Comparison of Chaos Driven PSO and Differential Evolution on the Selected PID Tuning Problem

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Abstract. This paper presents results of the utilization of selected discrete chaotic map, which is Dissipative standard map, as pseudo-random number generator for the differential evolution (DE) optimization algorithm and Particle Swarm Optimization (PSO) algorithm in the task of PID controller design for the selected 4[th] order dynamical system. The results are compared with previously published results; both chaos driven heuristics with each other and finally the obtained results are compared with canonical PSO and DE versions, which do not utilize the chaos in the place of pseudo-random number generator.

Keywords: Differential Evolution, PSO, Deterministic chaos, PID tuning.

1 Introduction

These days the methods based on soft computing such as neural networks, evolutionary algorithms, fuzzy logic, and genetic programming are known as powerful tool for almost any difficult and complex optimization problem.

In the past decades, PID controllers became a fundamental part of many automatic systems. The successful design of PID controller was mostly based on deterministic methods involving complex mathematics [1, 2].

Recently, different soft-computing methods were used with promising results for solving the complex task of PID controller design [3]. These techniques [5-8] use random operations and typically use various kinds of pseudo-random number generators (PRNGs) that depend on the platform the algorithm is implemented. More recently it was shown that chaotic systems could be used as PRNGs for various stochastic methods with great results. Some of these chaos driven stochastic methods were tested on the task of PID controller design in [4]. In [3] it was shown that Particle Swarm optimization (PSO) algorithm is able to deal with the task of PID controller design with very good results. Following that in [9 - 12] the performance of chaos driven PSO algorithm was tested on this task with great results.

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In this paper, the influence of promising discrete dissipative chaotic system to the performance of chaos driven heuristic algorithms, which is DE and PSO, are investigated and results are compared with previously published results of both canonical and chaos driven versions of evolutionary algorithm PSO [9 - 12] and with other techniques [3, 4] as well as with the canonical versions of DE (without chaotic pseudo-random number generator - CPRNG).

2 Motivation

Till now the chaos was observed in many of various systems (including evolutionary one) and in the last few years is also used to replace pseudo-number generators (PRGNs) in evolutionary algorithms (EAs).

This research is a continuation and extension of the previous successful initial application based experiments with chaos driven PSO and PID tuning task [9-12]. In this paper the DE/rand/1/bin strategy and PSO with inertial weight driven by Dissipative chaotic map (system) were utilized to solve the issue of evolutionary optimization of PID controller settings. Thus the idea was to utilize the hidden chaotic dynamics in pseudo random sequences given by chaotic Dissipative map system to help Differential evolution algorithm in searching for the best controller settings.

Recent research in chaos driven heuristics has been fueled with the predisposition that unlike stochastic approaches, a chaotic approach is able to bypass local optima stagnation. This one clause is of deep importance to evolutionary algorithms. A chaotic approach generally uses the chaotic map in the place of a pseudo random number generator [13]. This causes the heuristic to map unique regions, since the chaotic map iterates to new regions.

The primary aim of this work is to test, analyze and compare the implementation of different natural chaotic dynamics as the CPRNGs, thus to analyze and highlight the different influences to the system, which utilizes the selected CPRNG (including the evolutionary computational techniques).

3 PSO Algorithm

The PSO algorithm is inspired by the natural swarm behavior of animals (such as birds and fish). It was firstly introduced by Eberhart and Kennedy in 1995 [5]. Each particle in the population represents a possible solution of the optimization problem which is defined by the cost function (CF). In each iteration of the algorithm, a new location (combination of CF parameters) of the particle is calculated based on its previous location and velocity vector.

Within this research the PSO algorithm with global topology (GPSO) [14] was utilized. The CPRNG is used in the main GPSO formula (1), which determines a new "velocity", thus directly affects the position of each particle in the next iteration.

$$v_{ij}^{t+1} = w \cdot v_{ij}^{t} + c_1 \cdot Rand \cdot (pBest_{ij} - x_{ij}^{t}) + c_2 \cdot Rand \cdot (gBest_j - x_{ij}^{t}) \quad (1)$$

Where:

v_i^{t+1} - New velocity of the *ith* particle in iteration *t*+1.

w – Inertia weight value.
v_i^t - Current velocity of the *ith* particle in iteration t.
c_1, c_2 - Priority factors.
$pBest_i$ – Local (personal) best solution found by the *ith* particle.
gBest - Best solution found in a population.
x_{ij}^t - Current position of the *ith* particle (component j of the dim. D) in iteration t.
Rand – Pseudorandom number, interval ⟨0, 1⟩. CPRNG is applied only here.

