Function Inverse P-sets and the Hiding Information Generated by Function Inverse P-information Law Fusion

Kai-Quan Shi

School of Mathematics and System Sciences, Shandong University, Jinan, China
shikq@sdu.edu.cn

Abstract. Introducing the concept of function into inverse P-sets (inverse packet sets) and improving it, function inverse P-sets (function inverse packet sets) is obtained. Function inverse P-sets is the function set pair composed of function internal inverse P-set (function internal inverse packet set) $\overline{S}^F$ and function outer inverse P-set (function outer inverse packet set) $\overline{S}^{\bar{F}}$, or $(\overline{S}^F, \overline{S}^{\bar{F}})$ is function inverse P-sets. Function inverse P-sets, which have dynamic characteristic and law characteristic (or function characteristic), can be reduced to finite general function sets S under certain condition. Inverse P-sets is obtained by introducing dynamic characteristic to finite general element set X (Cantor set X) and improving it. Inverse P-sets is the element set pair composed of internal inverse P-set $\overline{X}^F$ (internal inverse packet set $\overline{X}^F$) and outer inverse P-set $\overline{X}^{\bar{F}}$ (outer inverse packet set $\overline{X}^{\bar{F}}$), or $(\overline{X}^F, \overline{X}^{\bar{F}})$ is inverse P-sets which has dynamic characteristic. In this paper, the structure of function inverse P-sets and its reduction, the inverse P-information law fusion generated by function inverse P-sets, and the attribute characteristics and attribute theorems of inverse P-information law are proposed. Using these theoretical results, the hiding image and its applications generated by inverse P-information law fusion are given, which is one of the important applications of function inverse P-sets.

Keywords: function inverse P-sets, inverse P-information law fusion, reduction theorem, attribute theorem, hiding information image, image camouflage, applications

1 Introduction

Shi (2008, 2009) indicated P-sets (packet sets), which has dynamic characteristic, are proposed by introducing dynamic characteristic to finite general element set X (Cantor set X) and improving it [1,2]. P-sets are a kind of mathematic structure using to research the information with dynamic characteristic. Function P-sets (function packet sets), which has dynamic characteristic and law (or function) characteristic, is put forward by introducing the concept of function to P-sets and improving it [3,4]. Function P-sets is a mathematic model used to research just the class of information law with dynamic characteristic. P-sets and function P-sets, are used in the theoretical and applicative research of dynamic information and dynamic information law

H. Li et al. (Eds.): I3E 2014, IFIP AICT 445, pp. 224–237, 2014.

respectively [1-10], and they have the same logic characteristic as following: If *X* is finite general element set, or *S* is finite general function set, α is the attribute set of *X*, or α is the attribute set of *S*, then $\forall x_i \in X$ whose attribute satisfies conjunctive normal form, where x_i has attribute $\wedge_{i=1}^{k}\alpha_i$ (or $\forall s_i \in S$ whose attribute satisfies conjunctive normal form , and s_i has attribute $\wedge_{i=1}^{k}\alpha_i$). Shi (2012) introduced dynamic characteristic into finite general element set *X* and improving it at the same time, inverse P-sets, which has dynamic characteristic, is put forward [12]. Inverse P-sets is the model to research the class of information with dynamic characteristic while it is a different class from that P-sets does, and inverse P-sets is also used in the theoretical and applicative research of a class of dynamic information. Shi (2013) introduced the concept of function to inverse P-sets and improving it at the same time, function inverse P-sets is proposed [13]. Function inverse P-sets, which have dynamic characteristic and law (or function) characteristic, is the mathematic model used to research the class of dynamic information law while it is a different class from function P-sets does. Inverse P-sets and function inverse P-sets have the same logical characteristic as following: If *X* is finite general element set, or *S* is finite general function set, α is the attribute set of *X*, or α is the attribute set of *S*, then $\forall x_i \in X$ whose attribute satisfies disjunctive normal form, where x_i has attribute $\vee_{i=1}^{k}\alpha_i$ (or $\forall s_i \in S$ whose attribute satisfies disjunctive normal form , and s_i has attribute $\vee_{i=1}^{k}\alpha_i$). In this paper, the structure and characteristic of function inverse P-sets, the inverse P-information law fusion of function inverse P-sets, the attribute characteristic and attribute theorems of inverse P-information law fusion, and the hiding information image generated by inverse P-sets and its applications are given.

In order to make readers accept the concept, structure and characteristic of function inverse P-sets easily, the characteristic and structure of inverse P-sets [12] are simple introduced to Appendix, where readers can compare function inverse P-sets with inverse P-sets. In Appendix, the existence fact of inverse P-sets and P-sets [1, 2, 4, 7, 8] and the proof are given respectively.

2 Function Inverse P-sets and Its Structure

Assumption. *U*(*x*) is the finite function universe, *V*(*α*) is the finite attribute universe, and $S(x)=\{S(x)_1, S(x)_2, \cdots, S(x)_n\}$ is the finite general function set on *U*(*x*), which is called function set for short. $\alpha=\{\alpha_1,\alpha_2,\cdots,\alpha_k\}$ is the finite attribute set on *V*(*α*), and *S*(*x*) and *r*(*x*) are both the function of *x*. *U*(*x*), *V*(*α*), *S*(*x*) and *r*(*x*) are respectively written as *U*, *V*, *S* and *r* for short.

