi_{H,t+1} + z_{t+1})] \\ & -\alpha(\tau+\lambda) h q_t \end{aligned} \tag{2}$$

where $0 < \alpha < 1$ denotes the share of foreign goods in the CPI and where $0 < h < 1$ measures the amount of habit formation. λ is a composite parameter equal to $\alpha(2-\alpha)(1-\tau)$ where τ, $0 < \tau < 1$, is the intertemporal elasticity of substitution in consumption. The restrictions on the parameters α and τ imply that $0 < \lambda < 1$. For $h = 0$, we obtain exactly the specification in DelNegro and Schorfheide (2008, equation (3)). Thus, according to the dynamic IS-equation, the contemporaneous output gap is negatively related to the real interest rate $R_t - \mathbb{E}_t \pi_{H,t+1}$, computed with respect to inflation of home produced goods $\pi_{H,t}$. If in addition $\alpha = 0$ we obtain the standard specification for the closed economy. Note that the terms of trade q_t, defined as the relative price of home produced goods in terms of imported goods, enter the dynamic IS-equation only through habit formation.[7]

The NKPC with indexation is defined in terms of $\pi_{H,t}$, the inflation with respect to home produced goods:

$$\pi_{H,t} = \gamma_b \pi_{H,t-1} + \gamma_f \mathbb{E}_t \pi_{H,t+1} + \frac{\kappa}{\tau+\lambda} x_t \tag{3}$$

7 The dynamic IS-equation above was actually derived by adding habit formation in DelNegro and Schorfheide (2008, equation (A41)) leaving all other equations unchanged.

In the NKPC the parameters are functions of the underlying preference and technology parameters:

$$\gamma_b = \frac{\omega}{1+\beta\omega},\ \gamma_f = \frac{\beta}{1+\beta\omega},\ \text{and } \kappa = \frac{(1-\theta\beta)(1-\theta)}{\theta},$$

where β and ω denote the subjective discount factor and the fraction of firms that index their prices to lagged inflation. Because $0<\beta<1$ and $0<\omega<1$, the sum of γ_b and γ_f is strictly smaller than one, but close to one if β takes the usual values close to one. Thus, our specification of the NKPC does not satisfy the natural rate hypothesis and is therefore not immune to McCallum's criticism.[8] The slope of the Phillips curve depends on κ which is itself a function of θ, the probability that a firm cannot reoptimize prices in the current period.[9] Thus if κ becomes large, i.e. $\theta \to 0$, the Phillips curve becomes vertical and the price rigidities vanish.

The third behavioral equation captures the conduct of monetary policy by the central bank and consists of a Taylor rule:

$$\begin{gathered} R_t = \rho_R R_{t-1} + \psi_\pi \pi_t + \psi_x(\Delta y_t + z_t) + \psi_q q_t + \varepsilon_{R,t}, \\ \varepsilon_{R,t} \sim \mathrm{WN}(0,\sigma_R^2). \end{gathered} \tag{4}$$

The rule is rather general as it allows the monetary authority to respond not only to inflation but also to output growth given by $\Delta y_t + z_t$ and the terms of trade. The rule is subject to a white noise disturbance term $\varepsilon_{R,t}$. Alternatively, one could assume that the central bank follows an optimal policy rule and therefore does not respond to output growth but instead to the output gap. We think that our specification is more realistic because $\Delta y_t + z_t$ corresponds more closely to traditional measures of output gap. Moreover, this policy function is based on observed variables only.[10]

The assumption of perfect risk-sharing links consumption at home with consumption of the rest of the world. Together with the market clearing conditions for the domestically produced good and the foreign good this allows us to express the deviation of marginal costs from its steady state as a function of domestic and foreign output (see DELNEGRO and SCHORFHEIDE, 2008, equation (A39)).

8 See MCCALLUM and NELSON (1999) and ANDRES et al. (2005) for details.

9 See DEL NEGRO and SCHORFHEIDE (2008) and SCHORFHEIDE (2008) for details.

10 This strategy was also followed in DELNEGRO and SCHORFHEIDE (2008), LUBIK and SCHORFHEIDE (2007), and JUSTINIANO and PRESTON (2009).

Note that the steady state marginal costs are a function of the mark-up charged by monopolistic producers of intermediary goods. The natural level of output y_t^n is defined as the output which makes the deviation of marginal costs from its steady state level equal to zero. This leads to an expression of y_t^n in terms of world output y_t^*. It is actually proportional to world output y_t^*:

$$y_t^n = -\frac{\lambda}{\tau} y_t^*. \tag{5}$$

Note that the natural level of output is negatively related to world output.

The model is closed by specifying the law for the exogenous processes. World output is taken as exogenous and is assumed to follow an AR(1) process:

$$y_t^* = \rho_{y^*} y_{t-1}^* + \varepsilon_{y^*,t}, \quad \varepsilon_{y^*,t} \sim \mathrm{WN}(0, \sigma_{y^*}^2). \tag{6}$$

Although the terms of trade are in principle endogenous in the model, we follow Del Negro and Schorfheide (2002) and treat $\{q_t\}$ as an exogenous AR(1) process:

$$q_t = \rho_q q_{t-1} + \varepsilon_{q,t}, \quad \varepsilon_{q,t} \sim \mathrm{WN}(0, \sigma_q^2). \tag{7}$$

This assumption is dictated by practical considerations as the terms of trade are hard to explain empirically. Finally, we relate $\pi_{H,t}$ to CPI inflation π_t and to changes in the nominal exchange rate:

$$\pi_t = \pi_{H,t} - \alpha q_t; \tag{8}$$

$$\Delta e_t = \pi_t - \pi_t^* - (1-\alpha) q_t \tag{9}$$

where π_t^* denotes foreign inflation. Again, we assume an AR(1) process for foreign inflation:

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \varepsilon_{\pi^*,t}, \quad \varepsilon_{\pi^*,t} \sim \mathrm{WN}(0, \sigma_{\pi^*}^2). \tag{10}$$

Note that despite the openness of the economy, the terms of trade do not play an independent role neither in the IS equation nor in the NKPC. This is a consequence of the restrictive assumptions of the law of one price for imported goods. A relaxation of the assumption results in a less than complete pass-through and breaks the identity to the canonical representation of the New Keynesian model. A detailed analysis of transmission mechanisms involved is given in CAVALIERE (2007). The implications for monetary policy is discussed in MONACELLI (2005).