The maximum velocity was limited to 0.2 times the range as it is usual. The new position of each particle is then given by (2), where x_i^{t+1} is the new particle position:

$$x_i^{t+1} = x_i^t + v_i^{t+1} \qquad (2)$$

Finally the linear decreasing inertia weight [14, 15] strategy was used in this work. The inertia weight has two control parameters w_{start} and w_{end}. The values used in this study were $w_{start} = 0.9$ and $w_{end} = 0.4$.

4 Differential Evolution

DE is a population-based optimization method that works on real-number-coded individuals [8]. For each individual $\vec{x}_{i,G}$ in the current generation G, DE generates a new trial individual $\vec{x}'_{i,G}$ by adding the weighted difference between two randomly selected individuals $\vec{x}_{r1,G}$ and $\vec{x}_{r2,G}$ to a randomly selected third individual $\vec{x}_{r3,G}$. The resulting individual $\vec{x}'_{i,G}$ is crossed-over with the original individual $\vec{x}_{i,G}$. The fitness of the resulting individual, referred to as a perturbed vector $\vec{u}_{i,G+1}$, is then compared with the fitness of $\vec{x}_{i,G}$. If the fitness of $\vec{u}_{i,G+1}$ is greater than the fitness of $\vec{x}_{i,G}$, then $\vec{x}_{i,G}$ is replaced with $\vec{u}_{i,G+1}$; otherwise, $\vec{x}_{i,G}$ remains in the population as $\vec{x}_{i,G+1}$. DE is quite robust, fast, and effective, with global optimization ability. It does not require the objective function to be differentiable, and it works well even with noisy and time-dependent objective functions. Description of used DERand1Bin strategy is presented in (3). See [8], [16] and [17] for the description of all other strategies.

$$u_{i,G+1} = x_{r1,G} + F \cdot \left(x_{r2,G} - x_{r3,G}\right) \qquad (3)$$

5 The Concept of CPRNG

The general idea of CPRNG is to replace the default PRNG with the chaotic system. As the chaotic system is a set of equations with a static start position, we created a random start position of the system, in order to have different start position for different experiments. This random position is initialized with the default PRNG, as a one-off randomizer. Once the start position of the chaotic system has been obtained, the system generates the next sequence using its current position.

Generally there exist many other approaches as to how to deal with the negative numbers as well as with the scaling of the wide range of the numbers given by the chaotic systems into the typical range 0 – 1:

- Finding of the maximum value of the pre-generated long discrete sequence and dividing of all the values in the sequence with such a maxval number.
- Shifting of all values to the positive numbers (avoiding of ABS command) and scaling.

5.1 Chaotic System for CPRNG

This section contains the description of discrete dissipative chaotic map, which was used as the CPRNG. The direct output iterations of the chaotic map were used for the generation of the both integer numbers and real numbers scaled into the typical range for random function: <0 - 1>.

The Dissipative Standard map is a two-dimensional chaotic map. The parameters used in this work are $b = 0.6$ and $k = 8.8$ as suggested in [18]. The map equations are given in (4).

$$\begin{aligned} X_{n+1} &= X_n + Y_{n+1} (\bmod 2\pi) \\ Y_{n+1} &= bY_n + k \sin X_n (\bmod 2\pi) \end{aligned} \quad (4)$$

The typical chaotic behavior of the utilized chaotic map, represented by the example of direct output for the variable x is depicted in Fig. 1.

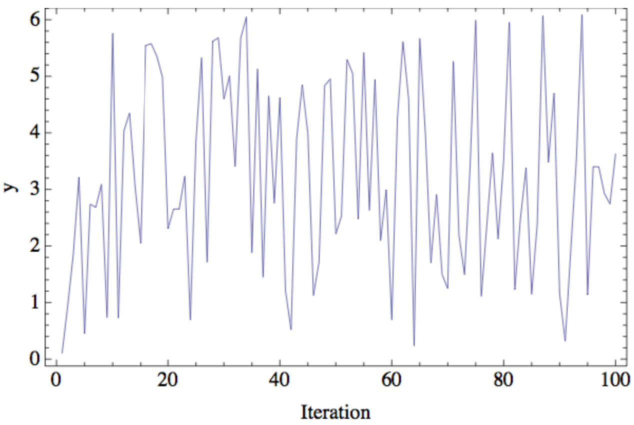

Fig. 1. Simulation of the chaotic behavior of Dissipative map (variable y – line-plot)

The illustrative histograms of the distribution of real numbers transferred into the range <0 - 1> generated by means of studied chaotic system is in Fig. 2.

Finally the Fig. 3 shows the example of dynamical sequencing during the generating of pseudo number numbers by means of studied CPRNG.