Definition 1. Given function set $S=\{s_1,s_2,\cdots,s_q\}\subset U$, if $\alpha=\{\alpha_1,\alpha_2,\cdots,\alpha_k\}\subset V$ is the attribute set of *S*, and then $\overline{S}^F$ is called function internal inverse P-set (function internal inverse packet set) of *S*, moreover

$$\overline{S}^F = S \cup S^+ \tag{1}$$

While S^{+} is called the F -function supplementary set of S, moreover

$$S^{+}=\{r \mid r\in U, r\,\overline{\in}\, S, f(r)=s'\in S, f\in F\} \quad (2)$$

If $\overline{S}^{F}$ has the attribute set α^{F}, which satisfies

$$\alpha^{F}=\alpha\cup\{\alpha' \mid f(\beta)=\alpha'\in\alpha, f\in F\} \quad (3)$$

Where $\beta\in V, \beta\,\overline{\in}\,\alpha$, and $f\in F$ can change β into $f(\beta)=\alpha'\in\alpha$ in expression (3). $S=\{s_1, s_2, \cdots, s_r\}$, $q<r$, and $q, r\in N^{+}$ in expression (1).

Definition 2. Given function set $S=\{s_1, s_2, \cdots, s_q\}\subset U$, if $\alpha=\{\alpha_1, \alpha_2, \cdots, \alpha_k\}\subset V$ is the attribute set of S, then $\overline{S}^{\overline{F}}$ is called the function outer inverse P-set (function outer inverse packet set), moreover

$$\overline{S}^{\overline{F}}=S-S^{-} \quad (4)$$

While S^{-} is called the $\overline{F}$ - function deleting set of S, moreover

$$S^{-}=\{s_i \mid s_i\in S, \overline{f}(s_i)=r_i\,\overline{\in}\, S, \overline{f}\in\overline{F}\} \quad (5)$$

If $\overline{S}^{\overline{F}}$ has the attribute set $\alpha^{\overline{F}}$, moreover

$$\alpha^{\overline{F}}=\alpha-\{\beta_i \mid \overline{f}(\alpha_i)=\beta_i\,\overline{\in}\,\alpha, \overline{f}\in\overline{F}\} \quad (6)$$

Where $\alpha_i\in\alpha$, $\overline{f}\in\overline{F}$ can change α_i into $\overline{f}(\alpha_i)=\beta_i\,\overline{\in}\,\alpha$ in expression (6); and $\overline{S}^{\overline{F}}\neq\phi$, $\alpha^{\overline{F}}\neq\phi$ in expression (4) while $\overline{S}^{\overline{F}}=\{s_1, s_2, \cdots, s_p\}$, $p<q$, and $p, q\in N^{+}$.

Definition 3. The function set pair composed of $\overline{S}^{F}$ and $\overline{S}^{\overline{F}}$, is called function inverse P-sets (function inverse packet sets) generated by function set S, moreover

$$(\overline{S}^{F}, \overline{S}^{\overline{F}}) \quad (7)$$

and finite function set S is called the ground set of function inverse P-sets $(\overline{S}^{F}, \overline{S}^{\overline{F}})$.

Using expression (3), we can get the following chain by adding attributes to α one after another,

$$\alpha_1^{F}\subseteq\alpha_2^{F}\subseteq\cdots\subseteq\alpha_{n-1}^{F}\subseteq\alpha_n^{F} \quad (8)$$

and function inter inverse P-set can be gotten from expression (8), moreover

$$\overline{S}_1^{F}\subseteq\overline{S}_2^{F}\subseteq\cdots\subseteq\overline{S}_{n-1}^{F}\subseteq\overline{S}_n^{F} \quad (9)$$

Using expression (6), we can get the following chain by deleting attributes from α one after another,

$$\alpha_n^{\overline{F}}\subseteq\alpha_{n-1}^{\overline{F}}\subseteq\cdots\subseteq\alpha_2^{\overline{F}}\subseteq\alpha_1^{\overline{F}} \quad (10)$$

and function outer inverse P-set can be gotten from expression (10), moreover

$$\overline{S}_n^{\bar{F}} \subseteq \overline{S}_{n-1}^{\bar{F}} \subseteq \cdots \subseteq \overline{S}_2^{\bar{F}} \subseteq \overline{S}_1^{\bar{F}} \tag{11}$$

Definition 4

$$\{(\overline{S}_i^F, \overline{S}_j^{\bar{F}}) | i \in \mathrm{I}, j \in \mathrm{J}\} \tag{12}$$

is called function inverse P-sets family generated by function set S, and expression (12) is the general form of function inverse P-sets, if $(\overline{S}_\lambda^F, \overline{S}_k^{\bar{F}}) \in \{(\overline{S}_i^F, \overline{S}_j^{\bar{F}}) | i \in \mathrm{I}, j \in \mathrm{J}\}$ is function inverse P-sets.

Using expressions (1) to (12), the following can be gotten.

Theorem 1. (The first reduction theorem of function inverse P-sets) Function inverse P-sets $(\overline{S}^F, \overline{S}^{\bar{F}})$ and function set S can satisfy that

$$(\overline{S}^F, \overline{S}^{\bar{F}})_{F=\bar{F}=\phi} = S \tag{13}$$

Theorem 2. (The second reduction theorem of function inverse P-sets) Function inverse P-sets $\{(\overline{S}_i^F, \overline{S}_j^{\bar{F}}) | i \in \mathrm{I}, j \in \mathrm{J}\}$ and function set S can satisfy that

$$\{(\overline{S}_i^F, \overline{S}_j^{\bar{F}}) | i \in \mathrm{I}, j \in \mathrm{J}\}_{F=\bar{F}=\phi} = S \tag{14}$$

Using the expressions (1) to (16) in part 2, part 3 is given as following.

3 Data Disassembly-Synthesis and the Generation of Inverse P-information Law Fusion

In reference [15], the following is given.

