5 Beam Manipulations in Photoinjectors

The design of an electron source is a challenging task. The designer must reconcile the contradictory requirements for a small emittances, a high charge, a high repetition rate, and, possibly, a high degree of beam polarization.

Electron beams can be generated in a variety of ways. Accordingly a number of different devices exist which can serve as electron sources for linear colliders: thermionic guns, dc guns with laser photocathodes (used at the SLC), or rf guns. In the future, also polarized rf guns may become available.

In this chapter, we first outline the general principle of an rf photoinjector, emphasizing the limits on the minimum emittance that it can produce. We then discuss two approaches for manipulating, shaping and preserving the transverse emittance of the beam generated by such a photoinjector, namely the compensation of space-charge induced emittance growth using a solenoid, and the flattening of the beam by the combined action of a solenoid and subsequent skew quadrupoles.

5.1 RF Photoinjector

In a laser-driven rf gun, or rf photoinjector, a high-power pulsed laser illuminates a photocathode placed on the end wall of an rf cavity. The emitted electrons are accelerated immediately in the rf field. The time structure of the electron beam is controlled by the laser pulse, and the rapid acceleration minimizes the effect of space-charge repulsion.

Several effects contribute to the normalized emittance attainable by such an rf gun [1]:

- The *thermal* emittance is determined by the initial transverse momenta of the electrons at the moment of their emission. It can be estimated as

$$\gamma\epsilon_{x,y}^{th}\ [\mathrm{mm\ mrad}] \approx \frac{1}{4}\sqrt{\frac{k_B T_\perp}{m_e c^2}}\ \sigma_{x,y}\ [\mathrm{mm}]\,, \tag{5.1}$$

  where $k_B T_e \approx 0.1$ eV represents the thermal emission temperature.
- An *rf emittance* arises from the time-dependent transverse focusing in the rf field. At the exit of the rf structure, it is approximately given by

M. G. Minty et al., *Measurement and Control of Charged Particle Beams*,
https://doi.org/10.1007/978-3-662-08581-3_5

$$\gamma\epsilon_{x,y}^{\mathrm{rf}}\ [\mathrm{mm\ mrad}] \approx \frac{eE_{\mathrm{rf}}}{\sqrt{8}m_E c^4}\ \sigma_{x,y}^2 \sigma_z^2 \omega_{\mathrm{rf}}^2\,, \tag{5.2}$$

where E_{rf} denotes the peak accelerating field.

- The space-charge emittance arises from the repelling force between the equally charged beam particles. Taking into account the focusing component of the rf field, the residual space-charge emittance is [2]

$$\gamma\epsilon_{x,y}^{\mathrm{sc}}\ [\mathrm{mm\ mrad}] \approx \frac{2N_b r_e}{7\sigma_{x,y}W}\ \exp\left(-3\sqrt{W\sigma_y}\right)\ \sqrt{\frac{\sigma_y}{\sigma_z}}\,, \tag{5.3}$$

where $W = eE_{\mathrm{rf}} \sin\phi_0/(2m_e c^2)$ and ϕ_0 is the rf phase at the beam center. Since the transverse space-charge force depends on the local charge density of the bunch, it disorients in phase space the transverse slices located at different longitudinal positions along the bunch. For round beams this dilution can be almost fully inverted by properly placed solenoids [3], as described in the following section.

5.2 Space-Charge Compensation

Nowadays, photoinjectors, rather than thermionic injectors, are used for all applications requiring the combination of high-peak current and low emittance [3]. After the electron emission from the cathode, at low energies, space charge forces are very important. Here we follow closely the work of B. Carlsten [3].

We first consider the case without compensation and also neglect rf focusing effects. In this case, scaling arguments, supported by simulations, show that the transverse emittance of a 'slug' beam of length L and radius a with peak current I grows to a value [3, 4]

$$\epsilon_{xN} \approx \frac{eIs}{16\pi\epsilon_0 m_0 c^3 \gamma^2 \beta^2}\ G\,, \tag{5.4}$$

provided that the bunch does not strongly deform over the drift distance s. The geometric factor G depends on the beam aspect ratio in the beam frame, $\left(\frac{\gamma L}{a}\right)$, and on the longitudinal distribution. In the long-bunch limit and assuming that the radial distribution is uniform, G can be calculated to be 0.556 for a Gaussian longitudinal distribution and 0.214 for a parabolic distribution.

