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A NOTE ON A PREVIOUS PAPER "NEW FOUNDATIONS OF PROJECTIVE AND AFFINE GEOMETRY"[1]

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1. This first paragraph is an erratum to the paper cited above. In §2, postulate IV (p. 458, referring to the symbol $\subseteq$ which is only afterwards introduced, has to be replaced by the two propositions (a) and (a′) of p. 458. Nothing else has to be changed in §2 since it is these hypotheses which are actually used. In §3 the remark of p. 461 that law 5 is equivalent to IV must be replaced then, by the statement that law 5, though stronger than IVa and IVa′, still holds in projective as well as in affine spaces.

2. The next two paragraphs are remarks by F. Alt on the paper cited above.

It is obvious that law 5 implies IVa and IVa′. It may be shown by an example that the converse is not true. We consider a class which besides V and 9 points 1, 2, 3, 4, 1′, 2′, 3′, 4′, 0, contains 12 "lines" consisting of the points (0, 1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4), (0, 1′, 2′), (1′, 3′), (1′, 4′), (2′, 3′), (2′, 4′), (3′, 4′), and 8 "planes" consisting of the points (0, 1, 2, 3), (0, 1, 2, 4), (1, 3, 4), (2, 3, 4), (0, 1′, 2′, 3′), (0, 1′, 2′, 4′), (1′, 3′, 4′), (2′, 3′, 4′) and U. For two elements A, B of this class we define $A + B$ as the element of least dimension containing all the points of A and of B (e.g. we have $1 + 1' = 1 + 2' = U$), and $A \cdot B$ as the element consisting of the points contained as well in A as in B. Then the class satisfies Postulates I, II, III, IVa, IVa′, V, while law 5 does not hold. This is seen for $A = 0$, $B = (0, 1, 2)$, $C = (0, 1, 2, 3)$.

Another remark may be added regarding this example: If we consider the subclass consisting of all parts of the plane (0, 1, 2, 3), i.e. V, 0, 1, 2, 3, (0, 1, 2), (1, 3), (2, 3), (0, 1, 2, 3) then we notice that this sub-class does not satisfy Postulate IVa. This is seen for $A = (0, 1, 2)$, $B = 0$. On the contrary, classes which satisfy laws 1–6 are "hereditary", i.e. for each element A the class of all parts of A satisfies laws 1–6 itself, A playing the rôle of U.

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[1] Annals of Mathematics, vol. 37, (1936), pp. 456–482.