The above model was kept very simple and deliberately omits additional complications arising from a thorough treatment of habit formation. Indeed a derivation from first principle would result in a more involved formula of real marginal costs which would depend on habit persistence. Additional features, like the deviation from the law of one price for the foreign good or a nonzero labor supply elasticity, would complicate the model even more.[11]

Equations (1) to (10) form a linear rational expectations model with parameter vector

$$\Theta = (\beta, \alpha, \tau, h, \kappa, \omega, \rho_R, \psi_\pi, \psi_x, \psi_q, \sigma_R, \rho_q, \rho_z, \rho_{y^*}, \rho_{\pi^*}, \sigma_q, \sigma_z, \sigma_{y^*}, \sigma_{\pi^*}).$$

Although it is possible to reduce the number of equations by making appropriate substitutions, we stick to the larger model because it proved to be numerically more stable. It should be noted that the model may have indeterminate solutions depending on the values of parameters. A necessary condition for determinacy is that the response of the monetary authority to inflation is actually strong enough, i.e that $\psi_\pi / (1 - \rho_R) > 1$. Although we will not restrict the parameter space to ensure determinacy, we will verify that the solution implied by our estimates is unique.

4. Estimation

4.1 Estimation Strategy

We pursue a Bayesian estimation strategy as exposed in AN and SCHORFHEIDE (2007) or FERNÁNDEZ-VILLAVERDE (2009).[12] In contrast to a classical approach, the Bayesian approach is more flexible in incorporating information not contained in the estimation sample. This flexibility proved to be necessary because the direct application of maximum likelihood estimates to DSGE models of this type often resulted in relatively flat likelihood functions and led to parameter estimates which are at odds with information from other sources.

11 In an earlier version of the paper we reported results from a model with an elastic labor supply. As the implemented model did not exactly correspond to the theoretical one, we abstained from reporting results related to this more general specification.

12 See DEJONG and DAVE (2007) for a textbook presentation.

The Bayesian framework aims at estimating the posterior distribution of the model parameters given a prior distribution of these parameters and the data. The posterior distribution $p(\Theta \mid Y)$ of the parameters given the data collected in Y factorizes according to Bayes' rule into the product of the likelihood function of the data, $\mathcal{L}(Y \mid \Theta)$, and the prior, $\pi(\Theta)$, subject to a normalization constant $p(Y)$:

$$
\begin{aligned}
p(\Theta \mid Y) &= \frac{\mathcal{L}(Y \mid \Theta)\pi(\Theta)}{p(Y)} \\
&\propto \mathcal{L}(Y \mid \Theta)\pi(\Theta)
\end{aligned}
$$

where $Y = \{Y_1, \ldots, Y_T\}$ denotes the sample at hand and $p(Y)$ its probability. In Bayesian analysis, the term $p(Y)$ is viewed as a constant given the distribution of Θ. The estimation of the posterior thus needs two ingredients: the likelihood function and a prior distribution for the parameters. In this section we take $\pi(\Theta)$ for granted and concentrate on the computation of the likelihood and posterior distribution. The specification of the prior is tackled in the next Section 4.2.

The first step consists of rewriting the rational expectations model so that it can be solved by Sims' algorithm (see Sims, 2001). Denoting the state of the system in period t by X_t, the model is rewritten as

$$\Gamma_0 X_{t+1} = \Gamma_1 X_t + \Psi Z_t + \Pi \eta_t$$

where the underlying shocks are pooled into

$$Z_t = (\varepsilon_{q,t+1}, \varepsilon_{z,t+1}, \varepsilon_{R,t+1}, \varepsilon_{y^*,t+1}, \varepsilon_{\pi^*,t+1})'$$

and where

$$\eta_t = (x_{t+1} - \mathbb{E}_t x_{t+1}, \pi_{H,t+1} - \mathbb{E}_t \pi_{H,t+1})'$$

represents the expectation errors. The state vector X_t has more components than endogenous variables to account for the lags due to habit formation and indexation. Sims' algorithm then casts the solution of the rational expectations model into a first-order stochastic difference equation:

$$X_{t+1} = G X_t + H V_{t+1}. \tag{11}$$

The matrices $G = G(\Theta)$ and $H = H(\Theta)$ are nonlinear functions of the parameters of the system.

The relation of the state to the data is given by the measurement equation:

$$Y_t = FX_t \tag{12}$$

where F may also be a function of Θ. There is no constant because all data are demeaned.[13] Moreover, we assume that there are no measurement errors, so that the measurement equation (12) includes no stochastic term. Such a specification only makes sense if the number of structural shocks, five in our case, equals the number of observed variables. Under this premise, the DSGE model generates a non-singular error covariance matrix. Hence, there is no obstacle to likelihood estimation.

Given a parametrization and the state space representation of the model solution in equations (11) and (12), we use the Kalman filter to compute the likelihood of the data. To obtain the posterior mode, i.e. the maximand of $\ln \mathcal{L}(Y|\Theta) + \ln \pi(\Theta)$, we make use of Sims' line search algorithm, which takes cliffs due to the non-existence or non-uniqueness of a rational expectations solution into account.[14] We are then in a position to use the Random Walk Metropolis-Hastings algorithm to approximate the posterior distribution.[15]

4.2 Specification of Prior

The appropriate specification of the prior distribution represents a critical stage in the estimation process. The specification used in this paper is represented in Table 1 and is justified below. We choose the form of the prior distribution to conform with the parameter range. Thus, we take a gamma distribution for parameters with positive support, a beta distribution for parameters with a support between zero and one and a normal distribution for parameters with the real line as a support. For the standard deviations we specify an inverted gamma distribution. In each case the distribution is pinned down by assigning values for the mean and the standard deviation.

