Fluctuations of Josephson Currents in Mesoscopic Systems

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Abstract. Various aspects of fluctuations in Josephson currents are studied for superconductor - normal metal - superconductor (SNS) systems with mesoscopic normal regions. In the system with diffusive normal region, relative magnitude of the fluctuation to the average is calculated. Effects of pair breaking on fluctuations are investigated especially in the systems with spin glass ordering, where it is found that the fluctuation remains finite even in the strong depairing limit although the average vanishes. In the system with Aharonov - Bohm ring geometry, we find hc/e oscillation in the critical current besides the well known $hc/2e$ oscillation.

1. Introduction

One of the most remarkable features in mesoscopic systems is the clear manifestation of interference effects of electron waves due to the persistence of the phase coherence of electrons. For example, the conductance undergoes random time - independent fluctuations as a function of the magnetic field that varies from sample to sample but is reproducible (magnetofingerprint)[1] and the amplitude of the fluctuation is of the order of e^2/h (Universal Conductance Fluctuation)[2,3]. Furthermore, the conductance in Aharonov - Bohm (AB) ring geometry shows the periodic oscillation with AB flux besides the aperiodic one mentioned above, which is known as the AB effect and the most remarkable example showing persistence of the phase coherence of electron waves in mesoscopic systems[1].

Although such fluctuations are usually of a fraction of the average, these can be comparable to or even larger than the average in some cases. The typical example is the fluctuation of orbital magnetism[4]. Even in good metallic samples, its fluctuation can be far bigger than the average (Landau diamagnetism) in isolated systems[5] and in systems attached to the perfect leads[6,7].

The other systems where the phase coherence is maintained are superconductors. Their phase coherence, however, is due to the macroscopic condensation of Cooper pairs and persists over a macroscopic scale. If a superconductor (S) has a boundary with a normal metal (N), the amplitude of Cooper pair condensate in the superconductor can penetrate into the normal region, which is known as proximity effects leading to finite Josephson currents in SNS systems[8,9]. Hence, if the N region in an SNS system is of mesoscopic scale, interesting interplay of two different kinds of phase coherence will be expected, which we will explore in this paper. This problem seems to be important as a first step to understand the relationship of phases between mesoscopic systems and superconductors.

The mesoscopic fluctuation in Josephson currents has been discussed by Al'tshuler *et al.*[10] and Beenakker[11] for wire geometry. In this paper, we will study a similar problem with particular emphasis on the effects of spin glass ordering. Moreover, a similar problem in the AB ring structure is also studied.

We take a unit of $k_B = \hbar = 1$.

2. Disordered N Region with Wire Geometry

First we will study the effect of mesoscopic fluctuations on the Josephson current in simple wire geometries shown in Fig.1.

2-1) The Case of Normal Impurity Scattering

We begin with the formula of Josephson currents given in the lowest order in $\Delta^l(\epsilon_n)$ and $\Delta^r(\epsilon_n)$ at S_l and S_r (S_l and S_r are superconductors attached to the left and the right side of the N region, respectively) :

$$I = 4ieT \sum_{\epsilon_n>0} \int d\vec{r}_{1\perp} d\vec{r}_{2\perp} \Big[X(\vec{r}_1, \vec{r}_2; \epsilon_n) \Delta^l(\epsilon_n) \Delta^{r*}(\epsilon_n) - X(\vec{r}_2, \vec{r}_1; \epsilon_n) \Delta^r(\epsilon_n) \Delta^{l*}(\epsilon_n) \Big], \tag{1}$$

where $\epsilon_n = (2n+1)\pi T$ and $\vec{r}_1 = (0, \vec{r}_{1\perp})$, $\vec{r}_2 = (L, \vec{r}_{2\perp})$ with 0 and L being the x coordinates of S_lN and NS_r interfaces, respectively. In eq.(1), $X(\vec{r}_1, \vec{r}_2; \epsilon_n) = G(\vec{r}_1, \vec{r}_2; \epsilon_n) G(\vec{r}_1, \vec{r}_2; -\epsilon_n)$ represents the penetration of Cooper pairs into the N region and $\Delta^{(l,r)}(\epsilon_n) = C\Delta \exp\{i\theta_{(l,r)}\} / \sqrt{\epsilon_n^2 + \Delta^2}$[12] is essentially the amplitude of the pair condensate at the S_lN and NS_r interface, $F^{(l,r)}(\vec{r}_{(1,2)}, \vec{r}_{(1,2)}; \epsilon_n)$, and C is a constant depending on the details of the SN interface. The Δ and $\theta_{(l,r)}$ are the gap and the phase of superconductors, respectively. Noting time reversal symmetries, we obtain the average, $< I(\varphi) >$, and the fluctuation, $< \delta I(\varphi)^2 >$, of Josephson currents as

$$< I(\varphi) > = 8eT \sum_{\epsilon_n>0} |\Delta^l(\epsilon_n)||\Delta^r(\epsilon_n)| \int d\vec{r}_{1\perp} d\vec{r}_{2\perp} < X(\vec{r}_1, \vec{r}_2; \epsilon_n) > \sin\varphi, \tag{2}$$

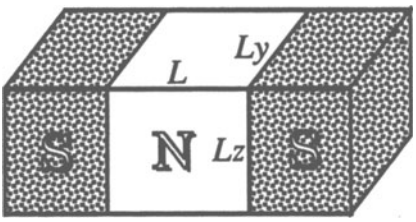

Fig.1
SNS junction
with simple wire geometry.