The Principle of Data Disassembly-Synthesis

Given finite data set $Y = \{y_1, y_2, \cdots, y_n\}$, there are finite sub data sets $y_i = \{y_{i,1}, y_{i,2}, \cdots, y_{i,n}\}$ while y_i is a disassembly of Y, and Y and y_i fulfill $Y = \{y_1, y_2, \cdots, y_n\} = \{\sum_{i=1}^{m} y_{i,1}, \sum_{i=1}^{m} y_{i,2}, \cdots, \sum_{i=1}^{m} y_{i,n}\}$, then Y is a synthesis of y_i. $\forall y_k, y_{k,i} \in R$, R is real number set, k=1, 2,⋯,n, i=1,2,⋯,m.

Using the principle of data disassembly-synthesis, the following can be gotten.

Definition 5. $w(x)$ is called the information law generated by function set $S = \{s_1, s_2, \cdots, s_q\}$, moreover

$$w(x) = \sum_{j=1}^{n} y_j \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \frac{x - x_i}{x_j - x_i} = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0 \tag{15}$$

If $w(x)$ is generated by Lagrange interpolation depending on the data points (x_1, y_1), $(x_2, y_2), \cdots, (x_n, y_n)$ composed by the discrete data set $y = \{y_1, y_2, \cdots, y_n\} = \{\sum_{i=1}^{q} y_{i,1}, \sum_{i=1}^{q} y_{i,2}, \cdots, \sum_{i=1}^{q} y_{i,n}\}$ of S, and $y_i = \{y_{i,1}, y_{i,2}, \cdots y_{i,n}\}$ is the discrete data set of $s_i \in S$, $i = 1, 2, \cdots, q$.

Definition 6. $\overline{w}(x)^F$ is called the inter inverse P-information law fusion of $w(x)$ generated by $\overline{S}^F$, moreover

$$\overline{w}(x)^F = b_{n-1}x^{n-1} + b_{n-2}x^{n-2} + \cdots + b_1 x + b_0 \tag{16}$$

If $\overline{w}(x)^F$ is generated by expression (16) depending on the data points $(x_1, y_1^f), (x_2, y_2^f), \cdots, (x_n, y_n^f)$ composed by the discrete data set $y^F = \{y_1^f, y_2^f, \cdots, y_n^f\} = \{\sum_{i=1}^{r} y_{i,1}, \sum_{i=1}^{r} y_{i,2}, \cdots, \sum_{i=1}^{r} y_{i,n}\}$ of $\overline{S}^F$.

Definition 7. $\overline{w}(x)^{\bar{F}}$ is called the outer inverse P-information law fusion of $w(x)$ generated by $\overline{S}^{\bar{F}}$, moreover

$$\overline{w}(x)^{\bar{F}} = c_{n-1}x^{n-1} + c_{n-2}x^{n-2} + \cdots + c_1 x + c_0 \tag{17}$$

If $\overline{w}(x)^{\bar{F}}$ is generated by expression (17) depending on the data points $(x_1, y_1^{\bar{f}})$, $(x_2, y_2^{\bar{f}}), \cdots, (x_n, y_n^{\bar{f}})$ composed by the discrete data set $y^{\bar{F}} = \{y_1^{\bar{f}}, y_2^{\bar{f}}, \cdots, y_n^{\bar{f}}\} = \{\sum_{i=1}^{p} y_{i,1}, \sum_{i=1}^{p} y_{i,2}, \cdots, \sum_{i=1}^{p} y_{i,n}\}$ of $\overline{S}^{\bar{F}}$.

Where p, q and r fulfill $p < q < r$ and $p, q, r \in N^+$ in definitions 5 to 7.

Definition 8. The information law fusion pair composed of $\overline{w}(x)^F$ and $\overline{w}(x)^{\bar{F}}$, is called the inverse P-information law fusion of $w(x)$ generated by function inverse P-sets $(\overline{S}^F, \overline{S}^{\bar{F}})$, and is called the inverse P-information law fusion for short, moreover

$$(\overline{w}(x)^F, \overline{w}(x)^{\bar{F}}) \tag{18}$$

Theorem 3. (The relation theorem between inter inverse P-information law fusion and information law) If there is a difference information law $\Delta w(x) \neq 0$, inter inverse P-information law fusion $\overline{w}(x)^F$ and information law $w(x)$ satisfy that

$$\overline{w}(x)^F - \Delta w(x) = w(x) \tag{19}$$

Theorem 4. (The relation theorem between outer inverse P-information law fusion and information law) If there is a difference information law $\nabla w(x) \neq 0$, outer inverse P-information law fusion $\overline{w}(x)^{\bar{F}}$ and information law $w(x)$ satisfy that

$$\overline{w}(x)^{\bar{F}} + \nabla w(x) = w(x) \tag{20}$$

Theorem 5. (The relation theorem between inverse P-information law fusion and information law) If there is a difference information law $(\Delta w(x), \nabla w(x))$, $\Delta w(x) \neq 0$, $\nabla w(x) \neq 0$, inverse P-information law fusion $(\overline{w}(x)^F, \overline{w}(x)^{\overline{F}})$ and informa-tion law $w(x)$ satisfy that

$$(\overline{w}(x)^F, \overline{w}(x)^{\overline{F}}) = (w(x) + \Delta w(x), w(x) - \nabla w(x)) \tag{21}$$

There are $\overline{w}(x)^F = w(x) + \Delta w(x)$ and $\overline{w}(x)^{\overline{F}} = w(x) - \nabla w(x)$ in expression (21).

It should be pointed out that the generation of information law $w(x)$ can use piecewise interpolation method, linear regression method and other methods, and the discussions are omitted.

