Foundations of Physics, Vol. 7, Nos. 11/12, 1977

Erratum and Addendum

In the paper, "New Four-Dimensional Symmetry," by J. P. Hsu *Found. Phys.* **6**, 317 (1976)], Eqs. (3a) and (3b) hold only for the two events, that satisfy $\mathbf{r}/t = \mathbf{r}_1/t_1$. In general, for two arbitrary events specified by (x, y, z, ct) and $(x_1, y_1, z_1, (ct)_1)$, Eqs. (3a) and (3b) should read

$$\begin{aligned} \Delta s^2 &= [ct - (ct)_1]^2 - (\mathbf{r} - \mathbf{r}_1)^2 = \Delta(ct)^2 - \Delta \mathbf{r}^2 \\ \Delta s'^2 &= [c't - (c't)_1]^2 - (\mathbf{r}' - \mathbf{r}_1')^2 = \Delta(c't)^2 - \Delta \mathbf{r}'^2 \end{aligned} \tag{E1}$$

When the interval between these two events is infinitesimal, we have $ds'^2 = d(c't)^2 - dr'^2$.

Equation (9) and the condition "$V(\Delta t) = 0$ for $\Delta t = 0$" in the note added at the end of the paper are unnecessary extra conditions and should be discarded. The velocity V in the space–light transformation (8) is a constant. Suppose universal clock systems in both f and f' frames are set up in such a way that c is a constant in the transformation (8), i.e., the speed of light is isotropic in the f frame. Then the interval Δs^2 in (El) can be written as $\Delta s^2 = c^2 \Delta t^2 - \Delta \mathbf{r}^2$ and c' will be a function of x/t, as shown in (8). Furthermore, the function $c'(x/t)$ corresponds to the value of the one-way speed of light measured in f' if and only if $\mathbf{r}$ and t satisfy the condition $r/t = c$, $r = |\mathbf{r}|$. When $x = \text{const} \neq 0$ and $t \to 0$, the function $c'(x/t)$ diverges, as one can see from (8). We stress that in this case the value $c'(x/t) \to \infty$ does not correspond to any speed of observable objects.

The frequency shift (26) and that in Section 9 should be understood as the radar-pulse frequency rather than the electromagnetic frequency shift. We note that the observer in the laboratory frame f cannot measure λ' and ω'/c' in Eq. (22). He can only compare the shifted quantities λ and ω/c with the unshifted quantities λ_u and ω_u/c associated with the same atom at rest in f in the Doppler shift experiment. Suppose the atom is at rest in the f' frame, and k' and ω' in (22) are the unshifted quantities observed in f'. In this case, one has $\lambda' = \lambda_u$ and $\omega'/c' = \omega_u/c$, and (22) leads to

$$\frac{1}{\lambda_u} = \frac{1}{\lambda} \frac{1 + \beta}{(1 - \beta^2)^{1/2}}, \qquad \omega_u = \omega \frac{1 + \beta}{(1 - \beta^2)^{1/2}} \tag{E2}$$

where $k = 2\pi/\lambda$ and $\cos\theta = -1$.

The decay width (31) or (33) of an unstable particle is the same for the particle at rest in any frame. Suppose the pions π_1 and π_2 are at rest in f and f', respectively; one then has $\Gamma(\pi_1) = \Gamma'(\pi_2)$. Their lifetimes $\tau(\pi_1)$ and $\tau(\pi_2)$ measured in f are related by $c\tau(\pi_1) = \bar{h}/\Gamma(\pi_1) = c'\tau'(\pi_2) = c'\tau(\pi_2)$. Suppose π_2 is at rest at the origin, for simplicity; one then has $c = \gamma c'$ and hence

$$\tau(\pi_2) = \gamma\tau(\pi_1), \qquad \gamma = (1 - v^2/c^2)^{-1/2} \tag{E3}$$

The time dilation experiment is explained directly by this relation (E3) rather than the relations (34) and (35). Note that this does not contradict the scalar property of the lifetime τ, which implies only that all observers in different frames measures the same value of τ when they observe the same unstable particle, e.g., $\tau(\pi_2) = \tau'(\pi_2)$, where $\tau'(\pi_2)$ is the pion lifetime measured in f'.

The concept of universal time discussed in this paper is actually different from Newtonian time. For a discussion of the relation between universal clocks in this paper and practical atomic clocks, we refer to a recent paper by C. B. Chiu, J. P. Hsu, and T. N. Sherry, *Phys. Rev. D*, in press (Univ. of Texas preprint, ORO 288, 1976).