13 The means can be interpreted as the steady state solution of the model and are as such a function of the underlying parameters. As we do not employ this information, we may well use a demeaned version of the model.

14 The algorithm was first used in LEEPER and SIMS (1994).

15 See GEWEKE (2005) for a detailed exposition.

In order to avoid a parameter assignment close to the boundary of the admissible range, we do not specify β directly, but set the long run value of the real interest rate r^*. β is then computed according to the relation $\beta = e^{-r^*/400}$. Given an average real interest rate of 1.57 over the sample, we derive a β equal to 0.9961. α denotes the share of foreign goods in the CPI and represents a crucial parameter for the model. It has steadily increased over the sample and reached at present a value just below 30 percent. We take this value as the mean of our prior distribution and take a relatively small standard deviation. τ is the intertemporal elasticity of substitution of households. Following the literature, we take its mean to be 0.50. This leads to an approximate average value of λ equal to 0.255 and therefore of $\tau + \lambda$ of 0.755. Following previous evidence, we have taken the mean for the habit persistence parameter h equal to 0.5 with standard deviation of 0.2.

Assuming that prices are fixed on average between 4 to 5 quarters leads to an average value of θ between ¾ and ⅘ and therefore of κ slightly smaller that 0.1.[16] The mean of the indexation parameter ω is taken to be 0.5 with standard deviation equal to 0.2.

The response of the central bank to inflation must be rather strong in order to avoid indeterminate solutions to the model. We therefore assume a value of 1.50. The responses to output growth and the terms-of-trade are assumed to be normally distributed around zero. This, perhaps unusual assumption, has the advantage that it does not exclude a priori the possibility that the SNB does just take inflation and not output and/or the terms-of-trade into account. On a priori grounds, the inflation goal has priority in the policy of the SNB. Although the three-month LIBOR clearly has a unit root, we restrict its autoregressive coefficient ρ_R to be smaller than 1.0.

For the world productivity we took over the specification of Lubik and Schorfheide (2007) and assumed a rather small autoregressive coefficient with an innovation standard deviation twice as high. The specification of $\{y_t^*\}$ was deduced from the analysis of the real GDP growth of the OECD.

4.3 Data

We use quarterly data over the period 1997:2 to 2009:2 to estimate our model. This rather short sample is dictated by the change in the policy of the SNB by the year 2000. Estimates which incorporate a longer sample showed some signs

16 See Kaufmann (2009) for the corresponding microeconomic evidence.

Table 1: Prior Distributions of Parameters

	distribution	mean	standard deviation
r^*	Gamma	1.57	0.05
α	Beta	0.30	0.05
τ	Beta	0.50	0.10
h	Beta	0.50	0.20
κ	Gamma	0.10	0.05
ω	Beta	0.50	0.20
ρ_R	Beta	0.50	0.10
ψ_π	Gamma	1.50	0.20
ψ_x	Normal	0.00	0.40
ψ_q	Normal	0.00	0.20
σ_R	Inv. Gamma	0.50	0.20
ρ_q	Beta	0.50	0.20
ρ_z	Beta	0.20	0.10
ρ_{y^*}	Beta	0.50	0.10
ρ_{π^*}	Beta	0.80	0.20
σ_q	Inv. Gamma	1.50	0.50
σ_z	Inv. Gamma	1.00	0.20
σ_{y^*}	Inv. Gamma	0.50	0.20
σ_{π^*}	Inv. Gamma	3.00	0.20

of structural breaks so that we felt more secure using the shorter sample.[17] Our data consist of CPI inflation, the three month LIBOR, the percentage change of the terms-of-trade computed as the price of exported goods over the price of imported goods, the quarterly growth rate of real GDP, and the percentage

17 See PERRUCHOUD (2009) for an analysis of regime changes in the SNB's Taylor rule. His analysis suggests that the Taylor rule of the SNB is less prone to regime switches in our sample period than in the period before.

change in the nominal exchange rate. If monthly data were available, we have taken monthly averages to obtain quarterly data. Data are seasonally adjusted. Details are reported in the appendix.

4.4 Estimation Results

4.4.1 The Full Model

Given the prior specified above we are now in a position to estimate the posterior distributions of the parameters by applying the random walk version of the Metropolis-Hastings algorithm. In Table 2 we report in columns 3 to 5 their modes, means and standard deviations.[18] To check the robustness of the specification, we draw randomly alternative parameter constellations from the prior distribution as starting values. It turned out that we converge to a solution for 97 percent of the drawn parameter values. While we do not converge to the exactly same solution in all the cases, the range of estimated parameter modes is relatively tight. Although the application of the Bayesian approach faces some technical difficulties[19], a detailed analysis of our results suggests that the estimation delivers sensible and robust results.

The mean coefficient on openness turned out to be low. Its mean value decreased from 0.30 for the prior to 0.23 for the posterior mean. This value is close to the sample mean of the weight of foreign goods in the CPI. The distribution of the habit formation coefficient changed considerably. Its mean value decreased from 1/2 to 0.23. In contrast, the intertemporal elasticity of substitution τ changed only slightly.

The coefficients of the Phillips curve show that price indexation is not an important issue in Switzerland. This is not surprising given the very low inflation rates observed over the sample period. The response of inflation to the output gap turned out to be higher than assumed a priori. This corresponds to an average value of price stickiness of less than three quarters. Note that the coefficient of the output gap is considerably higher than the one reported in Bignasca and Rossi (2007, Table 1 on page 9) .

The Taylor rule showed a strong response to inflation as expected. The posterior mean of ψ_π equals 0.73 so that the necessary stability condition ($\psi_\pi / (1 - \rho_R) > 1$)

18 In order to save space, we do not present the plots of the posterior distributions. They are, however, available upon request.