The Engineering Background and Engineering Significance of Law Fusion

$f(t)$ is a rectangular wave or rectangular function, and $f(t)$ can be decomposed to several $\sin k\omega t$, k=1,2,⋯,m; or there is another saying, $f(t) = \sin\omega t + \sin 2\omega t + \cdots + \sin\lambda\omega t + \sin m\omega t$. If $f(t)$ and $\sin k\omega t$ are defined as laws, it is obvious that $f(t)$ is gotten by the fusion of $\sin\omega t$, $\sin 2\omega t$, ⋯, $\sin m\omega t$. Another saying, law $f(t)$is gotten by the fusion of $\sin\omega t$, $\sin 2\omega t$, ⋯, $\sin m\omega t$. Conversely, $\sin\omega t$, $\sin 2\omega t$, ⋯, $\sin m\omega t$ are the law fusion of $f(t)$. In the general mathematics, Fourier's sine series of $f(t)$ are $f(t) = \sum_{n=1}^{\infty} b_n \sin n\omega t = b_1 \sin\omega t + b_2 \sin 2\omega t + \cdots + b_\lambda \sin\lambda\omega t + \cdots$, under certain conditions, where $f(t)$ and $b_k \sin\lambda\omega t$ are defined as laws. Apparently, law $f(t)$ is the law fusion of $b_1 \sin\omega t + b_2 \sin 2\omega t + \cdots + b_\lambda \sin\lambda\omega t$. $\mathrm{Sin}\,\omega t$ is called as fundamental wave in electric engineering, and $\sin 2\omega t$ and $\sin 3\omega t$,⋯ , are called as "second harmonic" , "third harmonic", and so on.

4 The Reduction Theorem of Inverse P-information Law Fusion

Theorem 6. (The reduction theorem of inter inverse P-information law fusion) If $F = \phi$, inter inverse P-information law fusion $\overline{w}(x)^F$ and information law $w(x)$ fulfill

$$\overline{w}(x)^F_{F=\phi} = w(x) \tag{22}$$

Theorem 7. (The reduction theorem of outer inverse P-information law fusion) If $\overline{F} = \phi$, outer inverse P-information law fusion $\overline{w}(x)^{\overline{F}}$ and information law $w(x)$ fulfill

$$\overline{w}(x)^{\overline{F}}_{\overline{F}=\phi} = w(x) \tag{23}$$

Theorem 8. (The reduction theorem of inverse P-information law fusion) If $F = \overline{F} = \phi$, inverse P-information law fusion $(\overline{w}(x)^F, \overline{w}(x)^{\overline{F}})$ and information law $w(x)$ fulfill

$$(\overline{w}(x)^F, \overline{w}(x)^{\overline{F}})_{F=\overline{F}=\phi} = w(x) \tag{24}$$

Corollary 1. Inverse P-information law fusion families satisfy

$$\{(\overline{w}(x)_i^F, \overline{w}(x)_j^{\overline{F}}) \mid i \in \mathrm{I}, j \in \mathrm{J}\}_{F=\overline{F}=\phi} = w(x) \tag{25}$$

5 The Attribute Characteristic of Inverse P-information Law Fusion

Theorem 9. (The attribute theorem of inter inverse P-information law fusion) $\overline{w}(x)^F$ is the inter inverse P-information law fusion of $w(x)$ if and only if there is attribute set $\Delta\alpha \neq \phi$, and the attribute set α^F of $\overline{w}(x)^F$ and the attribute set α of $w(x)$ fulfill

$$\alpha^F - (\alpha \cup \Delta\alpha) = \phi \tag{26}$$

Theorem 10. (The attribute theorem of outer inverse P-information law fusion) $\overline{w}(x)^{\overline{F}}$ is the outer inverse P-information law fusion of $w(x)$ if and only if there is attribute set $\nabla\alpha \neq \phi$, and the attribute set $\alpha^{\overline{F}}$ of $\overline{w}(x)^{\overline{F}}$ and the attribute set α of $w(x)$ fulfill

$$\alpha^{\overline{F}} - (\alpha - \nabla\alpha) = \phi \tag{27}$$

Theorem 11. (The attribute theorem of inverse P-information law fusion) $(\overline{w}(x)^F, \overline{w}(x)^{\overline{F}})$ is the inverse P-information law fusion of $w(x)$ if and only if there is attribute sets $\Delta\alpha \neq \phi$, $\nabla\alpha \neq \phi$, and the attribute sets $(\alpha^F, \alpha^{\overline{F}})$ of $(\overline{w}(x)^F,\ \overline{w}(x)^{\overline{F}})$ and the attribute set α of $w(x)$ fulfill

$$(\alpha^F, \alpha^{\overline{F}}) - ((\alpha \cup \Delta\alpha), (\alpha - \nabla\alpha)) = \phi \tag{28}$$

There are $\alpha^F - \alpha \cup \Delta\alpha = \phi$ and $\alpha^{\overline{F}} - (\alpha - \nabla\alpha) = \phi$ in expression (28).

Using the structure of function inverse P-set in part 2 and part 3 to 5, part 6 is given as following.

6 The Hiding Information Image Generated by Inverse P-information Law Fusion and Its Application

1. The Generation of Hiding Information Image and Its Structure

Definition 9. $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^F)$, which is called the information image with two boundary, is generated by function inverse P-sets $(\overline{S}_0^F, \overline{S}_0^{\overline{F}})$, while $\overline{w}(x)_0^{\overline{F}}$ and $\overline{w}(x)_0^F$ are respectively called the lower-boundary and upper-boundary of $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^F)$.

Where a and b are the common points of $\overline{w}(x)_0^{F}$ and $\overline{w}(x)_0^{\overline{F}}$, $a \neq b$; $a, b \in R^+$; $\overline{w}(x)_0^{F}$ is the inter inverse P-information law fusion, and it is generated by $\overline{S}_0^{F}$; $\overline{w}(x)_0^{\overline{F}}$ is the outer inverse P-information law fusion, and it is generated by $\overline{S}_0^{\overline{F}}$.

