Erratum to the paper « Impact Parameter Expansion of Fields ».

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In the indicated paper a computation error has been committed, therefore, sect. **4** should be replaced by the present one.

4. - Expansion of the Dirac field.

The Dirac field can be decomposed into « good » and « bad » components according to $\psi(x) = \psi^+(x) + \psi^-(x)$ with

$$\psi^+(x) = \frac{1+\alpha_3}{2}\,\psi(x)\,, \qquad \psi^-(x) = \frac{1-\alpha_3}{2}\,\psi(x)\,. \tag{4.1}$$

In terms of these components the Dirac equation takes the form

$$2i\frac{\partial}{\partial x^+}\,\psi^+(x) = \left(\frac{1}{i}\,\alpha^k\frac{\partial}{\partial x^k} + m\beta\right)\psi^-(x)\,, \tag{4.2}$$

$$2i\frac{\partial}{\partial x^-}\,\psi^-(x) = \left(\frac{1}{i}\,\alpha^k\frac{\partial}{\partial x^k} + m\beta\right)\psi^+(x) \qquad (k = 1, 2)\,. \tag{4.3}$$

Only the first of these equations is an equation of motion, because this contains a derivative with respect to time x^+ and, correspondingly, only $\psi^+(x)$ is considered as a dynamical variable. Equation (4.3) determines the subsidiary quantity $\psi^-(x)$, provided some boundary condition, *e.g.* $\psi^-(x)\to 0$ as $x^-\to-\infty$, is imposed. Then the equation of motion directly for $\psi^+(x)$ can be obtained by eliminating $\psi^-(x)$:

$$i\frac{\partial}{\partial x^+}\,\psi^+(x) = \frac{i}{4}\left(\frac{\partial^2}{\partial \boldsymbol{x}^2} - m^2\right)\int\limits_{-\infty}^{x^-}\mathrm{d}y^-\,\psi^+(x^+, y^-, \boldsymbol{x})\,. \tag{4.4}$$

In order to find impact parameter states, the generators of the Lorentz group are represented in the form of a sum of orbital and spin parts:

$$M_{\mu\nu} = M_{\mu\nu}^{\text{orb}} + M_{\mu\nu}^{\text{spin}}, \tag{4.5}$$

where $M_{\mu\nu}^{\text{orb}}$ is the generator used earlier for the scalar field and $M_{\mu\nu}^{\text{spin}} = (i/4)[\gamma^\mu, \gamma^\nu]$. In what follows Gell-Mann's representation of γ-matrices will be used:

$$\gamma^0 = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & i\sigma_1 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & -\sigma_3 \\ \sigma_3 & 0 \end{pmatrix}.$$

It has the advantage that good (bad) components are the two upper (lower) components of spinors.

According to (4.5), the operator of the impact parameter is also composed of orbital and spin parts:

$$B_k = \frac{2M_{+k}^{\text{orb}}}{\mu} + \frac{2M_{+k}^{\text{spin}}}{\mu} = B_k^{\text{orb}} + B_k^{\text{spi}} \qquad (k = 1, 2),$$

where

$$B_k^{\text{orb}} = i\frac{x^-}{\mu}\frac{\partial}{\partial x^k} + x^k$$

or in p-space

$$B_k^{\text{orb}} = i\frac{\partial}{\partial p^k}$$

and

$$B_1^{\text{spin}} = \frac{1}{2\mu}(-i\alpha_1 + \Sigma_2) = \frac{1}{\mu}\begin{pmatrix} 0 & i\sigma_1 \\ 0 & 0 \end{pmatrix}, \qquad B_2^{\text{spin}} = -\frac{1}{2\mu}(i\alpha_2 + \Sigma_1) = \frac{1}{\mu}\begin{pmatrix} 0 & i\sigma_2 \\ 0 & 0 \end{pmatrix}.$$

Therefore, in p-space the spinors ψ satisfy

$$\left(i\frac{\partial}{\partial p^k} + B_k^{\text{spin}}\right)\psi = (b_k^{\text{orb}} + b_k^{\text{spin}})\psi. \tag{4.6}$$