19 See Beltran and Draper (2008) for a model which is similar to ours and Canova and Sala (2009) for more general concerns, in particular regarding identification issues.

is fulfilled. In addition, there is also a strong reaction to output growth so that the nominal interest rate is increased during booms and decreased in recessions. There seems to be no reaction to the terms-of-trade.

Table 2: Statistics of the Posterior Distributions of Parameters

		full model			pure IT			benchmark		
	prior mean	mode	mean	s.d.	mode	mean	s.d.	mode	mean	s.d.
r^*	1.57	1.57	1.57	0.05	1.57[a]	1.57[a]		1.57[a]	1.57[a]	
α	0.30	0.20	0.23	0.06	0.24	0.53	0.07	0.23	0.27	0.04
τ	0.50	0.61	0.43	0.08	0.59	0.39	0.04	0.59	0.36	0.07
h	0.50	0.23	0.24	0.13	0.44	0.50	0.00	0[a]	0[a]	
κ	0.10	0.29	0.32	0.09	0.16	0.10	0.02	0.37	0.34	0.09
ω	0.50	0.06	0.08	0.06	0.05	0.02	0.01	0[a]	0[a]	
ρ_R	0.50	0.49	0.43	0.09	0.45	0.16	0.04	0.48	0.38	0.09
ψ_π	1.50	0.73	0.81	0.15	0.79	0.85	0.04	0.75	0.85	0.14
ψ_x	0.00	0.16	0.19	0.09	0[a]	0[a]		0.18	0.28	0.08
ψ_q	0.00	0.01	0.02	0.04	0[a]	0[a]		0.02	0.05	0.04
σ_R	0.50	0.02	0.04	0.05	0.02	0.44	0.02	0.02	0.04	0.02
ρ_q	0.50	0.68	0.71	0.12	0.88	0.98	0.01	0.57	0.64	0.08
ρ_z	0.20	0.75	0.71	0.07	0.71	0.32	0.02	0.74	0.68	0.06
ρ_{y^*}	0.50	0.77	0.74	0.08	0.80	0.75	0.04	0.75	0.64	0.11
ρ_{π^*}	0.80	0.31	0.33	0.12	0.31	0.16	0.08	0.31	0.33	0.12
σ_q	1.50	1.49	1.65	0.40	1.77	1.84	0.36	1.36	1.56	0.34
σ_z	1.00	0.05	0.17	0.25	0.05	0.67	0.02	0.05	0.24	0.27
σ_{y^*}	0.50	10.04	2.07	1.05	5.57	0.48	0.13	7.67	1.03	0.65
σ_{π^*}	3.00	3.61	3.92	0.87	3.50	2.78	0.54	3.54	3.82	0.85
log likelihood			−304.12			−316.49			−302.23	

a calibrated

4.4.2 Alternative Models

In order to check the robustness of our results we estimated a series of alternative models. Here we report the results for two special cases. The first one is a model with pure inflation targeting (pure IT) where the SNB is supposed to react only to inflation. This implies a specification with $\psi_x = \psi_q = 0$. The second alternative is the benchmark model estimated by Lubik and Schorfheide (2007) and DelNegro and Schorfheide (2008). There is no room for habit persistence nor indexation in this model. Thus, in the benchmark model h and ω equal zero. The results for these alternative models are reported in columns 6 to 11 of Table 2.

There are only minor differences between the full model and the benchmark model. Most notably, the intertemporal elasticity of substitution τ is lower and the reaction to output growth in the Taylor rule is higher in the benchmark model compared to the full model. However, the differences are of minor magnitude and, as reported in Figure 4, have almost no effect on the output gap estimates.

The posterior means of the pure inflation targeting model differ quite substantially from the full model estimates. The degree of openness equals 0.53 and is distinctly higher than in the full model. Furthermore, the habit persistence parameter equals 0.50 and thus is more than twice of the value in the full model. The response to the output gap in the NKPC equals 0.10 and thus is more in line with micro-level studies. Nevertheless, even with these pronounced differences in parameters, the estimated output gap does not change substantially as reported in Figure 4. We also report the log likelihood of the models in Table 2. The higher likelihood of the benchmark model compared to the full model may seem surprising at first. Note, however, that the maximization is performed over the objective $\ln \mathcal{L}(Y|\Theta) + \ln \pi(\Theta)$ and not over $\ln \mathcal{L}(Y|\Theta)$.

4.5 Impulse Responses

In addition, it is instructive to examine the responses of the endogenous variables to various shocks. Selected impulse responses are displayed in Figures 1 and 2. These figures show the median response together with a 95-percent confidence interval. The confidence interval was computed by randomly drawing parameters from the posterior distribution, then solving the corresponding model by Sims' algorithm and using the Kalman filter representation to compute the impulse response. Repeating this 40'000 times allowed us to compute the appropriate confidence interval for each horizon separately.

As expected, a positive productivity shock raises significantly the output gap and output growth. The positive output gap then increases inflation. This development is counteracted with a lag by the SNB which raises the short term interest rate. The shock is more or less absorbed after 4 quarters.

An unexpected increase of the interest rate also shows the awaited responses. The output gap and growth decline on impact as does the inflation rate. This negative influence is reduced as the SNB brings the interest rate back to its original level. The cut in the interest rate then leads to positive output growth which brings back the system to its steady state. The effect of the shock disappears after 4 quarters.

5. Discussion of the Estimated Output Gap

Having estimated the posterior distribution, we are now in a position to provide estimates for the output gap. For this purpose we ran the Kalman filter with the parameters given by the mode of the posterior distribution. This procedure delivers the filtered values of the state vector $X_{t|t}$, i.e. the best linear prediction of X_t, in the mean squared error sense, given the observation up to and including period t. For comparison purposes we provide two alternative measures. The first one is derived from the standard Hodrick-Prescott filter (HP filter).[20] The second alternative is constructed from the Beveridge-Nelson decomposition (BN decomposition) implied by an AR(1) model for ΔlnGDP. This decomposition delivers the trend of the series (see MILLS, 2003, chapter 3.5; and NEUSSER, 2009, chapter 7.1) and thus the cycle can be constructed as the difference to the original series.[21] These three estimates are plotted in Figure 2. To check the reliability of our estimates we have also plotted the two alternative estimates together with the filtered output (see Figure 2). As a further alternative we examined the estimates derived from the production function approach as described by STALDER (2002)[22] and the unemployment rate. We report descriptive statistics in Table 3 and Table 4. In order to assess the robustness of our model-based estimates, we plot the output gap estimates for all three models in Figure 4.