Definition 10. $O^*(a, \overline{w}(x)_k^{\overline{F}}, b, \overline{w}(x)_\lambda^{F})$ is called the hiding information image of $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^{F})$, if its lower-boundary and upper-boundary respectively satisfy

$$\overline{w}(x)_0^{\overline{F}} - \overline{w}(x)_k^{\overline{F}} \geq 0 \tag{29}$$

$$\overline{w}(x)_0^{F} - \overline{w}(x)_\lambda^{F} \leq 0 \tag{30}$$

Figure 1 shows $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^{F})$ and $O^*(a, \overline{w}(x)_k^{\overline{F}}, b, \overline{w}(x)_\lambda^{F})$ visually in the form of folder line.

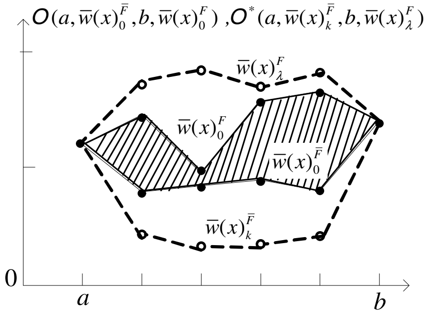

Fig. 1. $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^{F})$ is information law with two boundary, $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^{F})$ is shown in real line; $O^*(a, \overline{w}(x)_k^{\overline{F}}, b, \overline{w}(x)_\lambda^{F})$ is the hiding information image of $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^{F})$, and $O^*(a, \overline{w}(x)_k^{\overline{F}}, b, \overline{w}(x)_\lambda^{F})$ is shown in broken line; a, b are common points, and $a \neq b$; $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^{F})$ is shown in shade.

Theorem 12. (The non-unique existence theorem of hiding information image) If $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^{F})$ is the information image with two boundary, then there are some $O^*(a, \overline{w}(x)_n^{\overline{F}}, b, \overline{w}(x)_m^{F})$, and any $O^*(a, \overline{w}(x)_k^{\overline{F}}, b, \overline{w}(x)_\lambda^{F})$ in them is one of the hiding information images of $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^{F})$, $k \in (1, 2, \cdots, n)$, $\lambda \in (1, 2, \cdots, m)$.

The proof can be gotten by definitions 9 and 10, theorems 2 to 5, and corollaries 2 to 4 , and it is omitted.

2. The Application of Hiding Information Image in the Information Image Camouflage

Definition 11. $O^*(a, \overline{w}(x)_i^{\overline{F}}, b, \overline{w}(x)_j^{F})$ is called an information image camouflage of $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^{F})$, if $O^*(a, \overline{w}(x)_i^{\overline{F}}, b,$ $\overline{w}(x)_j^{F})$ is a hiding information image of $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^{F})$.

Using definitions 9 to 11, the following can be gotten.

Stealth Camouflage Principle of Information Image

Any hiding information image $O^*(a, \overline{w}(x)_p^{\overline{F}}, b, \overline{w}(x)_q^F)$ is the stealth camouflage of real information image $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^F)$, and $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^F)$ is hidden in $O^*(a, \overline{w}(x)_p^{\overline{F}}, b, \overline{w}(x)_q^F)$, or

$$O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^F) \subset O^*(a, \overline{w}(x)_p^{\overline{F}}, b, \overline{w}(x)_q^F) \tag{31}$$

In expression (31),“ $\subset$ ”expresses that $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^F)$ is surrounding by $O^*(a, \overline{w}(x)_p^{\overline{F}}, b, \overline{w}(x)_q^F)$, and $p \in (1, 2, \cdots, n)$, $q \in (1, 2, \cdots, m)$.

The example in this part is from a sub-image of an important information image, which is a two-boundary image. In order to keep easy and not lose generality, the lower-boundary and upper-boundary of sub-image $O(a, \overline{w}(x)_0^{\overline{F}}, b, \overline{w}(x)_0^F)$ and those of its hiding information images $O^*(a, \overline{w}(x)_k^{\overline{F}}, b, \overline{w}(x)_\lambda^F)$ are all expressed in folder line (here $\overline{w}(x)_0^{\overline{F}}$, $\overline{w}(x)_0^F$, $\overline{w}(x)_k^{\overline{F}}$ and $\overline{w}(x)_\lambda^F$ are all the information law fusion in the form of folder line), which can not make misunderstanding. Table one shows the discrete data of $\overline{w}(x)_0^{\overline{F}}$ and $\overline{w}(x)_0^F$, while Table two shows the discrete data of $\overline{w}(x)_k^{\overline{F}}$ and $\overline{w}(x)_\lambda^F$. The data in Table one come from the real measured value of the sub-image.

Table 1. The discrete distribution of inter inverse P-information law fusion $\overline{w}(x)_0^F$ and outer inverse P- information law fusion $\overline{w}(x)_0^{\overline{F}}$

k	1	2	3	4	5	6
$\overline{w}(x)_o^F$	1.20	1.35	0.94	1.55	1.63	1.38
$\overline{w}(x)_o^{\overline{F}}$	1.20	0.76	0.83	0.92	0.80	1.38

Table 2. The discrete distribution of inter inverse P-information law fusion $\overline{w}(x)_k^F$ and outer inverse P- information law fusion $\overline{w}(x)_\lambda^{\overline{F}}$

k	1	2	3	4	5	6
$\overline{w}(x)_k^F$	1.20	1.70	1.83	1.69	1.76	1.38
$\overline{w}(x)_\lambda^{\overline{F}}$	1.20	0.46	0.35	0.37	0.44	1.38

It should be pointed out that the data in Table 2 is gotten depending on the principle of data disassembly-synthesis and the disassembly-synthesis rule given (the coefficient of data extension and contraction is the random number on (0,1)). The real values of the coefficient of data extension and contraction are omitted for some reason.