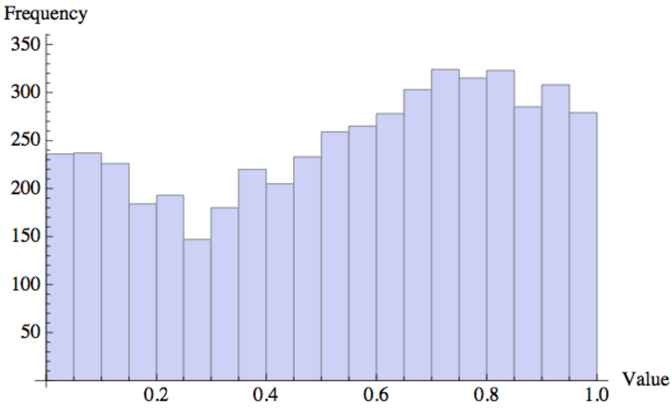

Fig. 2. Histogram of the distribution of real numbers transferred into the range <0 - 1> generated by means of the chaotic Dissipative standard map – 5000 samples

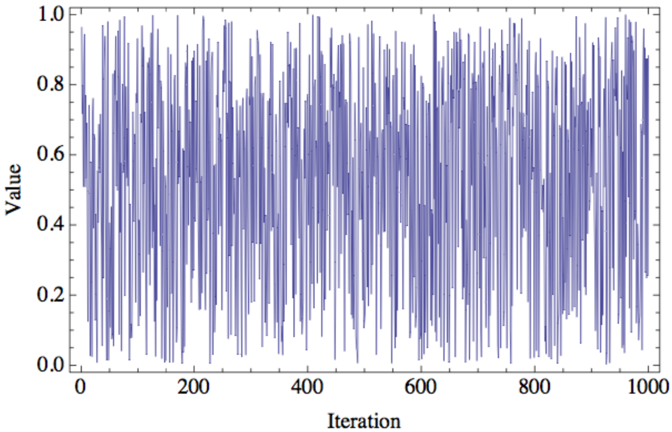

Fig. 3. Example of the chaotic dynamics: range <0 - 1> generated by means of the chaotic Dissipative standard map

6 Problem Design

6.1 PID Controller

The PID controller contains three unique parts; proportional, integral and derivative controller [1-4]. A simplified form in Laplace domain is given in (5).

$$G(s) = K\left(1 + \frac{1}{sT_i} + sT_d\right) \tag{5}$$

The PID form most suitable for analytical calculations is given in (6).

$$G(s) = k_p + \frac{k_i}{s} + k_d s \tag{6}$$

The parameters are related to the standard form through: $k_p = K$, $k_i = K/T_i$ and $k_d = KT_d$. Acquisition of the combination of these three parameters that gives the lowest value of the test criterions was the objective of this research. Selected controlled system was the 4th order system that is given by Eq. 7.

$$G(s) = \frac{1}{s^4 + 6s^3 + 11s^2 + 6s} \tag{7}$$

6.2 Cost Function

Test criterion measures properties of output transfer function and can indicate quality of regulation [1-4]. Following four different integral criterions were used for the test and comparison purposes: IAE (Integral Absolute Error), ITAE (Integral Time Absolute Error), ISE (Integral Square Error) and MSE (Mean Square Error). These test criterions (given by Eq. 8 – 11) were minimized within the cost functions for the enhanced PSO algorithm.

1. Integral of Time multiplied by Absolute Error (ITAE)

$$I_{ITAE} = \int_0^T t\left|e(t)\right| dt \tag{8}$$

2. Integral of Absolute Magnitude of the Error (IAE)

$$I_{IAE} = \int_0^T \left|e(t)\right| dt \tag{9}$$

3. Integral of the Square of the Error (ISE)

$$I_{ISE} = \int_0^T e^2(t) dt \tag{10}$$

4. Mean of the Square of the Error (MSE)

$$I_{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(e(t)\right)^2 \tag{11}$$

7 Results

In this section, the results obtained within experiments with ChaosDE and Chaos PSO algorithms are compared with each other and with previously published works [3, 4, 9 - 12]. Table 1 shows the typical used settings for the both ChaosDe and Canonical DE, whereas Table 2 contains the settings for both Chaos PSO and canonical PSO.