From these figures we can see that our estimate of the output gap is very close to zero for most of the sample period. The estimates vary in a narrow range of ± 0.5 percent, the last two to three years and the end of the 1990's being an exception. In the end of the 1990's the output gap was slightly negative and turned into

20 Similar results are obtained when a band pass filter is used instead.

21 Both the HP filter and BN decomposition have been estimated over the period 1980:2 to 2009:2.

22 We thank the SNB for providing us with the corresponding data.

positive around the year 2000. This positive period was followed by a movement around the zero line which lasted until 2007. In this year the output gap dropped sharply, but became strongly positive in the following period. The most recent episode was marked by a sharp fall. However, by mid 2009 the output gap was again very close to zero. This is in sharp contrast to the other estimates which show a drastic fall during the financial crisis.

Thus, the volatility of the DSGE output gap turns out to be quite small. This suggests that, perhaps with the exception of the last two years, price rigidities did not play a major role in the Swiss business cycle. However, one could also interpret this finding as a sign for good monetary policy by the SNB or as a sign for the lack of bad shocks. Nevertheless, this result should not come as a surprise. Given that Swiss inflation was very low and relatively stable, the Phillips curve would predict an output gap close to zero.[23]

In Table 3 we provide some descriptive statistics of the four estimates and the unemployment rate. The main difference between our model-based estimate and conventional statistical estimates is the much lower volatility of the former. Whereas the model-based estimate has a standard deviation of only 0.21, the alternative estimates have a value which is more than four times as high. This reduced volatility goes together with a lower persistence as reflected in the autocorrelation coefficients.[24]

Table 4 shows the cross-correlation coefficients of our alternative estimates. The DSGE based measure of output and the HP filtered output are positively correlated. The correlation with the BN decomposition is also positive but less pronounced. The DSGE based output gap shows no correlation with the HP filtered estimate and the DSGE based output. However, it is negatively correlated with the BN decomposition. As already mentioned, the production function based measure is strongly correlated with the HP filter. The correlation with the unemployment rate is substantially negative for all estimates, except for the BN decomposition. These results suggest that the alternative output gap measures are more closely related to variations in potential output.

23 See Goodfriend (2007) and Goodfriend and King (1997) for a rationale of this finding.

24 Note, however, that the high autoregressive correlation may be spurious as shown by Harvey and Jäger (1993).

Table 3: Some Descriptive Statistics of the Output Gap Estimates (DSGE Estimates Based on Mode)

	DSGE based output gap	DSGE based output	HP filter	BN decomp.	production function	unemploy-ment rate
std	0.21	0.59	1.17	0.90	1.30	0.93
$\rho(1)$	0.49	0.80	0.84	0.72	0.90	0.88
$\rho(2)$	0.40	0.55	0.59	0.39	0.72	0.74

Sample: 1997:2 to 2009:2.

Table 4: Cross Correlations between Output Gap Estimates (DSGE Estimates Based on Mode)

DSGE based output gap	DSGE based output	HP filter	BN decomp.	unemploy-ment rate	production function
1.00	0.15	0.01	−0.51	−0.29	0.16
	1.00	0.78	0.52	−0.27	0.80
		1.00	0.32	−0.58	0.90
			1.00	0.22	0.23
				1.00	−0.55
					1.00

Sample: 1997:2 to 2009:2.

6. Conclusion

For the first time, to our knowledge, we provided an estimate of the output gap in a small open economy based on an explicitly specified DSGE model. While one can raise many objections concerning the details of the model or the specifics of estimation strategy, we claim that the model-based approach is the correct way to proceed. It provides a clear definition of the output gap and does not rely on ad hoc statistical identification schemes, like smoothness.[25]

25 See HALL (2005) for an explicit criticism of this assumption.

Our paper shows that the application of the Bayesian DSGE approach to Switzerland not only delivers convincing and promising results, but also allows the estimation of latent variables like the natural level of output or of the output gap. The estimate revealed that the output gap as defined in our model was, with the exception of the last two years, almost closed. As argued by Goodfriend (2007) and Goodfriend and King (1997) such a result has to be expected within the paradigm of the New Keynesian model for countries which were able to achieve price stability for a prolonged period of time.

From an empirical point of view, a closer investigation of the Taylor rule seems worthwhile. In particular, the reaction to output gap growth as given by the parameter ψ_x is at stake. One might also question the responsiveness of the SNB with regard to the terms of trade and/or the exchange rate. More generally, one should take up in more detail the concerns raised by Beltran and Draper (2008) and Canova and Sala (2009) about the identification of parameters and the inferential validity of the estimation process. As previously mentioned, the data were not informative with respect to all parameters. However, this does not call into question our main results.

To get a deeper understanding of the Swiss business cycle a more elaborate model is necessary. Such a model should give a more prominent role to openness by breaking the correspondence to the canonical New Keynesian model, thereby allowing for more interesting transmission mechanisms of terms of trade or exchange rate shocks.[26] Alternatively, one could relax the assumption of perfect risk sharing. The incorporation of such features would break or at least attenuate the negative relation of the natural output level and foreign output in equation (5). A further issue for future research relates to the adequate representation of the peculiarities of the Swiss labor market. In particular, the openness of the Swiss labor market together with the handling of quotas and regulations must be incorporated.