Basing on table one and two, it can be gotten that the stealth camouflages $O^*(a, \overline{w}(x)_k^{\bar{F}}, b, \overline{w}(x)_\lambda^F)$ and $O(a, \overline{w}(x)_0^{\bar{F}}, b, \overline{w}(x)_0^F)$ of sub-image $O(a, \overline{w}(x)_0^{\bar{F}}, b, \overline{w}(x)_0^F)$ fulfill expression (31). It is difficult to finding out $O(a, \overline{w}(x)_0^{\bar{F}}, b, \overline{w}(x)_0^F)$ from hiding information image $O^*(a, \overline{w}(x)_k^{\bar{F}}, b, \overline{w}(x)_\lambda^F)$, and it is difficult to steal $O(a, \overline{w}(x)_0^{\bar{F}}, b, \overline{w}(x)_0^F)$ from $O^*(a, \overline{w}(x)_k^{\bar{F}}, b, \overline{w}(x)_\lambda^F)$, too. But it is easy for the image transmission to reduce $O^*(a, \overline{w}(x)_k^{\bar{F}}, b, \overline{w}(x)_\lambda^F)$ to $O(a, \overline{w}(x)_0^{\bar{F}}, b, \overline{w}(x)_0^F)$ by using the rule of data disassembly-synthesis. Figure 1 gives out $O(a, \overline{w}(x)_0^{\bar{F}}, b, \overline{w}(x)_0^F)$ generated by table one and $O^*(a, \overline{w}(x)_k^{\bar{F}}, b, \overline{w}(x)_\lambda^F)$ generated by table two. In order to keep simple, the lower-boundary $\overline{w}(x)_0^{\bar{F}}$ and upper-boundary $\overline{w}(x)_0^F$ of $O(a, \overline{w}(x)_0^{\bar{F}}, b, \overline{w}(x)_0^F)$, and the lower-boundary $\overline{w}(x)_k^{\bar{F}}$ and upper-boundary $\overline{w}(x)_\lambda^F$ of $O^*(a, \overline{w}(x)_k^{\bar{F}}, b, \overline{w}(x)_\lambda^F)$ are all shown in the form of folder line.

7 Discussion

Function inverse P-sets are gotten by introducing dynamic characteristic into finite general function set S and improving it. In other words, introducing the concept of function into inverse P-sets and improving it, function inverse P-sets is gotten. Function inverse P-sets has dynamic characteristic and law characteristic, and it is a new model to research the characteristic and application of a class of dynamic information laws which is different from that of function P-sets does. Function inverse P-sets have the dynamic characteristic and law characteristic, which are contrary to that of function P-sets [3-4]. And function P-sets are also a new model to research the characteristic and applications of some one class of dynamic information laws, which is a different class from the one function inverse P-sets does. Function inverse P-sets and function P-sets are two separate dynamic models, which can't be replaced by each other and can only be used separately.

In order to understand the characteristic of function inverse P-sets and compare it with inverse P-sets, the structure of inverse P-sets in expressions (1*)-(14*) are given in appendix.

Acknowledgements. This work was supported by National Natural Science Foundation of China (61273277).

References

1. Shi, K.Q.: P-sets. Journal of Shandong University (Natural Science) 43(11), 77–84 (2008) (in Chinese)
2. Shi, K.Q.: P-sets and its applications. An International Journal Advances in Systems Science and Application 2, 209–219 (2009)

3. Shi, K.Q.: Function P-sets. Journal of Shandong University (Natural Science) 2, 62–69 (2011)
4. Shi, K.Q.: P-sets and its applied characteristics. Computer Science 8, 1–8 (2010)
5. Shi, K.Q.: P-reasoning and P-reasoning discovery-identification of information. Computer Science 7, 1–9 (2011)
6. Lin, H.K., Fan, C.X., Shi, K.Q.: Backward P-reasoning and attribution residual discovery-application. Computer Science 10, 189–198 (2011)
7. Fan, C.X., Lin, H.K.: P-sets and the reasoning-identification of disaster information. International Journal of Convergence Information Technology 1, 337–345 (2012)
8. Lin, H.K., Fan, C.X.: The dual form of P-reasoning and identification of unknown attribute. International Journal of Digital Content Technology and its Applications 1, 121–131 (2012)
9. Shi, K.Q., Zhang, L.: Internal P-sets and data outer-recovery. Journal of Shandong University (Natural Science) 4, 8–14 (2009)
10. Fan, C.X., Huang, S.L.: Inverse P-reasoning discovery identification of inverse P-information. International Journal of Digital Content Technology and its Applications 20, 735–744 (2012)
11. Zhang, L., Xiu, M., Shi, K.Q.: P-sets and applications of internal-outer data circle. Quantitative Logic and Soft Computing 2, 581–592 (2010)
12. Shi, K.Q.: Inverse P-sets. Journal of Shandong University (Natural Science) 1, 98–109 (2012)
13. Shi, K.Q.: Function inverse P-sets and information law fusion. Journal of Shandong University (Natural Science) 8, 73–80 (2012)
14. Shi, K.Q.: P-sets, inverse P-sets and the intelligent fusion-filter identification of information. Computer Science 4, 1–13 (2012)
15. Shi, K.Q.: Function S-rough sets, function rough sets and separation-composition of law for information systems. Computer Science 10, 1–10 (2010)
16. Robinson, G.D., Harry, N.: Evaluation of two application of spectral mixing models to image fusion. Remote Sens Environ. 71, 272–281 (2000)
17. Varshney, P.K.: Multisensor data fusion. Journal of Electronics Computer Engineering 6, 245–253 (1999)

