10 Komplexe Zahlen

10.1 a) $x_1 = -3 + i\,,\, x_2 = -3 - i$ b) $x_1 = 3 + 2\,i\,,\, x_2 = 3 - 2i$ c) $x_1 = -1 + 3i,\ x_2 = -1 - 3i$

10.2 a) $z_S = -1 + 5i, z_D = 5 - i$ b) $z_S = -9 - i, z_D = -1 + 9i$ c) $z_S = -6i, z_D = -4$

10.3 a) $1 - i$ b) $1 + i$ c) $1 - 5i$ d) $5 + 5i$ e) $1 + 5i$ f) $-5 - i$

10.4 a) $|z_1| = \sqrt{6^2 + 8^2} = 10$ b) $|z_2| = \sqrt{12^2 + 5^2} = 13$ c) $|z_3| = \sqrt{20^2 + 15^2} = 25$

10.5 a) $z_1 + z_2 = (a + bi) + (c + di) = (a + c) + (b + d)i$

$$\Rightarrow \overline{z_1 + z_2} = (a + c) - (b + d)i \text{ oder}$$

$$\overline{z_1 + z_2} = (a - bi) + (c - di)$$

$$\Rightarrow \overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

b) $z_1 \cdot z_2 = (a + bi) \cdot (c + di) = ac + ad \cdot i + bc \cdot i + bd \cdot i^2$

$$z_1 z_2 = (ac - bd) + (ad + bc)i$$

$$\Rightarrow \overline{z_1 z_2} = (ac - bd) - (ad + bc)i \text{ oder}$$

$$\overline{z_1 z_2} = ac - ad \cdot i - bc \cdot i - bd = ac - ad \cdot i - bc \cdot i + bd \cdot i^2 \text{ oder}$$

$$\overline{z_1 z_2} = (a - b \cdot i) \cdot (c - d \cdot i)$$

$$\Rightarrow \overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$$

10.6 a) $-12 - 5i$ b) $2 + 23i$ c) $-7 + 22i$

10.7 $z_1 \cdot z_2 = (a + bi) \cdot (c + di) = ac + ad \cdot i + bc \cdot i + bd \cdot i^2$ oder

$$z_1 z_2 = ac + bc \cdot i + ad \cdot i + bd \cdot i^2 \text{ oder}$$

$$z_1 z_2 = (c + di)(a + bi)$$

$$\Rightarrow z_1 z_2 = z_2 z_1$$

10.8

a)	b)	c)
$\frac{z_1}{z_2} = -i$	$\frac{z_1}{z_2} = -\frac{22}{13} - \frac{7}{13}i$	$\frac{z_1}{z_2} = \frac{23}{41} + \frac{2}{41}i$
$\frac{z_2}{z_1} = i$	$\frac{z_2}{z_1} = -\frac{22}{41} + \frac{7}{41}i$	$\frac{z_2}{z_1} = \frac{23}{13} - \frac{2}{13}i$

10.9 $\frac{z_2}{z_1} = \frac{(c + di)(a - bi)}{(a + bi)(a - bi)} = \frac{ac - bc \cdot i + ad \cdot i + bd}{a^2 + b^2} \Rightarrow \frac{z_2}{z_1} = \frac{ac + bd}{a^2 + b^2} + \frac{ad - bc}{a^2 + b^2} \cdot i$

K.-H. Pfeffer, *Lösungsbuch zur Analysis für technische Oberschulen*, DOI 10.1007/978-3-658-06380-1_10