manuscripta math. 71, 337 - 338 (1991)

manuscripta mathematica

CORRECTION TO THE GONALITY OF SMOOTH CURVES WITH PLANE MODELS

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In our paper [1], we have made an error in the proof of the main lemma. It is that $L(kH - D) \neq 0$ does not imply the assertion of the main lemma. The same problem arises in the proof of Lemma 3.1. So we shall give a corrected proof of the main lemma. We shall use the notations from our paper. The proof of Lemma 3.1 can be corrected quite similarly. The authors would like to thank Professor M. Homma who pointed out this error.

Proof of the main lemma. Let V_{k-1} be the $\frac{k(k+1)}{2}$-dimensional affine linear space parametrizing affine equations of plane curves of degree $k-1$. Let h be the equation of a line and let H be the corresponding divisor on C. Let $D \in g_n^1$ and let s be a nonconstant element of $L(D)$. Consider the cup-product-homomorphism

$$\mu : V_{k-1} \otimes \langle 1; s \rangle \to L((k-1)H + D).$$

defined by $\mu(f \otimes t) = \frac{ft}{h^{k-1}}$ for $f \in V_{k-1}, t \in \langle 1; s \rangle$.

Assume that $\ker \mu \neq 0$. Then, there exist $f_0, f_1 \in V_{k-1}$ such that $\mu(f_0 \otimes 1 + f_1 \otimes s) = 0$. Hence $s = -\frac{f_0}{f_1}$ and g_n^1 is induced by the pencil $\mathbf{P} \subset \mathbf{P}_{k-1}$ defined by $\langle f_0; f_1 \rangle$.

Assume that $\ker \mu = 0$. Then $\ell((k-1)H + D) \geq k(k+1)$ and we have $\ell((d-2-k)H - B - D) \geq 1$ (see [1]). Let $E \in |(d-2-k)H - D - B|$ and let $L_1, \ldots, L_{k-1}$ be general lines on $\mathbf{P}^2$ and let $H_1, \ldots, H_{k-1}$ be the associated elements of $|H|$ on C. For $D' \in g^1_n$ one has $E + D' + H_1 + \cdots + H_{k-1} \in |(d-3)H - B| = |K_C|$, hence $E + D' + H_1 + \cdots + H_{k-1}$ is cut out by an adjoint curve of degree $d-3$, i. e. a plane curve Γ' of degree $d-3$ containing the singularities of Γ. Because of Bezout's theorem, we have $\Gamma' = L_1 + \cdots + L_{k-1} + \Gamma''$ where Γ'' is an adjoint curve of degree $d-2-k$. Hence $D' + (E+B) = p^*(\Gamma''.\Gamma)$. This implies that g^1_n is cut out by a pencil $\mathbf{P} \subset \mathbf{P}_m$ for some $m \leq d-2-k$. As in [1], by Lemma 1.2 and the assumption of the lemma we have

$$(m-k)(m-d+k) > 0$$

Since $m - d + k \leq -2 < 0$, we have $m < k$. This completes the proof.

References

1. Coppens, M. and Kato, T.: The gonality of smooth curves with plane models *Manuscripta Math.*, **70** (1990) 5–25

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(Received March 8, 1991)