Appendix

Inverse P-sets and Its Structure

Given $X=\{x_1,x_2,\cdots,x_q\}\subset U$ while $\alpha=\{\alpha_1,\alpha_2,\cdots,\alpha_k\}\subset V$ is the attribute set of *X*, $\overline{X}^F$ is called the inter inverse P-sets generated by *X*, and called $\overline{X}^F$ is inter inverse P-sets for short, moreover

$$\overline{X}^F=X\cup X^+ \tag{1*}$$

X^+ is called *F*-element supplementary set of *X*, moreover

$$X^+=\{u\,|\,u\in U, u\,\overline{\in}\, X, f(u)=x'\in X, f\in F\} \tag{2*}$$

If $\overline{X}^F$ has attribute set α^F, moreover

$$\alpha^F=\alpha\cup\{\alpha'\,|\,\beta\in V,\beta\,\overline{\in}\,\alpha, f(\beta)=\alpha'\in\alpha, f\in F\} \tag{3*}$$

Here in expression (1*), $\overline{X}^{F}=\{x_1,x_2,\cdots,x_r\}$, $q,r\in N^{+}$, $q\le r$.

Given $X=\{x_1,x_2,\cdots,x_q\}\subset U$, $\alpha=\{\alpha_1,\alpha_2,\cdots,\alpha_k\}\subset V$ is the attribute set of *X*, and $\overline{X}^{\overline{F}}$ is the outer inverse P-sets of *X*, and $\overline{X}^{\overline{F}}$ is called outer inverse P-sets for short, moreover

$$\overline{X}^{\overline{F}}=X-X^{-} \tag{4*}$$

X^{-} is called the $\overline{F}$ -element deleting set of *X*, moreover

$$X^{-}=\{x\,|\,x\in X,\overline{f}(x)=u\,\overline{\in}\,X,\overline{f}\in\overline{F}\} \tag{5*}$$

If $\overline{X}^{\overline{F}}$ has attribute set $\alpha^{\overline{F}}$, moreover

$$\alpha^{\overline{F}}=\alpha-\{\beta_i\,|\,\alpha_i\in\alpha,\overline{f}(\alpha_i)=\beta_i\,\overline{\in}\,\alpha,\overline{f}\in\overline{F}\} \tag{6*}$$

Here in expression (4*), $\overline{X}^{\overline{F}}=\{x_1,x_2,\cdots,x_p\}$, $p,q\in N^{+}$, $p\le q$, $\overline{X}^{\overline{F}}\ne\phi$, and in expression (6*), $\alpha^{\overline{F}}\ne\phi$.

The element set pair composed by inters inverse P-sets $\overline{X}^{F}$ and outer inverse P-sets $\overline{X}^{\overline{F}}$, is called inverse P-sets generated by *X*, and called inverse P-sets for short, moreover

$$(\overline{X}^{F},\overline{X}^{\overline{F}}) \tag{7*}$$

and finite general element set *X* is called the ground set of inverse P-sets $(\overline{X}^{F},\overline{X}^{\overline{F}})$.

By using expression (3*), the following can be gotten

$$\alpha_1^{F}\subseteq\alpha_2^{F}\subseteq\cdots\subseteq\alpha_{n-1}^{F}\subseteq\alpha_n^{F} \tag{8*}$$

According to expression (8*), inter inverse P-sets $\overline{X}^{F}$ fulfill

$$\overline{X}_1^{F}\subseteq\overline{X}_2^{F}\subseteq\cdots\subseteq\overline{X}_{n-1}^{F}\subseteq\overline{X}_n^{F} \tag{9*}$$

By using expression (6*), the following can be gotten

$$\alpha_n^{\overline{F}}\subseteq\alpha_{n-1}^{\overline{F}}\subseteq\cdots\subseteq\alpha_2^{\overline{F}}\subseteq\alpha_1^{\overline{F}} \tag{10*}$$

According to expression (10*), outer inverse P-sets $\overline{X}^{\overline{F}}$ fulfill

$$\overline{X}_n^{\overline{F}}\subseteq\overline{X}_{n-1}^{\overline{F}}\subseteq\cdots\subseteq\overline{X}_2^{\overline{F}}\subseteq\overline{X}_1^{\overline{F}} \tag{11*}$$

By using expressions (9*) and (11*), the following can be gotten:

$$\{(\overline{X}_i^{F},\overline{X}_j^{\overline{F}})\,|\,i\in \mathrm{I},j\in \mathrm{J}\} \tag{12*}$$

Expression (12*) is called inverse P-sets family, and it is the general form of inverse P-sets.

Using expressions (1*) to (12*), the following can be gotten:

Theorem 1*. If $F=\overline{F}=\phi$, then inverse P-sets $(\overline{X}^F,\overline{X}^{\overline{F}})$ and finite general element set X fulfill

$$(\overline{X}^F,\overline{X}^{\overline{F}})_{F=\overline{F}=\phi}=X \tag{13*}$$

Theorem 2*. If $F=\overline{F}=\phi$, then inverse P-sets $\{(\overline{X}_i^F,\overline{X}_j^{\overline{F}})\,|\,i\in \mathrm{I}, j\in \mathrm{J}\}$ and finite general element set X fulfill

$$\{(\overline{X}_i^F,\overline{X}_j^{\overline{F}})\,|\,i\in \mathrm{I}, j\in \mathrm{J}\}_{F=\overline{F}=\phi}=X \tag{14*}$$

Figure 2 shows inverse P-sets $(\overline{X}^F,\overline{X}^{\overline{F}})$ directly.