Finally, the model could benefit from the inclusion of more variables in our observation vector Y_t, like world output or a measure of labor input. As the number of variables would then be greater than the number of structural shocks, we would run into the problem of a singular prediction error covariance matrix which would make the computation of the likelihood function impossible. Although literature proposes some ways around this problem (for example by introducing measurement errors or by allowing more structural shocks), it remains open how to proceed in this particular case.

26 See Monacelli (2005) or Justiniano and Preston (2009) for examples of such models.

Appendix

Data

Series	Source	Description	Transformation	Relation to model
LIBOR	SNB	3 Month LIBOR CHF (publicly available only from 1989 on)		$4R$
CPI	SNB	Consumer Price Index (Total)	$100 \cdot \Delta \log(CPI_t / CPI_{t-4})$	4π
Prices of exported goods	SECO	Implicit chain price indexes of seasonally adjusted data (without valuables)		
Prices of imported goods	SECO	Implicit chain price indexes of seasonally adjusted data (without valuables)		
Terms of Trade (ToT)		Prices of exported goods over prices of imported goods	$100 \cdot \Delta \log(ToT)$	q
GDP	SECO	Chained values of seasonally adjusted Quarterly GDP in Mio. Swiss Francs, at prices of preceding year. The reference year is 2000.	$100 \cdot \Delta \log GDP$	$y_t - y_{t-1} + z_t$
Nom. exchange rate (ex)	SNB	Nominal Exchange rate index against 24 trade partners	$\Delta(ex)$	Δe_t
SNB:	Swiss National Bank			
SECO:	State Secretariat for Economic Affairs SECO			

Figure 1: Impulse Responses to a One Standard Deviation Productivity Shock with 95-Percent Confidence Interval

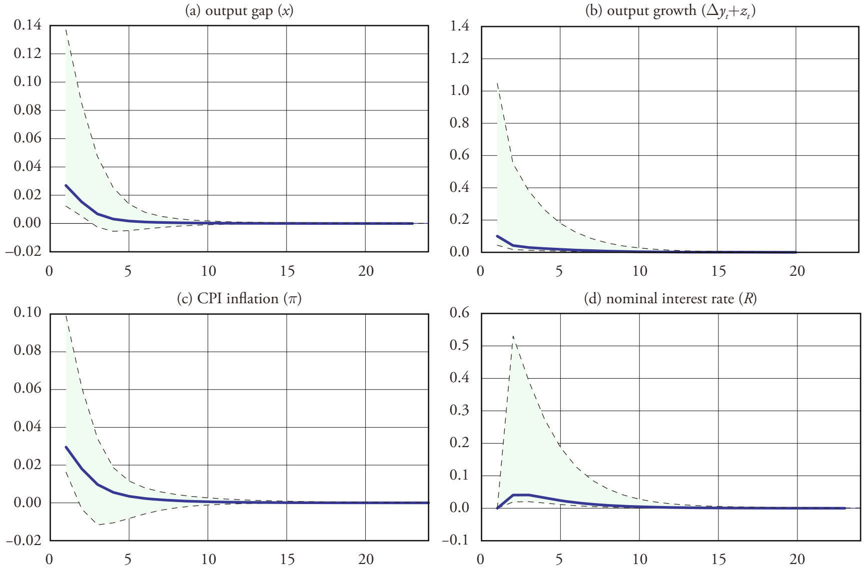


Figure 2: Impulse Responses to a One Percent Interest Rate Shock with 95-Percent Confidence Interval

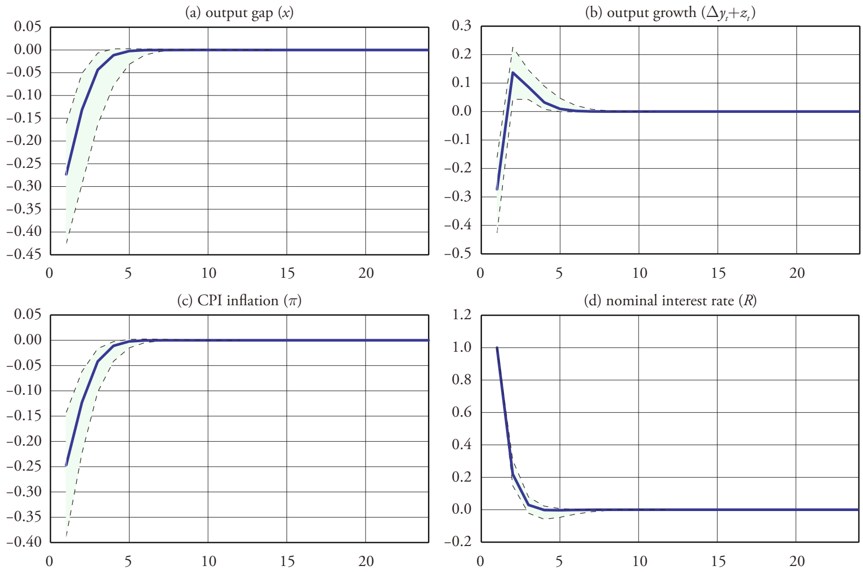


Figure 3: Comparison of DSGE Output Gap and DSGE Output of the Full Model (both Evaluated at the Mode of the Posterior Distribution) with Alternative Measures of Business Cycles