Table 1. DE settings

DE Parameter	Value
PopSize	20
F	0.8
CR	0.8
Generations	50
Max. CF Evaluations (CFE)	1000

Table 2. PSO settings

DE Parameter	Value
PopSize	20
v_{max}	0.2•Range
w_{start}	0.9
w_{end}	0.4
Priority factors c_1 and c_2	2
Iterations	50
Max. CF Evaluations (CFE)	1000

Table 3. Comparisons of results for other heuristics – 4^{th} order system PID controller design

Criterion	ZN Step Response	Canonical DE	Chaos DE	Chaos SOMA	PSO	Chaos PSO
IAE	34.9413	12.3262	**12.3260**	12.3305	12.3738	12.3479
ITAE	137.5650	15.1935	**15.1919**	15.3846	16.4079	15.5334
ISE	17.8426	**6.40515**	**6.40515**	6.41026	6.40538	6.40516
MSE	0.089213	**0.032026**	**0.032026**	0.032027	0.032030	**0.032026**

Table 4. Statistical results of all 50 runs of Both Chaos heuristics versions

DE Version	Avg CF	Median CF	Max CF	Min CF	StdDev
ITAE Criterion					
Chaos PSO Dissipative Map	12.4184	12.3959	12.6140	12.3479	0.072049
Chaos DE Dissipative Map	**12.3274**	**12.327**	**12.3314**	**12.3262**	**0.001216**
IAE Criterion					
Chaos PSO Dissipative Map	17.6267	17.4012	21.5345	15.5334	1.594303
Chaos DE Dissipative Map	**15.2251**	**15.2127**	**15.3212**	**15.1919**	**0.033799**
ISE Criterion					
Chaos PSO Dissipative Map	6.4059	6.4057	6.4083	6.40516	0.000841
Chaos DE Dissipative Map	**6.40516**	**6.40516**	**6.40517**	**6.40515**	**0.000025**
MSE Criterion					
Chaos PSO Dissipative Map	0.03203	**0.03202**	0.03206	**0.03202**	$8 \cdot 10^{-6}$
Chaos DE Dissipative Map	**0.03202**	**0.03202**	**0.03202**	**0.03202**	$\mathbf{9 \cdot 10^{-9}}$

Best results obtained for each method are given in Table 3. The statistical results of the experiments for all criterions are shown in Table 4, which represent the simple statistics for cost function (CF) values, e.g. average, median, maximum values, standard deviations and minimum values representing the best individual solution for all 50 repeated runs of canonical DE and ChaosDE. The bold values within the all Tables 3 and 4 depict the best obtained results.

Furthermore an example of the step responses of the system with PID controllers designed by means of Chaos DE is depicted in Fig 4.

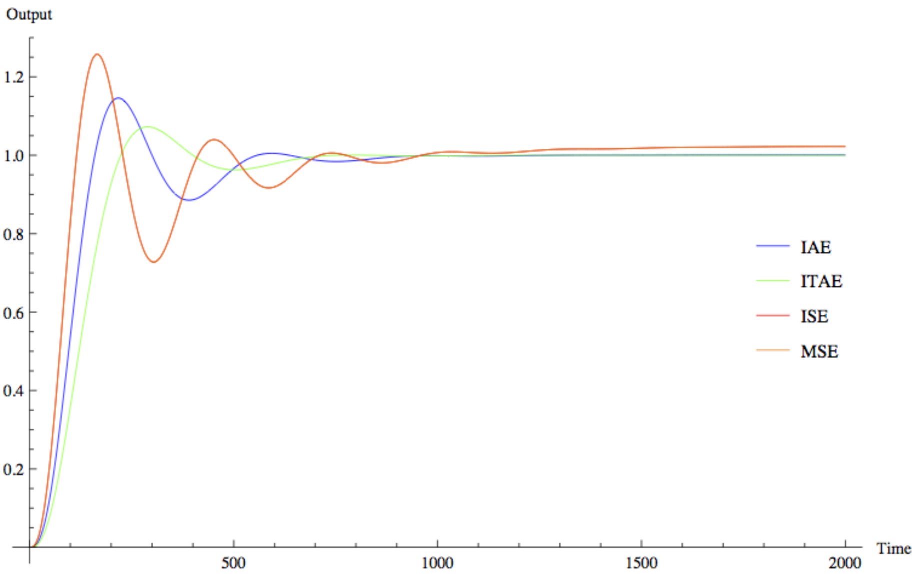

Fig. 4. Comparison of system responses – 4th order system; Chaos DE

8 Conclusion

In this paper the chaotic dissipative standard map was presented and investigated over their capability of enhancing the performance of DE and PSO algorithms in the task of PID controller design.

From the comparisons, it follows that through the utilization of chaotic systems; the best overall results were obtained and entirely different statistical characteristics of CPRNGs-based heuristic can be achieved. Thus the different influence to the system, which utilizes the selected CPRNG, can be chosen through the implementation of particular inner chaotic dynamics given by the particular chaotic system. When comparing both chaos driven heuristics, DE has outperformed PSO.

Promising results were presented, discussed and compared with other methods of PID controller design. More detail experiments are needed to prove or disprove these claims and explain the effect of the chaotic systems on the optimization and controller design.

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