The Modified Mumford-Shah Model Based on Nonlocal Means Method for Textures Segmentation

Jingge Lu, Guodong Wang(✉), and Zhenkuan Pan

College of Information Engineering, Qingdao University, Qingdao, China
cgjingge730@163.com, doctorwgd@gmail.com

Abstract. Textures segmentation is a very important subject in the fields of computer vision. In order to segment the textures, a new method is achieved. The traditional Mumford-Shah model is modified. In detail, a smoothness term is added which used the nonlocal means method. The traditional Mumford-Shah model can be used to segment the conventional images. The modified Mumford-Shah model can dispose the textures well. What's more, in order to improve the computation efficiency, this paper designs the Split-Bregman algorithm. At last, our performance is demonstrated by segmenting many real texture images.

Keywords: Textures segmentation · Nonlocal means method · Modified Mumford-Shah model · Split-Bregman algorithm

1 Introduction

Texture is an aspect so easy to be visually recognized but hard enough to be mathematically characterized. Various methods of extracting textural features from images have been designed. The filters, geometric, statistical models have shown promising results. But these methods have different shortcomings more or less.

The classical filter forms are Gabor and Wavelet [1]. The summation of the filter responses can synthesize a new image without texture which is the basic idea of the texture segmentation using filter theory. Such active contour models need to search for help from the edge detecting methods .And these methods are time consuming.

In recent years, local binary pattern method achieves good result in texture segmentation [2]. The LBP characteristic achieve better result than Gabor and Wavelet features. It needs lower computational burden [3].

The segmentation methods mentioned above rely on the choice of the initial condition. And all these methods can achieve good results especially for easy images. But the results are sometimes not good for complicated images. Because the texture on an object is very similar to its boundary, it always leads to the wrong segmentation result. But we can get good results to use our method.

The outline of the paper is as follows. The next section introduces the conventional Mumford-Shah model for image segmentation. Then the modified Mumford-Shah model based on the nonlocal means method is proposed. What's more, we design the

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Split-Bregman algorithm. At last, we make some numerical experiments and obtain the final conclusion.

2 The Mumford-Shah Model

The model of piecewise smooth is the basis of variational image segmentation research. For the gray image f(x) of defining on the rectangular open area $\Omega:\Omega\to R, x\in\Omega$,Mumford-shah model [4] is

$$\underset{u,K}{Min}\left\{E(u,K)=\int_{\Omega/K}(u-f)^2dx+\lambda\int_{\Omega/K}|\nabla u|^2dx+\gamma H^{N-1}(K)\right\} \quad (1)$$

The original image will be segmented into the area of smooth image $u(x)$ by the edge point set K. The first item means the proximity between the original image with the approximate image of the smooth regions. The second item is the penalty item of piecewise smooth image. The third item $H^{N-1}(K)$is the length of segmentation-line. λ,γ are the penalty parameters. The larger the λ is, the more smooth the piecewise image becomes. The larger the γ is, the shorter the segmentation-line made by the edge point set becomes. For avoiding the optimization problem because of the coexistence of the different dimensions space variable, u is defined as the function of the special function space of bounded variation in the document [5],the formula(1) is rewritten

$$\underset{u}{Min}\left\{E(u)=\int_{\Omega}\left((u-f)^2+\lambda|\nabla u|^2\right)dx+\gamma H^{N-1}(S_u)\right\} \quad (2)$$

S_u is the set of the discrete points.

Vese et al [6] use sign distance function $\phi(x)$ zero level set to express the continuous contour line of divisiory area, they use variational level set method to make formula(2) being similar with the formula(3).

$$\underset{u}{Min}\left\{E(u)=\alpha_1\int_{\Omega}\left((u_1-f)^2+\lambda_1|\nabla u_1|^2\right)H(\phi)dx+\alpha_2\int_{\Omega}\left((u_2-f)^2+\lambda_2|\nabla u_2|^2\right)(1-H(\phi))dx+\gamma\int_{\Omega}|\nabla H(\phi)|dx\right\} \quad (3)$$

If Ω_1 means the prospect of the image, $\Omega_2=\Omega\setminus\Omega_1$ means the background of the image, Γ means the contour line. When u_1,u_2 means the piecewise smooth image of prospect and background, the first two items are the estimated data of the piecewise smooth images, the third item is similar with the third item of the formula(2). $\alpha_1,\alpha_2,\lambda_1,\lambda_2,\gamma$ are the penalty parameters. The level set function $\phi(x)$ which be defined by the sign distance function is

$$\phi(x)=\begin{cases} d(x,\Gamma), & x\in\Omega_1 \\ 0, & x\in\Gamma \\ -d(x,\Gamma), & x\in\Omega_2 \end{cases} \quad (4)$$

$d(x,\Gamma)$ is the minimum distance between arbitrary point with contour line in the image space. $H(\phi)=\begin{cases}1, & \phi\geq 0\\ 0, & otherwise\end{cases}$ is Heaviside function.