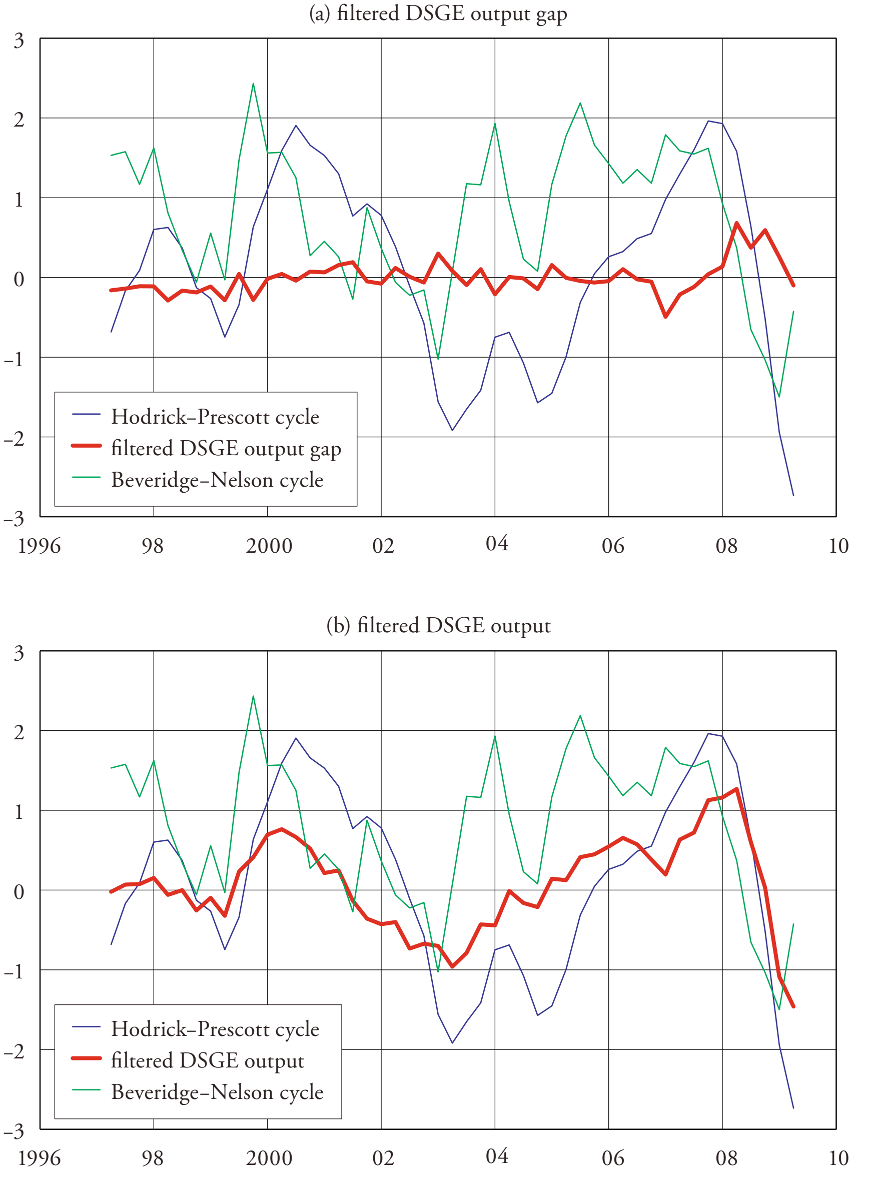
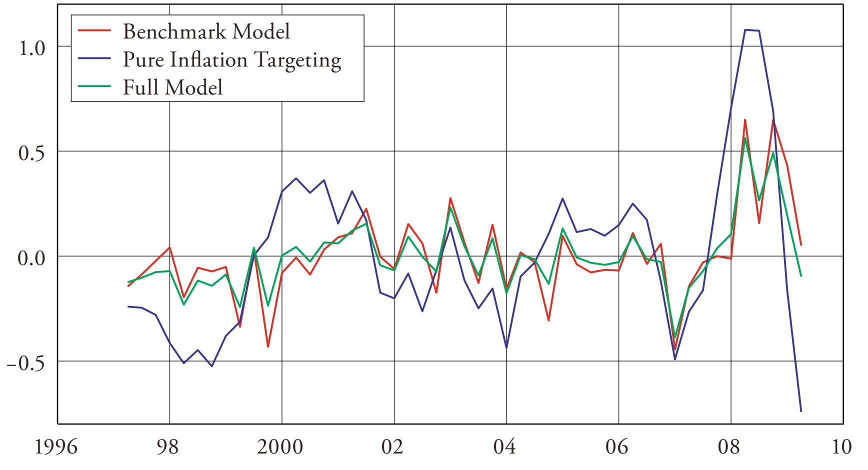


Figure 4: Median DSGE Output Gaps of All Three Models



References

An, S., and F. Schorfheide (2007), "Bayesian analysis of DSGE models", *Econometric Reviews*, 26, pp. 113–172.

Andrés, J., D. López-Salido, and E. Nelson (2005), "Sticky-Price Models and the Natural Rate Hypothesis", *Journal of Monetary Economics*, 52, pp. 1025–1053.

Assenmacher-Wesche, K., and S. Gerlach (2008), "Money Growth, Output Gaps and Inflation at Low and High Frequency: Spectral Estimates for Switzerland", *Journal of Economic Dynamics and Control*, 32, pp. 411–435.

Bäurle, G., and T. Menz (2008), *Monetary Policy in a Small Open Economy Model: A DSGE-VAR Approach for Switzerland,* Ph.D. dissertation, Department of Economics, University of Bern, Bern, Switzerland.

Beltran, D. O., and D. Draper (2008), "Estimating the Parameters of a Small Open Economy DSGE Model: Identifiability and Inferential Validity", International Finance Discussion Paper 955, Board of Governors of the Federal Reserve System.

Bignasca, F., and E. Rossi (2007), "Applying the Hirose-Kamada Filter to Swiss Data: Output Gap and Exchange Rate Pass-Through Estimates", Swiss National Bank Working Papers 2007-10, Swiss National Bank, Zürich.

Canova, F., and L. Sala (2009), "Back to Square One: Identification Issues in DSGE Models", *Journal of Monetary Economics*, 56, pp. 431–449.

Cavaliere, M. (2007), *Exchange Rate Pass-Through to Process: Characteristics and Implications*, Dissertation Series 8, Study Center Gerzensee.

DeJong, D. N., and C. Dave (2007), *Structural Macroecometrics*, Princeton University Press, Princeton and Oxford.

Del Negro, M., and F. Schorfheide (2008), "Inflation Dynamics in a Small Open Economy Model under Inflation Targeting – Some Evidence from Chile, in: K. Schmidt-Hebbel and C. Walsh (eds), *Monetary Policy under Uncertainty and Learning*, Central Bank of Chile, Santiago, Chile.

