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Erratum

A Unified Approach to String Scattering Amplitudes

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In my paper cited above, I constructed a certain holomorphic line bundle

$$\lambda_2 \otimes \lambda_1^{-13} \otimes \left(\bigotimes_{\nu=1}^{13} \langle \mathcal{O}(D^\nu), \mathcal{O}(D^\nu) \rangle \right)^{-1} \tag{1}$$

on a generalized moduli space $\mathcal{M}_{g,n,B}$ of complex compact algebraic curves X of genus g with n punctures $Q_1, \ldots, Q_n$ being contained in a disc B on the curve. (The curves were considered up to an isomorphism identical on the punctures, and homotopically equivalent disks on the punctured curve were also identified.) That bundle was provided with a canonical hermitian metric, and I claimed that this metric was flat (Proposition 2.2), that is not true: actually, one can prove that this metric is relatively admissible with respect to the natural projection $\mathcal{M}_{g,n,B} \to \mathcal{M}_g$, i.e., its curvature is proportional to a canonical (1, 1)-form on the fibers of this projection (see 4.4). This error makes it necessary to define a generalized Mumford form $\mu_{g,n,B}$ as an arbitrary local holomorphic section of bundle (1) and to include its norm $\|\mu_{g,n,B}\|$ in the formulation of the generalized Belavin-Knizhnik theorem in the amplitudic case (Theorem 2 from the introduction) as follows:

Theorem. *The Polyakov measure $d\pi_{g,n}$ is equal to $\mu_{g,n,B} \wedge \bar{\mu}_{g,n,B} / \|\mu_{g,n,B}\|^2$, where $\mu_{g,n,B}$ is a local holomorphic section of the hermitian line bundle*

$$\lambda_2 \otimes \lambda_1^{-13} \otimes \left(\bigotimes_{\nu=1}^{13} \langle \mathcal{O}(D^\nu), \mathcal{O}(D^\nu) \rangle \right)^{-1}$$

over the moduli space $\mathcal{M}_{g,n,B}$ of the data $(X, Q_1, \ldots, Q_n, B)$. Here $D^\nu = \sum_{i=1}^{n} p_i^\nu \cdot Q_i$ is the complex divisor with the momentum components as coefficients. The section $\mu_{g,n,B}$ is defined locally up to a holomorphic factor.

Similar changes need to be made in Sect. 5 of the introduction and in Sect. 4.6 with the bundle

$$\lambda_2 \otimes \lambda_1^{-13} \otimes \mathcal{C}^{\boxtimes 13}$$

on J^{13}, which becomes canonically isometric to (1) under the Jacoby map $\varphi: \mathcal{M}_{g,n,B} \to J^{13}$. Correspondingly, a universal Mumford form μ_U must be defined as an arbitrary local holomorphic section of this bundle. Then it will be connected with $d\pi_{g,n}$ by the formula $d\pi_{g,n} = \varphi^*(\mu_U) \wedge \overline{\varphi^*(\mu_U)} / \varphi^*(\|\mu_U\|^2)$.

To complete these corrections I also have to make the following changes.

1. In 2.2 the bundles $\mathcal{O}(D^\nu)$, $\nu = 1, \ldots, 13$, over a family $X \to S$ of complex curves must be provided with relatively flat hermitian metrics, instead of flat.
2. In 3.1 the unique hermitian metrics on the bundles $\mathcal{P}$ and $\mathcal{B}$ over $J \times J^t$ and $J \times J$, correspondingly, must be defined by the conditions:
(a) their curvatures vanish on $J \times j$ for any $j \in J^t$ for $\mathcal{P}$ and $j \in J$ for $\mathcal{B}$,
(b) they are compatible with the corresponding trivializations – $\mathcal{P}$ at $e \times J^t$ and $\mathcal{B}$ at $e \times J$.
Such metrics exist, because the restrictions of $\mathcal{P}$ and $\mathcal{B}$ on the first multiplier are topologically trivial.
3. The same changes in the relative case for the bundles $\mathcal{P}$ and $\mathcal{B}$ in 4.2 and 4.6.
4. Omit the assertions on flatness in Proposition 4.2 and Lemma 4.2.
5. Lemma 4.3 must be formulated as follows:

Lemma. *Let $\pi: X \to S$ be a smooth proper map of complex manifolds of relative dimension* 1 *with connected fibers. Let $\mathcal{L}$ and $\mathcal{M}$ be two relatively flat hermitian holomorphic line bundles. Then the canonical metric on $\langle \mathcal{L}, \mathcal{M} \rangle$ does not depend on the choice of such metrics on $\mathcal{L}$ and $\mathcal{M}$.*

Proof. Let $\| \ \|_1$ and $\| \ \|_2$ be two relatively flat metrics on $\mathcal{L}$. Then the metric $\| \ \|_3 := \| \ \|_1 \cdot \| \ \|_2^{-1}$ on the trivial line bundle $\mathcal{O} = \mathcal{L} \otimes \mathcal{L}^{-1}$ is constant fiberwise due to properness. But for the corresponding canonical metrics on $\langle \mathcal{L}, \mathcal{M} \rangle$ and $\langle \mathcal{L}, \mathcal{M} \rangle \otimes \langle \mathcal{O}, \mathcal{M} \rangle$ there holds the equality

$$\|\langle l, m \rangle\|_1 = \|\langle l, m \rangle\|_2 \cdot \|\langle 1, m \rangle\|_3,$$

where l and m are (local by S) holomorphic sections of $\mathcal{L}$ and $\mathcal{M}$ with nonintersecting divisors. According to the definition (see [1]),

$$\|\langle 1, m \rangle\|_3 = \exp\left(\int_{X/S} \mathbf{c}_1(\mathcal{O}) \cdot \log \|m\| + \log(\|1\|_3(\operatorname{div} m)) \right).$$

The Chern form $\mathbf{c}_1(\mathcal{O})$ of $\mathcal{O}$ with respect to metric $\| \ \|_3$ vanishes fiberwise, yielding the integral to vanish. Next, $\|1\|_3$ is constant fiberwise and $\deg \mathcal{M} = 0$, whence $\log(\|1\|_3(\operatorname{div} m)) = 0$. Thus, $\|\langle 1, m \rangle\|_3 = 1$. □

6. In Point 3 of the definition of a multivalued holomorphic function (Sect. 1.2) replace "any sequence $\{a_m\}$ in $\widetilde{X \backslash m}$" by "any sequence $\{a_m\}$ in a domain of univalence in $\widetilde{X \backslash m}$".
7. Replace "$0 < \operatorname{Re} A < 1$" by "$0 < \operatorname{Re} A \leqq 1$" in Lemma 1.2.

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References

1. Deligne, P.: Le determinant de la cohomologie. Contemp. Math. **67**, 93–178 (1987)

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