The radial space-charge force is a function of position within the bunch. Following [3] we introduce cylindrical coordinates ρ and ξ within the bunch, $\rho = 1$ defining the radial edge, and $\xi = \pm 1$ the longitudinal ends. There is no emittance growth if the radial force is linear in ρ and independent of ξ [3], or, equivalently, if

$$\Lambda(\rho, \xi, t) \equiv \frac{eE_r(\rho, \xi, t)}{m_0\gamma^3\beta^2c^2} = \rho_0\Lambda_0(t)\,, \tag{5.5}$$

where we have introduced the normalized force Λ, and E_r is the radial electric field in the laboratory frame.

If the longitudinal bunch density is not a constant, this condition is not fulfilled, and there will be a growth in the transverse emittance because different slices of the beam experience different radial space-charge forces. It is the projected emittance that increases, while the emittance of each short slice remains constant. In phase space the slices rotate against each other.

Now there exists an elegant method by use of a focusing solenoid to realign the different slices in the same phase space direction, and thus to recover the original emittance.

We consider again a slug beam. For simplicity, we assume that the space-charge force does not vary in time. If initially the beam at location $z = 0$ is non-divergent and has a radius r_0, a point in the slug at coordinates (ρ, ξ) will execute a non-relativistic transverse motion, so that after a distance z its radial coordinates will be

$$r(\rho, \xi, z) = \rho r_0 + \Lambda(\rho, \xi)\frac{z^2}{2} \tag{5.6}$$

and

$$r'(\rho, \xi, z) = \Lambda(\rho, \xi)z \tag{5.7}$$

after a distance z. We now place a lens (in practice, this lens is a solenoid) at the position $z = z_l$, and choose its focal length equal to [3]

$$f = \frac{z_d^2}{2(z_l + z_d)}\,, \tag{5.8}$$

where z_d denotes the distance from the lens to a point downstream. At this point, the ratio of the beam divergence to its radius becomes

$$\frac{r'(\rho, \xi)}{r(\rho, \xi)} = \frac{2(z_l + z_d)}{z_d(z_d + 2z_l)}\,, \tag{5.9}$$

which is independent of the particle's motion within the bunch. Thus the effect of the lens was to back-rotate the slices along the bunch with respect to each other so that they are again re-aligned after the total distance (z_l+z_d). The normalized emittance can be written as [3]

$$\epsilon_{x,y} = \frac{1}{2}\beta\gamma\sqrt{\langle\Lambda^2\rangle\langle\rho^2\rangle - \langle\Lambda\rho\rangle^2}\,\left(2r_0(z_l + z_d) - \frac{z_d^2 r_0}{f}\right)\,, \tag{5.10}$$

where r_0 is the initial beam radius and the angular brackets indicate an average over the beam distribution. Equation (5.10) confirms that the emittance vanishes with the proper choice of lens (focal length f). The compensation recipe is illustrated schematically in Fig. 5.1.

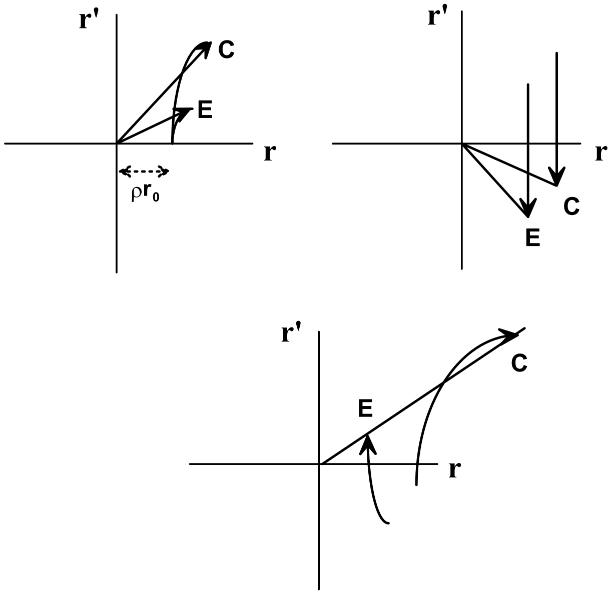

Fig. 5.1. Space charge compensation in photoinjectors. The two arrows illustrate the particle motion at the center (C) and at the end (E) of the bunch: (1) after initial drift, (2) after solenoid focusing, (3) after final drift until slice emittances are realigned [3]

