CORRECTION TO "ON QUASICONFORMAL MAPPINGS"

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(This Volume, pp. 1—58)

While the paper was in proof it was pointed out to me that the mappings of class $\overline{Q}_m(K)$ are not known to be topological, whereas in Theorems 10 and 11 this is tacitly assumed. This inconsistency is eliminated by noting that the theorems in question can be interpreted and remain true for arbitrary mappings in $\overline{Q}_m(K)$, topological or not.

For arbitrary $\zeta(z) \in \overline{Q}_1(K)$ the relation (33) is true if $S(r)$ is interpreted as the measure of the image of $|z| < r$. The proof goes through with one change. It can still be maintained that

$$\lim_{n \to \infty} \iint\limits_{|z|<r} (|p_n|^2 - |q_n|^2)\, dx\, dy = S(r),$$

provided that the image of $|z| = r$ has the area zero. From the fact that

$$\iint\limits_{C} (|p_n|^2 + |q_n|^2)\, dx\, dy$$

is bounded it follows that

$$\underline{\lim}_{n \to \infty} \int\limits_{|z|<r} (|p_n| + |q_n|)\, d\varphi < \infty$$

for a. e. r, and when this is so the image of $|z| = r$ has finite length and hence zero area. One concludes as in the text that (33) holds for a. e. r, and by continuity for every r.

Because of (33) null-sets are mapped into null-sets, and hence measurable sets go into measurable sets. Moreover, the set-function $m'(E)$, defined as the measure of the image of E, is additive on rectangles, say. This could not be so unless the set of points ζ with more than one inverse image were a null-set. It follows that $m'(E)$ is additive and absolutely continuous with the density $|p|^2 - |q|^2$ a. e.

From these remarks it is concluded that the integral $I_m(\zeta)$ is meaningful for any $\zeta(z) \in \overline{Q}_m(K)$, and Theorem 11 can be proved as before. At the end of Ch. VII it should be pointed out that a formal solution is eo ipso topological.

Another mistake occurs on top of page 41. In order to derive the third formula it is necessary to prove the second formula with a remainder of the form $o((|p|^2 - |q|^2)\varepsilon)$. In this form it does not follow from (39), but it is an immediate consequence of the relation

$$|p'|^2 - |q'|^2 = (|p|^2 - |q|^2)\left(\left|\frac{\partial H}{\partial z'}\right|^2 - \left|\frac{\partial H}{\partial \bar{z}'}\right|^2\right).$$

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CORRIGENDUM

This Volume, page 7, line 7: For $m \geqq a$ read $m \geqq \frac{a}{b}$.