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Erratum

Parallelizability of Homogeneous Spaces, II

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In the list of stably parallelizable quotients [2, Theorem 2] case (vi) is wrongly stated. It should read:

(vi) $\mathrm{Sp}(n)/(SU(2)\times\ldots\times SU(2))=Z_{n,k}$ where k denotes the number of factors $SU(2)$, with $2k\leqq n$.

As a consequence, Theorem 1 and the ensuing discussion require the additional hypothesis that G is not isomorphic to $\mathrm{Sp}(n)$, $n\geqq 4$. In the general case, we have instead:

Theorem 1*. *Let G be a simple 1-connected compact Lie group and H a closed connected subgroup. Denote by I the ideal of $RO(H)$ which is generated by the elements*

$$(\psi|H-\dim\psi)\quad \text{where } \psi\in RO(G), \text{ and}$$
$$\varphi\cdot(\lambda|H-\dim_{\mathbb{C}}\lambda)\quad \text{where } \varphi\in RSp(H),\ \lambda\in RSp(G).$$

Then G/H is stably parallelizable if and only if

$$(\mathrm{Ad}_H-\dim H)\in I.$$

The error comes from Sect. 7 which should be replaced by the following text:

Observe that the tensor product of two symplectic representations is real. Therefore, if H is a closed subgroup of any compact Lie group G, we have a natural homomorphism

$$\text{(7.1)*}\qquad \bar{\alpha}: RO(H)\otimes_{RO(G)\oplus RSp(H)\cdot\widetilde{RSp}(G)}\mathbb{Z}\to KO(G/H).$$

Here, $\widetilde{RSp}(G)$ denotes the module of all virtual symplectic representations of dimension 0.

(7.3)* **Corollary.** *The manifolds $Z_{n,k}=\mathrm{Sp}(n)/(SU(2))^k$ are parallelizable.*

Proof. Write $R(\mathrm{Sp}(n)) = \mathbb{Z}[\lambda_1, \ldots, \lambda_n]$ where λ_i are the fundamental representations, with λ_{2i} orthogonal and λ_{2i+1} symplectic; $R(SU(2)^k) = \mathbb{Z}[\varphi_1, \ldots, \varphi_n]$ where φ_i is the projection onto the i-th factor. Writing $H = SU(2)^k$, we have

$$\begin{aligned}\mathrm{Ad}_H &= \sum_{i=1}^{k} (\varphi_i^2 - 1) \\ &= (\sum \varphi_i) \cdot (\lambda_1 | H - 2n) + \lambda_2 | H + (2n - 2k) \cdot 2\lambda_1 | H + l, l \in \mathbb{Z}.\end{aligned}$$

As φ_i is symplectic and $2\lambda_1$ is real $\mathrm{Sp}(n)/H$ is stably parallelizable by (7.1)*. To obtain parallelizability, observe that there is a free $\mathbb{Z}_2 \times \mathbb{Z}_2$-operation on $Z_{n,k}$ for $k \geqq 2$ (the case $k = 1$ has been dealt with in [1, 4.5]):

Let R be the multiplication by $\mathrm{diag}(i, i, 1, 1, \ldots, 1)$ and S by $\mathrm{diag}(1, 1, i, i, 1, \ldots, 1)$ from the right. The group generated by R and S gives the desired action.

(7.4)* **Lemma.** *Let H be a subgroup of* $\mathrm{Sp}(n)$ *such that* $SU(2n)/H$ *is stably parallelizable. Then* $\mathrm{Sp}(n)/H$ *is diffeomorphic to* $\mathrm{Sp}(2)/SU(2)$ *or* $\mathrm{Sp}(n)/(\mathrm{Sp}(1) \times \ldots \times \mathrm{Sp}(1))$.

Proof. This follows from [2, 6.5].

(7.5)* **Lemma.** *Let H be a subgroup of* $\mathrm{Sp}(n)$ *such that* $\mathrm{Sp}(n)/H$ *is stably parallelizable, but* $SU(2n)/H$ *is not. Then* $\mathrm{Sp}(n)/H$ *is diffeomorphic to* $Z_{n,k}$ *or to* $\mathrm{Sp}(n)/\mathrm{Sp}(k)$.

This follows by considering Pontrjagin and Stiefel-Whitney classes.

Finally [2, 7.6] remains unchanged.

References

1. Singhof, W.: Parallelizability of homogeneous spaces. I. Math. Ann. **260**, 101–116 (1982)
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