

Erratum

Small Deformation of Normal Singularities

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This note corrects a mistake in the proof of Lemma 3.1 in [1]. The isomorphism

$$\mathrm{Ext}_{\mathcal{O}_{E^*}}^i(\mathcal{O}_{E^*}(-L_r + K_{E^*}), \mathcal{O}(K_{E^*})) \simeq \mathrm{Hom}_{\mathcal{O}_{C^*}}(R^{n-i} f_* \mathcal{O}_{E^*}(-L_r + K_{E^*}), \mathcal{O}_{C^*}),$$

which was described at 1.9 in p. 145 does not follow Theorem 11.2, (f) (III, in [2]) immediately. Because we cannot apply the theorem to \mathcal{O}_{C^*} which is not an injective module.

However the lemma remains true. In fact, in the original proof, the preceding isomorphism was used only to prove the assertions from 1.11 to 1.13 "there exists an integer r_0 such that for any $r \geq r_0$, $f_* \mathcal{O}_{E^*}(L_r) = 0$ and $R^1 f_* \mathcal{O}_{E^*}(L_r)$ is zero (resp. torsion free) if $n \geq 3$ (resp. $n = 2$)." So, we will show the above assertions by a correct method below.

Applying the relative duality theorem (III, 11.1 in [2]) to the flat morphism $E^* \rightarrow C^*$ of relative dimension $n-1$, we get

$$\mathbb{R} f_* \mathcal{O}_{E^*}(L_r)[n-1] = \mathbb{R} \mathcal{H}om_{\mathcal{O}_{C^*}}(\mathbb{R} f_* \mathcal{O}(-L_r + K_{E^*}), \mathcal{O}_{C^*}).$$

Note that there exists an integer r_0 such that, for any $r \geq r_0$, $R^i f_* \mathcal{O}_{E^*}(-L_r + K_{E^*})$ is 0 if $i > 0$ and locally free if $i = 0$ respectively. Since for $r \geq r_0$, $\mathcal{E}xt_{\mathcal{O}_{C^*}}^j(f_* \mathcal{O}_{E^*}(-L_r + K_{E^*}), \mathcal{O}_{C^*})$ is 0 if $j > 0$ and locally free if $j = 0$, which yields $R^i f_* \mathcal{O}_{E^*}(L_r)$ is 0 for $i < n-1$ and locally free for $i = n-1$ respectively. This completes the proof of our assertions.

References

1. Ishii, S.: Small deformation of normal singularities. Math. Ann. 275, 139–148 (1986)
2. Hartshorne, R.: Residues and duality. Lect. Notes Math. 20. Berlin, Heidelberg, New York: Springer 1966

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