In reality the physics is not quite so simple. In particular, the space-charge force is not constant in time. This complication results in a residual nonzero emittance. Nevertheless, already in the first beam experiments performed at Los Alamos [3] the above compensation scheme was shown to reduce the normalized rms emittance by up to an order of magnitude.

Let us assume the beam is focused to a beam-radius minimum. If the space-charge forces are weak compared with the external focusing, all particles cross through the beam's center. This can be called a *crossover* [3]. On the other hand, for strong space-charge forces, the particles will be deflected away from the center. This may be called a *waist* [3], but be careful not to confuse this with the notion of beam waist used to describe a generic position of minimum beam radius. In general, parts of the bunch will have a high density and particles there will experience a waist, while particles in the other parts will crossover. Indeed there exist particles at the border between these two regions, which are initially extremely close together and later on will be a finite distance apart. This is called a phase-space *bifurcation* [3].

The space-charge induced emittance growth can only be compensated for those particles which do not cross over, and only for those do the above approximations apply. Therefore, one of the most important design criteria for photoinjectors is to minimize the fraction of the beam crossing over.

The technique described here may be generalized to other situations where one wants to correct a correlated growth in the projected emittances that is induced by a nonlinear force.

5.3 Flat-Beam Transformation

Linear colliders require flat electron beams at the collision point, in order to maximize the luminosity while limiting the amount of synchrotron radiation emitted during the collision in the field of the opposing beam (this radiation is called *beamstrahlung*). Unfortunately, electron guns usually produce round beams.

A scheme by which one can transform a round beam ($\epsilon_x = \epsilon_y$) into a flat beam ($\epsilon_x \gg \epsilon_y$) was proposed by Y. Derbenev, R. Brinkmann, and K. Flottmann in 1999 [5, 6, 7]. We describe the idea following Edwards [8]. The basic scheme consists of two parts:

- the beam from a cathode immersed in a solenoidal field develops an angular momentum at exit from the solenoid;
- subsequently this beam is passed through a quadrupole (or skew quadrupole) channel with 90° phase advance difference between the two planes, and length scale defined by the solenoid field.

Consider electrons moving parallel to a solenoid field whose axis is oriented in the z direction. Maxwell's equations imply the presence of a radial magnetic field at the exit of the solenoid. This radial field gives rise to a transverse deflection, which depends on the distance from the solenoid axis. For example, the vertical deflection at the solenoid exit is

$$\Delta y' = \frac{1}{B\rho} \int B_x \, \mathrm{d}z = \frac{1}{B\rho} \frac{x_0}{2} B_z \,, \tag{5.11}$$

where B_z is the longitudinal field inside the solenoid and x_0 the horizontal offset. A similar expression holds for $\Delta x'$. Abbreviating, we write $\Delta y' = kx_0$, $\Delta x' = -ky_0$ with $k = B_z/(2B\rho)$. After leaving the solenoid, the beam takes on a clock-wise rotation

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_0 = \begin{pmatrix} x_0 \\ -ky_0 \\ y_0 \\ kx_0 \end{pmatrix} . \tag{5.12}$$

We have neglected any initial uncorrelated momenta, assuming that these are much smaller than kx_0 or ky_0. However, actually these terms are important, as they determine the final flat-beam emittance. We will see this below.

Suppose now that the quadrupole channel behind the solenoid produces an I matrix in x and an additional 90° phase advance in y:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & -1/\beta & 0 \end{pmatrix} \begin{pmatrix} x_0 \\ -ky_0 \\ y_0 \\ kx_0 \end{pmatrix} = \begin{pmatrix} x_0 \\ -ky_0 \\ k\beta x_0 \\ -\frac{1}{\beta} y_0 \end{pmatrix} . \tag{5.13}$$

If we choose $\beta = 1/k$, the final phase-space vector becomes

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_1 = \begin{pmatrix} x_0 \\ -ky_0 \\ x_0 \\ -ky_0 \end{pmatrix} . \tag{5.14}$$

This is a flat beam inclined at 45°. If one uses a skew quadrupole channel instead of quadrupole channel, the beam can be made flat in the vertical plane, as shown next.