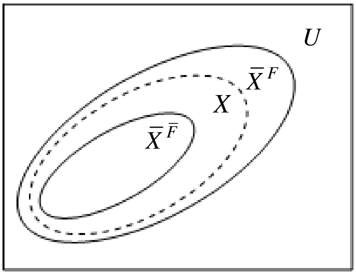

Fig. 2. U is finite element universe, X is the finite general element set on U, which has the attribute set α, and X can be shown in broken line. $\overline{X}^F$ is inter inverse P-sets, it has the attribute set α^F, and $\overline{X}^F$ can be shown in real line. $\overline{X}^{\overline{F}}$ is outer inverse P-sets, it has the attribute set $\alpha^{\overline{F}}$, and $\overline{X}^{\overline{F}}$ can be shown in real line. Inverse P-sets $(\overline{X}^F,\overline{X}^{\overline{F}})$ is composed of $\overline{X}^F$ and $\overline{X}^{\overline{F}}$.

The Existence Fact of Inverse P-sets and Its Proof

A company can produce m kinds of productions which can be named $x_1,x_2,\cdots,x_m$; $x_1,x_2,\cdots,x_m$ can compose to production universe U; $x_1,x_2,\cdots,x_q$ have the purchase contract (written as "Contract") $\alpha_1,\alpha_2,\cdots,\ \alpha_q$ respectively; and if $x_i\neq x_j$, then $\alpha_i\neq\alpha_j$. If $x_1,x_2,\cdots,x_q$ are defined to element set $X=\{x_1,x_2,\cdots,x_q\}\subset U$, and "Contract" $\alpha_1,\alpha_2,\cdots,\alpha_q$ are defined to the attribute sets of $x_1,x_2,\cdots,x_q$ respectively, apparently, set $X=\{x_1,x_2,\ \cdots,x_q\}$ has the attribute set $\alpha=\{\alpha_1,\alpha_2,\cdots,\alpha_q\}$ where $q<m, q,m\in \mathrm{N}^+$. If new attributes written as $\alpha_q,\alpha_{q+1},\cdots,\alpha_r$ are adding to α, and α can be changed to $\alpha^F=\alpha\bigcup\{\alpha_q,\alpha_{q+1},\cdots,\alpha_r\}=\{\alpha_1,\alpha_2,\cdots,\alpha_r\}$, then set $X=\{x_1,x_2,\cdots,x_q\}$ can be change to inter inverse P-sets $\overline{X}^F=X\bigcup\{x_{q+1},x_{q+2},\cdots,x_r\}=\{x_1,x_2,\cdots,x_r\}$. If $\alpha_{p+1},\alpha_{p+2},\cdots,\alpha_q$ are deleting from α, and α can be changed to $\alpha^{\overline{F}}=\alpha-\{\alpha_{p+1},\alpha_{p+2},\cdots,\alpha_q\}=\{\alpha_1,\alpha_2,\ \cdots,\alpha_p\}$, then

set $X=\{x_1,x_2,\cdots,x_q\}$ can be changed to outer inverse P-sets $\overline{X}^{\bar{F}}=X-\{x_{p+1},x_{p+2},\cdots,x_q\}=\{x_1,x_2,\cdots,x_p\}$; $p<q<r$, $p,q,r\in \mathrm{N}^+$. By promoting this fact, we can get that if some new attributes are adding to α while other attributes are deleting from α , then set X can be changed to set pair $(\overline{X}^F,\overline{X}^{\bar{F}})$, and $(\overline{X}^F,\overline{X}^{\bar{F}})$ is inverse P-sets. It is easy to get that in inverse P-sets, element x_i and attribute α_i satisfy (disjunctive normal form), and x_i has the attribute set $\vee_{i=1}^k \alpha_i$.

The Existence Fact of P-sets and Its Proof

$x_1, x_{2,}x_3, x_4, x_5$ have the same attributes α_1 = red and α_2 = sweet apple. If $x_1, x_{2,}x_3, x_4, x_5$ are defined to element set $X=\{x_1, x_{2,}x_3, x_4, x_5\}$ and both α_1 and α_2 are defined to the attributes of $x_1, x_{2,}x_3, x_4, x_5$, it is apparent that set $X=\{x_1, x_{2,}x_3, x_4, x_5\}$ has the attribute set $\alpha=\{\alpha_1,\alpha_2\}$. If the new attribute α_3 = weight 150g are adding to α , and α can be changed to $\alpha^F=\alpha\cup\{\alpha_3\}=\{\alpha_1,\alpha_2,\alpha_3\}$, then set X can be changed to inter P-sets $X^{\bar{F}}=X-\{x_1, x_{2,} x_4\}=\{x_3, x_5\}$. If α_2 is deleting from α and α is changed to $\alpha^{\bar{F}}=\alpha-\{\alpha_2\}=\{\alpha_1\}$, then set X is changed to outer P-sets $X^F=X\cup\{x_6, x_7\ x_8\}=\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\ x_8\}$. By promoting this fact, we can get that if some new attributes are adding to α while other attributes are deleting from α , then set X can be changed to set pair $(X^{\bar{F}},X^F)$, and $(X^{\bar{F}},X^F)$ is P-sets [1,2]. It is easy to get that element x_i and attribute α_i satisfy conjunctive normal form in P-sets, and x_i has the attribute set $\wedge_{i=1}^k \alpha_i$.

From the upper facts and proofs, we can get that inverse P-sets and P-sets [1,2,5,8,9] are two dynamic mathematics models which has the oppose dynamic characteristic and logical relation from each other, inverse P-sets and P-sets are two separate model, and they can not be replaced by each other.