The solution can be found by the ansatz $\psi = \varphi\chi$, where $\varphi \equiv \varphi_{\mu,b}$ is the orbital part of impact parameter states and χ is some bispinor. It follows then that χ has the form

$$\chi = \exp[-i\boldsymbol{b}^{\text{spin}}\boldsymbol{p}]\exp[i\boldsymbol{B}^{\text{spin}}\boldsymbol{p}]\chi_0,$$

where χ_0 is again some bispinor. Furthermore, we have the eigenvalue equation (2.5) for the spin projection w^-/p^-. This can be expressed as

$$\frac{w^-}{p^-} = \frac{1}{2}\Sigma_3 - (\boldsymbol{B}^{\text{spin}} \times \boldsymbol{p})_3. \tag{4.7}$$

Thus the spin part of eigenfunctions takes the form in p-space

$$\chi_{\sigma,\mu,b} = \exp[-i\boldsymbol{b}^{\text{spin}}\boldsymbol{p}]\left(m, \frac{p^1 + ip^2}{\mu}, 1, 0\right), \qquad \text{for } \sigma = +\frac{1}{2},$$

$$\chi_{\sigma,\mu,b} = \exp[-i\boldsymbol{b}^{\text{spin}}\boldsymbol{p}]\left(-\frac{p^1 - ip^2}{\mu}, -m, 0, 1\right), \qquad \text{for } \sigma = -\frac{1}{2}.$$

These states are normalized according to $\bar{\chi}\chi = 2m$. With this the positive-energy $\sigma = \frac{1}{2}$ eigenfunctions in x-space can be written as

$$\text{(4.8)} \qquad \psi_{\frac{1}{2},\mu,b}(x) = \\ = (2\pi)^{-\frac{3}{2}}\left(m, \frac{1}{\mu}\left(\frac{1}{i}\frac{\partial}{\partial x^1} + \frac{\partial}{\partial x^2}\right), 1, 0\right)\int \exp[-ip^\mu x_\mu]\exp[-i\boldsymbol{p}(\boldsymbol{b}^{\text{orb}} + \boldsymbol{b}^{\text{spin}})]\,\mathrm{d}^2\boldsymbol{p},$$

and a similar expression for the $\sigma = -\frac{1}{2}$ component. Therefore, in the scalar eigenfunction as given by (3.4) b should be replaced by the total impact parameter $\boldsymbol{b}^{\text{total}} = \boldsymbol{b}^{\text{orb}} + \boldsymbol{b}^{\text{spin}}$. The spinor part as given above can be written as

$$\text{(4.9)} \qquad \begin{cases} \psi = \psi_{\frac{1}{2},\mu,b}(x) = \left(m, \dfrac{1}{\mu}\left(\dfrac{1}{i}\dfrac{\partial}{\partial x^1} + \dfrac{\partial}{\partial x^2}\right), 1, 0\right)\varphi^{\text{total}}_{\mu,b}(x), \\[2ex] \psi = \psi_{-\frac{1}{2},\mu,b}(x) = \left(-\dfrac{1}{\mu}\left(\dfrac{1}{i}\dfrac{\partial}{\partial x^1} - \dfrac{\partial}{\partial x^2}\right), -m, 0, 1\right)\varphi^{\text{total}}_{\mu,b}(x). \end{cases}$$

For the negative-energy part the complex conjugate $\varphi^*_{\mu,b}(x)$ is needed.

Expansion of $\psi^+(x)$ now reads

$$\psi^+(x) = \int_0^\infty \frac{\mathrm{d}\mu}{2\mu}\int \mathrm{d}^2\boldsymbol{b}\sum_{\sigma=\pm\frac{1}{2}}\left(a(\mu, \boldsymbol{b}, \sigma)\,\psi_{\sigma,\mu,b}(x) + d^\dagger(\mu, \boldsymbol{b}, \sigma)\,\tilde{\psi}_{\sigma,\mu,b}(x)\right),$$

where $a(\mu, \boldsymbol{b}, \sigma)$ absorbs a particle and $d^\dagger(\mu, \boldsymbol{b}, \sigma)$ emits an antiparticle with energy μ and impact parameter b. The $\sim$ indicates that the complex conjugate of the corresponding $\varphi_{\mu,b}(x)$ should be taken. The good component satisfies the following equal-x^+ anticommutation relation:

$$\{\psi^+(x), [\psi^+(y)]^\dagger\}_{x^+=y^+} = \frac{1+\alpha_3}{2}\,\delta(x^- - \gamma y^2)\,\delta^2(\boldsymbol{x} - \boldsymbol{y}),$$

where † denotes the adjoint spinor.