The general 4×4 transport matrix from the end of the solenoid through the skew quadrupole channel can be written as

$$M = R^{-1} T R \tag{5.15}$$

with

$$R = \frac{1}{\sqrt{2}} \begin{pmatrix} I_2 & I_2 \\ -I_2 & I_2 \end{pmatrix} , \tag{5.16}$$

where I_2 is 2×2 identity, and the matrix T represents a normal quadrupole channel:

$$T = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} . \tag{5.17}$$

Combining the above, we write M as

$$M = \frac{1}{2} \begin{pmatrix} A+B & A-B \\ A-B & A+B \end{pmatrix} . \tag{5.18}$$

The initial state after the solenoid exit is

$$X \equiv \begin{pmatrix} x_0 \\ -ky_0 \end{pmatrix} \quad \text{and} \quad Y \equiv \begin{pmatrix} y_0 \\ kx_0 \end{pmatrix} , \tag{5.19}$$

which we write more elegantly as

$$Y = SX \quad \text{using} \quad S \equiv \begin{pmatrix} 0 & -1/k \\ k & 0 \end{pmatrix} . \tag{5.20}$$

The final state is then

$$\begin{pmatrix} X \\ Y \end{pmatrix}_1 = \frac{1}{2} \begin{pmatrix} \{A+B+(A-B)S\}X \\ \{A-B+(A+B)S\}X \end{pmatrix} , \tag{5.21}$$

and the condition for a flat beam is $Y_1 = 0$, or $I = -(A-B)^{-1}(A+B)S$.

Using the Courant–Snyder parametrization [9] $A = \exp(J\mu)$, $B = \exp(J(\mu + \Delta))$, where J denotes the matrix

$$J = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} , \tag{5.22}$$

the flat-beam condition becomes

$$I = -\frac{\cos(\Delta/2)}{\sin(\Delta/2)} \begin{pmatrix} k\beta & \alpha/k \\ -k\alpha & \gamma/k \end{pmatrix} . \tag{5.23}$$

This is fulfilled for $\Delta = -\pi/2$, $\alpha = 0$ and $\beta = 1/k$.

Finally, adding a random component to the slope of the initial vector, so that (5.12) is replaced by

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_0 = \begin{pmatrix} x_0 \\ -ky_0 + x'_0 \\ y_0 \\ kx_0 + y'_0 \end{pmatrix} , \tag{5.24}$$

we can apply the same transformation M as above, (5.18), and, assuming that the beam at the source is round with $\sigma_{x0} = \sigma_{y0}$, $\sigma'_{x0} = \sigma'_{y0}$, and no initial correlation between the two transverse planes (e.g., $\langle x'_0 y'_0 \rangle = 0$), we find [6]

$$\epsilon_{y,1} = \frac{1}{2} \frac{{\sigma'_{y0}}^2}{k} \tag{5.25}$$

and

$$\epsilon_{x,1}/\epsilon_{y,1} = 1 + 4k^2 \frac{\sigma_{x0}^2}{{\sigma'_{x0}}^2} . \tag{5.26}$$

The larger the value of k, i.e., the stronger the solenoid field, the flatter the beam becomes.

First experimental tests of a flat beam electron source at Fermilab have demonstrated the viability of this scheme [10]. A similar application, which employs the inverse (flat-to-round) transformation, is the matching of a flat electron beam to a round proton beam, e.g., for electron cooling [5].

Exercises

5.1 Solenoidal Focusing

Verify that the ratio of the beam divergence to its radius in an rf photoinjector is given by (5.9).

5.2 Flat-Beam Transformer

a) Calculate the explicit form of the matrix M, (5.18), for $\mu = 2\pi$, $\Delta = -\pi/2$, $\alpha = 0$ and $\beta = 1/k$. See also the definition of A and B above (5.22).

b) Using the result, verify (5.25) and (5.26).