The document [7] use binary tag function $\phi(x)=\begin{cases}1, & x\in\Omega_1\\ 0, & x\in\Omega_2\end{cases}$, $\Omega=\Omega_1\cup\Omega_2$, $\Omega_1\cap\Omega_2=\phi$,the formula(3) is rewritten the constrained optimization problem:

$$\underset{u_1,u_2,\phi\in\{0,1\}}{Min}\left\{E(u_1,u_2,\phi)=\alpha_1\int_\Omega\left((u_1-f)^2+\lambda_1|\nabla u_1|^2\right)\phi dx+\alpha_2\int_\Omega\left((u_2-f)^2+\lambda_2|\nabla u_2|^2\right)(1-\phi)dx+\gamma\int_\Omega|\nabla\phi|dx\right\} \quad (5)$$

3 The Modified Mumford-Shah Model Based on Nonlocal Means Method

3.1 Nonlocal Operators

Gillboa and Osher defined in [8] the variational formulation of nonlocal means, with a non-local partial differential equation (PDE).The nonlocal means algorithm can be seen as the generalization of the Yaroslavsky filter [9] and bilateral filters [10] to intensity patch feature instead of single pixel feature. See also the paper on Texture Synthesis [11]. The nonlocal means can be used the image denoising and other aspects [12].

The traditional local operator means the relationship between the current pixels with the consecutive pixels. The nonlocal method is based on the image slice similarity. It defines as follows:

$$w(x,y)=\exp\left\{-\frac{G_\sigma*\left(\|f(x+\bullet)-f(y+\bullet)\|\right)^2}{h^2}\right\} \quad (6)$$

$x\in\Omega$, $y\in\Omega$, in which G_σ is the Gaussian kennel function. σ is the width parameter of the gaussian kernel function,h is the threshold value for similarities between two patch windows. So the nonlocal gradient for two points x and y in the image is defined as

$$\nabla_{NL}u(x,y)=(u(y)-u(x))\sqrt{w(x,y)} \quad (7)$$

$x,y\in\Omega$ .The nonlocal gradient is not the general vector but the map of $\Omega\times\Omega\rightarrow R$.The nonlocal vector is v(x,y): $\Omega\times\Omega\rightarrow R$,the inner product is defined

$$\langle v_1\bullet v_2\rangle(x)=\int_\Omega v_1(x,y)v_2(x,y)dx \quad :\Omega\rightarrow R \quad (8)$$

The module is

$$|v|(x)=\sqrt{\int_\Omega(v(x,y))^2dy} :\Omega\rightarrow R \quad (9)$$

With the inner product , The nonlocal divergence is defined as

$$(\nabla_{NL}\bullet v)(x)=\int_\Omega(v(x,y)-v(y,x))\sqrt{w(x,y)}dy \quad (10)$$

Then the nonlocal laplacian can be defined as

$$\Delta_{NL}u(x)=\frac{1}{2}(\nabla_{NL}\bullet(\nabla_{NL}u))(x)=\int_\Omega(u(y)-u(x))w(x,y)dy \quad (11)$$

With the above mentioned definitions ,we can write the norm of the nonlocal gradient for a function u as follows.

$$|\nabla_{NL}u|(x)=\sqrt{\int_{\Omega}(u(y)-u(x))^2 w(x,y)dy}:\Omega\rightarrow R \tag{12}$$

3.2 The Proposed Model

Based on the above definitions, the modified Mumford-Shah model can be defined as

$$\underset{u,\phi\in\{0,1\}}{Min}\left\{E(u,\phi)=\sum_{i=1}^{2}\alpha_i\int_{\Omega}\left((u_i-f)^2+\beta_i|\nabla u_i|^2+\lambda_i|\nabla_{NL}u_i|^2\right)\chi_i(\phi)dx+\sum_{i=1}^{1}\gamma\int_{\Omega}|\nabla\phi_i|dx\right\} \tag{13}$$

$\chi_i(\phi)$ can be defined as follows.

$$\chi_i(\phi)=\phi_i\prod_{j=0}^{i-1}(1-\phi_j)(\phi_0\equiv 0,\phi_{n+1}\equiv 1) \tag{14}$$

When λ_i is zero, the formula (13) is the conventional Mumford-Shah model. When β_i is zero, the formula (12) is the Mumford-Shah model with nonlocal means method. But when λ_i and β_i are not equal to the zero, this model can segment not only the conventional images but also the texture images. The answer in detail of the formula (12) is defined as follows.