Edge, R. M., M. T. Kiley, and J.-P. Laforte (2008), "Natural Rate Measures in an Estimated DSGE Model of the U.S. Economy", *Journal of Economic Dynamics and Control*, 32, pp. 2512–2535.

Fernández-Villaverde, J. (2009), "The Econometrics of DSGE Models", PIER Working Paper 09-008, Penn Institute for Economic Research, Philadelphia, 2009.

Friedman, M. (1968), "The Role of Monetary Policy", *American Economic Review*, 58, pp. 1–17.

Galí, J., and T. Monacelli (2005), "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", *Review of Economic Studies*, 72, pp. 707–734.

Gerlach; S.; and F. Smets; (1999), "Output Gaps and Monetary Policy in the EMU Area", *European Economic Review*, 43; pp. 801–812.

Geweke, J. (2005), *Contemporary Bayesian Econometrics and Statistics*, Wiley-Interscience, Hoboken, New Jersey.

Goodfriend, M. (2007), "How the World Achieved Consensus on Monetary Policy", *Journal of Economic Perspectives*, 21, pp. 47–68.

Goodfriend, M., and R. G. King (1997), "The New Neoclassical Synthesis and the Role of Monetary Policy", in: B. S. Bernanke and J. J. Rotemberg (eds), *Macroeconomics Annual 1997*, volume 12, National Bureau of Economic Research, MIT Press, Cambridge, Massachusetts, pp. 231–283.

Hall, R. E. (2005), "Separating the Business Cycle from other Economic Fluctuations", in: *The Greenspan Era: Lessons for the Future*, Federal Reserve Bank of Kansas City, pp. 133–179.

Harvey, A. C., and A. Jäger (1993), "Detrending, Stylized Facts and the Business Cycle", *Journal of Applied Econometrics*, 8, pp. 231–247.

Ireland, P. (2004), "Technology Shocks in the New Keynesian Model", *Review of Economics and Statistics*, 86, pp. 923–936.

Justiniano, A., and B. Preston (2009), "Monetary Policy and Uncertainty in an Empirical Small Open Economy Model", *Journal of Applied Econometrics*, forthcoming.

Justiniano, A., and G. E. Primiceri (2008), Potential and Natural Output, 2008, manuscript.

Kaufmann, D. (2009), "The Price Setting Behaviour in Switzerland – Evidence from CPI Micro Data", *Swiss Journal of Economics and Statistics*, 145: forthcoming, 2009.

Leeper, E. M., and C. A. Sims (1994), "Toward a Modern Macroeconomic Model Usable for Policy Analysis", in: S. Fischer and J. J. Rotemberg (eds), *NBER Macroeconomics Annual 1994*, volume 9, National Bureau of Economic Research, Inc, MIT Press, Cambridge, Massachusetts, pp. 81–118.

Lubik, T. A. and F. Schorfheide (2007), "Do Central Banks Respond to Exchange Rate Movements? A Structural Investigation", *Journal of Monetary Economics,* 54, pp. 1069–1087.

McCallum, B. T. (2001), "Should Monetary Policy Respond Strongly to Output Gaps?", *American Economic Review (Papers and Proceedings)*, 91, pp. 258–262.

McCallum, B. T., and E. Nelson (1999), "Performance of Operational Policy Rules in an Estimated Semi-Classical Structural Model", in: J. Taylor (ed.), *Monetary Policy Rules*, University of Chicago Press, Chicago, pp. 15–45.

Mills, T. C. (2003), *Modelling Trends and Cycles in Economic Time Series*, Palgrave Texts in Econometrics, Palgrave MacMillan, New York.

Monacelli, T. (2005), "Monetary Policy in a Low Pass-Through Environment", *Journal of Money Credit and Banking*, 37, pp. 1047–1066.

Neiss, K. S., and E. Nelson (2005), "Inflation Dynamics, Marginal Cost, and the Output Gap: Evidence from Three Countries", *Journal of Money, Credit, and Banking*, 37, pp. 1019–1045.

Neusser, K. (2009), *Zeitreihenanalyse in den Wirtschaftswissenschaften*, Vieweg +Teubner, Wiesbaden, 2nd edition.

Orphanides, A., and S. van Norden (2002), "The Unreliability of Output-Gap Estimates in Real Time", *Review of Economics and Statistics*, 84, pp. 569–583.

Perruchoud, A. (2009), "Estimating a Taylor Rule with Markov Switching Regimes", *Swiss Journal of Economics and Statistics*, 145, pp. 187–220.

Schorfheide, F. (2008), "DSGE Model-Based Estimation of the New Keynesian Phillips Curve", *Economic Quarterly*, 94(4), pp. 397–433.

Sims, C. A. (2001), "Solving Linear Rational Expectations Models", *Computional Economics,* 20, pp. 1–20.

STALDER, P. (2007), "The Changing Role of Foreign Labor and Female Participation: Impacts on Wage-Price Dynamics and Unemployment in Switzerland", mimeo.

WATSON, M. F. (2007), "How Accurate Are Real-Time Estimates of Output Trends and Gaps?", *Economic Quarterly*, 2, pp. 143–161.

WOODFORD, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton, New Jersey.

SUMMARY

The output gap plays an important role in the assessment and conduct of monetary policy. Most of the current literature, however, relies on filtering procedures which use ad hoc smoothness arguments for identification. Furthermore, they are subject to end-of-sample problems and do not provide estimates of the output gap based on economic theory. In contrast, our model-based approach relies on a precise definition of the natural level of output and consequently of the output gap. This paper provides, for the first time, an estimate of the output gap based on a DSGE small open economy model for Switzerland. We use Bayesian econometrics to derive an estimate of the output gap, which is then compared to some alternatives. Except for the last years, our measure of the output gap is close to zero most of the time. This suggests that price rigidities play a minor role in the propagation of the Swiss business cycle. We show that our estimate produces sensible and robust results which encourage further research.