manuscripta math. 81, 223 - 224 (1993)

manuscripta mathematica

ERRATUM TO
ON CONTINUOUS DYNAMICS OF AUTOMORPHISMS OF $\mathbf{C}^2$

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In the proof of Theorem 1.4 of [1] a couple of coefficients was missing by a misprint. This sort of misprint does not change the final results of the paper. To recover the missing coefficients substitute the definition of E_1 with

$$E_1=\left\{\begin{pmatrix}x\\y\end{pmatrix}\mapsto\begin{pmatrix}cx+f(y)\\c^{-1}y+\beta\end{pmatrix},\ \middle|\ c\in\mathbf{C}^*,\ \beta\in\mathbf{C},\ \ f\in\operatorname{Hol}(\mathbf{C},\mathbf{C})\right\}.$$

(the coefficient c was missing). Then the intersection $A_1\cap E_1$ is given by

$$A_1\cap E_1=\left\{\begin{pmatrix}x\\y\end{pmatrix}\mapsto\begin{pmatrix}cx+\alpha y+a\\c^{-1}y+\beta\end{pmatrix},\ \middle|\ c\in\mathbf{C}^*,\ \alpha,\beta,a\in\mathbf{C}\right\}.$$

In order to prove that $L_1\cap L_3=\{0\}$, in the proof of of Theorem 1.4 of [1] we must take

$$g_1\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x+f_1(y)\\y\end{pmatrix},\ g_2\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}cx+\alpha y\\\beta x+dy\end{pmatrix}\text{ and }g_3\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x+f_3(y)\\y\end{pmatrix},$$

where f_1 and f_3 are non-linear elements of $\mathcal{H}_0(\mathbf{C})$, $cd-\alpha\beta=1$ and $\beta\neq0$ (the coefficients c and d were missing). Then the correct expression for $g_3\circ g_2\circ g_1$ is given by

$$g_3\circ g_2\circ g_1\begin{pmatrix}x\\y\end{pmatrix}=g_3\begin{pmatrix}c(x+f_1(y))+\alpha y\\\beta x+\beta f_1(y)+dy\end{pmatrix}=$$
$$\begin{pmatrix}c(x+f_1(y))+\alpha y+f_3(\beta x+\beta f_1(y)+dy)\\\beta x+\beta f_1(y)+dy\end{pmatrix}.$$

If $\langle x,y\rangle\ \cap\ \langle g_3\circ g_2\circ g_1\begin{pmatrix}x\\y\end{pmatrix}\cdot e_1,g_3\circ g_2\circ g_1\begin{pmatrix}x\\y\end{pmatrix}\cdot e_2\rangle\neq\{0\}$ there exist $\gamma,\delta\in\mathbf{C}$ such that $|\gamma|+|\delta|>0$ and

$$\gamma[c(x+f_1(y))+\alpha y+f_3(\beta x+\beta f_1(y)+dy)]+\delta[\beta x+\beta f_1(y)+dy]$$

is linear in x and y. Then $(c\gamma+\delta\beta)f_1(y)+\gamma f_3\big(\beta x+dy+\beta f_1(y)\big)$ is linear in x and y, and therefore taking the derivative with respect of x we find that $\gamma\beta f_3'\big(\beta x+$

$\beta f_1(y)+dy)$ is constant. As f_3 is non-linear and $\beta x+\beta f_1(y)+dy$ is non-constant, then $\gamma\beta=0$, and since $\beta\neq 0$, $\gamma=0$. Thus $\delta\beta f_1(y)$ is linear in x and y against the fact that $\delta\beta\neq 0$ and f_1 is non-linear.

Proposition 2.6 of [1] must be replaced by the new

Proposition 2.6. *All the one-parameter groups in E_1 are expressed (up to conjugation) by*

$$\Phi_t\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}e^{ta}x+f_t(y)\\e^{-ta}y\end{pmatrix}\qquad \Phi_t\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x+f_t(y)\\y+ts\end{pmatrix},$$

where $a\in\mathbf{C}$ and in the first case f_t satisfies $f_{t+\tau}(y)=e^{\tau a}f_t(y)+f_\tau(e^{ta}y)$, while in the second it satisfies f_t satisfies $f_{t+\tau}(y)=f_t(y+\tau s)+f_\tau(y)$.

Proof. The fact that $\Phi_t\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}\alpha_t x+f_t(y)\\\alpha_t^{-1}y+\beta_t\end{pmatrix}$ satisfies the composition rule is equivalent to the fact that $\alpha_t^{-1}y+\beta_t$ is a one parameter group of affine transformations of $\mathbf{C}$, hence it can be conjugated to obtain $y\mapsto e^{ta}y$ or $y\mapsto y+t$, the relation on f follows immediately. □

With these replacements all other statements and proofs remain as they are.

[1] C. de Fabritiis, On continuous dynamics of automorphisms of $\mathbf{C}^2$, Manuscripta Mathematica, 77, 337-359 (1992)

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(Received January 25, 1993;
in revised form October 17, 1993)