The energy function about u is

$$\underset{u_i}{Min}\left\{E(u_i)=\sum_{i=1}^{2}\alpha_i\int_{\Omega}\left((u_i-f)^2+\beta_i|\nabla u_i|^2+\lambda_i|\nabla_{NL}u_i|^2\right)\chi_i(\phi)dx\right\} \tag{15}$$

The Euler-Lagrange equation solved by the variational method about u is

$$(u_i-f)\chi_i(\phi)-\beta_i\Delta u_i-\lambda_i\nabla_{NL}\bullet\left(\chi_i(\phi)|\nabla_{NL}u_i|\right)=0 \tag{16}$$

Considering the equation (15), we can obtain $u_i(x)$ as follows

$$u_i(x)=\frac{f(x)\chi_i(\phi)+\beta_i*C+\lambda_i\int_{\Omega}(\chi_i(\phi(y))+\chi_i(\phi(x)))u_i(y)w(x,y)dy}{\chi_i(\phi)+4*\beta_i+\lambda_i\int_{\Omega}(\chi_i(\phi(y))+\chi_i(\phi(x)))w(x,y)dy} \tag{17}$$

Without the loss of generality, we represent a gray image as an N×N matrix. Where m, n=1,...,N, C is defined as follows.

$$C=u_{m+1,n}+u_{m-1,n}+u_{m,n+1}+u_{m,n-1} \tag{18}$$

The energy function about ϕ is

$$\underset{\phi_i}{Min}\left\{E(\phi_i)=\sum_{i=1}^{2}\int_{\Omega}\left((u_i-f)^2+\beta_i|\nabla u_i|^2+\lambda_i|\nabla_{NL}u_i|^2\right)\chi_i(\phi)dx+\sum_{i=1}^{1}\gamma|\nabla\phi_i|dx\right\} \tag{19}$$

This paper designs to use the split Bregman iteration method, the instrumental variable v_i and Bregman iteration parameter b_i are introduced, then the formula(16) is transformed into the iterative optimization format.

$$\min_{\phi_i \in \{0,1\}} \left\{ (\phi, v) = \sum_{i=1}^{2} \alpha_i \int_\Omega Q_i(u) \chi_i(\phi) dx + \sum_{i=1}^{1} \gamma \int_\Omega |v_i| dx + \sum_{i=1}^{1} \frac{\theta}{2} \int_\Omega \left(v_i - \nabla \phi - b_i^{k+1}\right)^2 dx \right\} \tag{20}$$

And the constrains is defined by

$$b_i^{k+1} = b_i^k + \nabla_{NL} u_i^k - v_i^k, b_i^0 = v_i^0 = 0 \tag{21}$$

The form of $Q_i(u)$ is

$$Q_i(u) = (u_i - f)^2 + \beta_i |\nabla u_i|^2 + \lambda_i |\nabla_{NL} u_i|^2 \tag{22}$$

The Euler-Lagrange equation about ϕ and the approximate generalized soft threshold formula about v_i based on the alternating iterative optimization strategy and variational method can be obtained.

$$\begin{cases} \nabla \bullet \left(v_i - \nabla \phi - b_i^{k+1}\right) + \sum_{i=1}^{2} Q_i(u) \frac{\partial \chi_i(\phi)}{\partial \phi} = 0 & in\Omega \\ \left(v_i^k - \nabla \phi_i - b_i^{k+1}\right) \bullet \vec{n} = 0 & on\partial\Omega \end{cases} \tag{23}$$

In the equation (20), the form of $\frac{\partial \chi_i(\phi)}{\partial \phi_k}$ is

$$\frac{\partial \chi_i(\phi)}{\partial \phi_k} = \begin{cases} \prod_{j=0}^{i-1} [1 - \phi_j] & k = i \\ - \prod_{j=0,k \neq i}^{i-1} [1 - \phi_j] \phi_i & k \neq i \end{cases} \tag{24}$$

With the equation (21),we can obtain v_i^{k+1} as follows.

$$v_i^{k+1} = Max\left(\left|\nabla \phi_i^{k+1} + b_i^{k+1}\right| - \frac{\gamma}{\theta}, 0 \right) \frac{\nabla \phi_i^{k+1} + b_i^{k+1}}{\left|\nabla \phi_i^{k+1} + b_i^{k+1}\right|} \tag{25}$$

Where

$$\phi_i = Max\left(Min(\phi_i, 1), 0\right) \tag{26}$$

4 Experiment and Analysis

In the section, several numerical experiments is presented to show the performance of the model proposed in the paper for texture image segmentation. And we make comparison with the method which is referenced from literature [13].This method combines image decomposition model and active contour model.

