1 Introduction

1.1 On the Goal of this Book

The works of Hertz (1882) and Boussinesq (1885) are generally considered the beginning of classical contact mechanics. The solutions for the pressure distribution under a cylindrical flat punch and a sphere that are featured in those works certainly enjoy a high level of prominence. Yet multiple exact solutions exist which are of similar technical relevance to the Hertzian contact problems, but only a limited number of specialists know about these. Among other reasons, this is due to the fact that many individual contact mechanical solutions were published in relevant journals, however, a generalized representation in any complete monograph is lacking. Exceptions to this can be found in the books by Galin (2008) and Gladwell (1980), yet even these were written with scientific usage in mind rather than as a handbook for technical applications. This book aims to provide a compendium of exact solutions for rotationally symmetric contact problems which are suitable for practical applications.

Mathematically the terms “rotationally symmetric” and “contact problem” are quite straightforward to define. But what is an “exact solution”? The answer to this question is dual-faceted and involves an aspect of modeling; consideration must also be given to the final structure of what one accepts as a “solution”. The first aspect is unproblematic: any model represents a certain degree of abstraction of the world, and makes assumptions and simplifications. Any solution derived from this model can, of course, only be as exact as the model itself. For example, all solutions in this book operate under the assumption that the resulting deformations and gradients of contacting surfaces within the contact area are small.

The second aspect is tougher to define. A “naïve” approach would be that an exact solution can be derived and evaluated without the aid of a computer. However, even the evaluation of trigonometric functions requires computation devices. Does a solution in the form of a numerically evaluated integral or a generalized, perhaps hypergeometric function count as “exact”? Or is it a solution in the form of a differential or integral equation? In exaggerated terms, assuming the valid-

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ity of a respective existence and uniqueness theorem, simply stating the complete mathematical description of a problem already represents the implicit formulation of its solution. Recursive solutions are also exact but not in closed form. Therefore, distinguishing between solutions to be included in this compendium and those not "exact enough" remains, for better or worse, a question of personal estimation and taste. This is one of the reasons why any encyclopedic work cannot ever—even at the time of release—make a claim of comprehensiveness.

The selection of the problems to be included in this book were guided by two main premises: the first one being the technical relevance of the particular problem, and secondly, their place in the logical structure of this book, which will be explained in greater detail in the next section.

1.2 On Using this Book

Mechanical contact problems can be cataloged according to very different aspects. For instance, the type of the foundational material law (elastic/viscoelastic, homogeneous/inhomogeneous, degree of isotropy, etc.), the geometry of the applied load (normal contact, tangential contact, etc.), the contact configuration (complete/incomplete, simply connected/ring-shaped, etc.), the friction and adhesion regime (frictionless, no-slip, etc.), or the shape of contacting bodies are all possible categories for systematization. To implement such a multi-dimensional structure while retaining legibility and avoiding excessive repetition is a tough task within the constraints of a book.

We decided to dedicate the first five chapters to the most commonly used material model: the linear-elastic, homogeneous, isotropic half-space. Chapters 7 through to 9 are devoted to other material models. Chapter 10 deals with ring-shaped contact areas. The chapters are further broken down into sub-chapters and sections, and are hierarchically structured according to load geometry, the friction regime, and the indenter shape (in that order). While each section aims to be understandable on its own for ease of reference, it is usually necessary to pay attention to the introductory sentences of e.g. Chap. 4 and Sect. 4.6 prior to Sect. 4.6.5.

Furthermore, many contact problems are equivalent to each other, even though it may not be obvious at first glance. For example, Ciavarella (1998) and Jäger (1998) proved that the tangential contact problem for an axially symmetric body can be reduced under the Hertz–Mindlin assumptions to the respective normal contact problem. In order to avoid these duplicate cases cross-references are provided to previously documented solutions in the book which can be looked up. Where they occur, these references are presented and explained as clearly and unambiguously as possible.

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