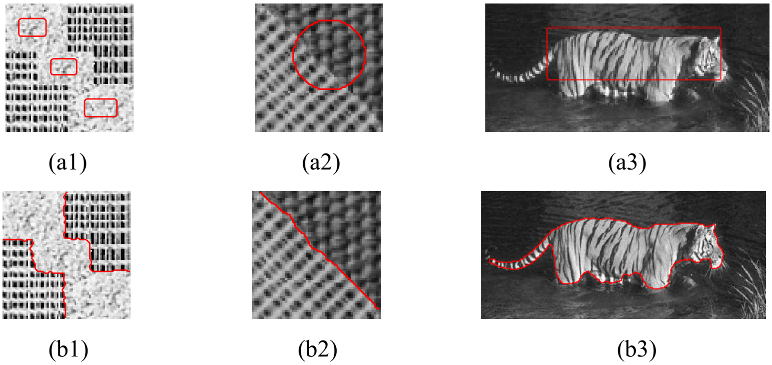

Fig. 1. Texture images for segmentation: (a1), (a2) and(a3) are the initialized images.(b1) ,(b2) and (b3) are the segmentation results.

We first present the initialized contour line, the second line is the segmentation result in Fig. 1. For the first picture(a1), there are two cut-off rules between the two opposite angles texture areas with the middle area. And the cut-off rule is very long. But the cut-off rule is close to the edge good. The second picture (a2) is similar. The third picture (a3) is a tiger. We know the body of the tiger has some texture streaks. We can segment the tiger from the whole picture very well. No matter the simple complex texture images or the real texture image, we can obtain very good results. The segmentation line can be fully fit with the edge of the segmentation area.

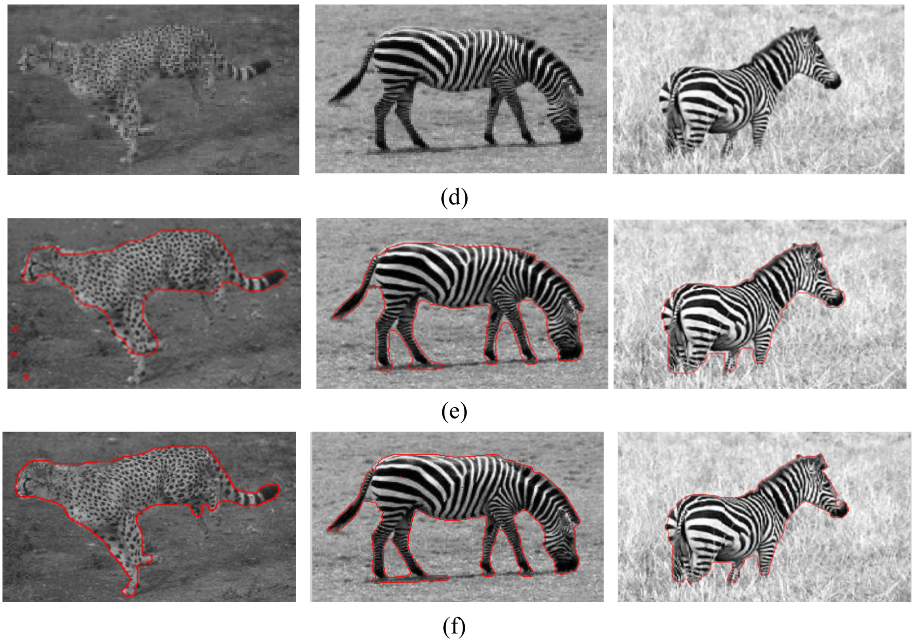

Fig. 2. Comparison results between our method and the method in [13]: (d) original image,(e)shows the results using the method in [13], (f) shows the results of our method.

These images are typical. Compared with the method [13], the results of our method are exact. For the first picture, the leopard has many spots. The foreground leopard and the background has similar hue meanwhile. It is difficult to segment this picture which used the conventional method well. The following two zebras have quite a lot of texture streaks and it is the greatest interference factor for segmenting the zebra. Because we are based on the nonlocal the image slice similarity but the image pixel. So the zebra is segmented well. From the first picture, we can see that the whole body of the leopard is fully fit with the segmentation line, especially the head and the legs. The second picture shows the great performance of the legs' segmentation. The segmentation line of the old method has a certain gap with the zebra's legs. Our method has better performance which makes the segmentation line fully fit with the zebra's legs. The third picture shows the good segmentation result of the legs and the head. Meanwhile the segmentation effect of the zebra's ears is good too.

5 Conclusions

In this paper, by using the relevant concepts of nonlocal operators and the modified Mumford-shah model, we proposed this model for texture image segmentation. In this model, we design a Split Bregman algorithm and provide the implementations. Numerical experiments confirm the performance of the proposed model for texture image segmentation.

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