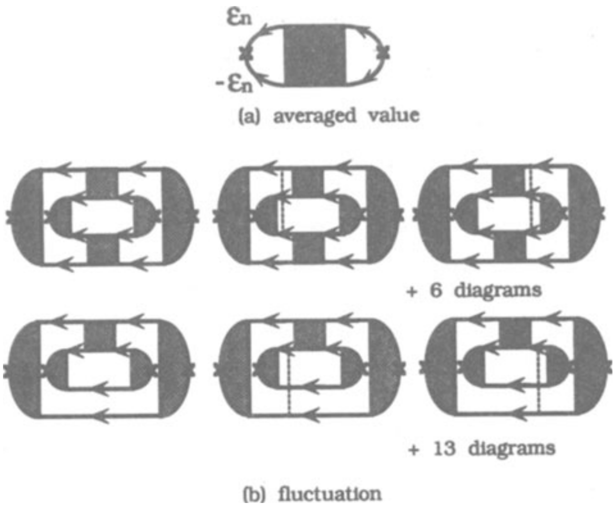

Fig.2 Diagrams for averaged value and fluctuation.
A dotted line means a single impurity scattering.

$$< \delta I(\varphi)^2 >= 64e^2T^2 \sum_{\epsilon_n,\epsilon'_n>0} |\Delta^l(\epsilon_n)||\Delta^r(\epsilon_n)||\Delta^l(\epsilon'_n)||\Delta^r(\epsilon'_n)|$$

$$\int d\vec{r}_{1\perp}d\vec{r}_{2\perp}d\vec{r}'_{1\perp}d\vec{r}'_{2\perp} < X(\vec{r}_1,\vec{r}_2;\epsilon_n)X(\vec{r}'_1,\vec{r}'_2;\epsilon'_n) > \sin^2\varphi, \quad (3)$$

where φ is the phase difference between S_l and S_r, $\varphi \equiv \theta_l - \theta_r$, and $< \cdots >$ implies impurity averaging. Here the phase difference, φ, is taken to be fixed. To the lowest order in $1/\epsilon_F\tau$, ϵ_F and τ being the Fermi energy and the life time of elastic scattering in the N region, the average and the fluctuation are given by the diagrams shown in Figs.2 (a) and (b), respectively.

In this calculation, two kinds of cooperons are needed; one connecting the same system and the other connecting different systems. In the high temperature regime, where $L > \xi_D$ with $\xi_D = \sqrt{D/2\pi T}$ being the coherence length in the N region in the diffusive regime with the diffusion constant, D, the average current and the relative intensity of the fluctuation to the average are given as follows:

$$< I >= 32eT\sin\varphi|\Delta^l(\pi T)||\Delta^r(\pi T)|\pi N_0 S\frac{\xi_D}{D}\exp(-L/\xi_D), \quad (4)$$

$$\frac{< \sqrt{\delta I^2} >}{< I >} \simeq \frac{1}{N_0 SD/L}\sqrt{c_d}, \quad (5)$$

where N_0 is the density of states per unit volume in the N region and $S = L_yL_z$ represents the cross sectional area. The c_d's are constants depending on the dimensionality of the cooperons and are given as follows: $c_1 \simeq 1$ for $L > \xi_D > L_y, L_z$; $c_2 \simeq L_y/\sqrt{\xi_D L}$ for $L, L_y > \xi_D > L_z$ and $c_3 \simeq S/(\xi_D L)$ for $L, L_y, L_z > \xi_D$. The factor N_0SLD/L^2 is the effective channel number $N_{c,eff}$, so L/N_0SD is of the order of $1/<g>$ with g being the conductance of the N region in units of e^2/h.

Finally we will discuss the observability of this fluctuation. The fluctuations have been calculated for the fixed phase difference, which will be realized, *e.g.*, by changing the gate voltage which controls the carrier density. On the other hand, the fluctuation will be introduced by the applied magnetic field as well. However, it will be difficult to observe the fluctuation since the correlation range in the fluctuation is comparable to the period of the damped oscillation in the Fraunhofer pattern.

2-2) Effects of Pair Breaking due to Localized Impurity Spins

We discuss effects of magnetic scattering on mesoscopic fluctuation of the Josephson current. This scattering is known as a pair breaking mechanism of superconductivity and generally reduces supercurrent.

In the case where impurity spins are in a paramagnetic state, there is no magnetic correlation between two measurements because the spin configuration will vary in a time scale of an experiment[13]. Formally speaking, the two measurements cannot be connected by the spin scattering. Then the spin scattering rate, $1/\tau_s$, can be regarded as an effective dephasing rate also for the fluctuation. Namely, this rate becomes the mass of cooperon and diffuson: $1/(D\vec{q}^2+\omega_n) \to 1/(D\vec{q}^2+\omega_n+1/\tau_s)$. Therefore, magnetic scatterings play the role of pair breaking on fluctuation and then the fluctuations as well as the averages will be reduced in strong magnetic scattering.

In spin glass systems where the spin configurations are frozen, however, the situation is different. This is because there now exist magnetic correlations between the different measurements by the fixed spin orientation[14] and then the two measurements are connected by the spin scattering. Namely, if the spin configuration remains fixed on the time scale of one measurement, the random spins act as random field. In this case the cooperons and the diffusons shown in Figs.3 are given by[15]:

$$C^{++++} = \frac{1}{2\pi N_0\tau^2}\frac{1}{D\vec{q}^2+\omega_n+2/3\tau_s},$$
$$C^{++--} = \frac{1}{4\pi N_0\tau^2}\left[\frac{1}{D\vec{q}^2+\omega_n+2/3\tau_s}+\frac{1}{D\vec{q}^2+\omega_n+2/\tau_s}\right],$$
$$C^{+--+} = \frac{1}{4\pi N_0\tau^2}\left[\frac{1}{D\vec{q}^2+\omega_n+2/3\tau_s}-\frac{1}{D\vec{q}^2+\omega_n+2/\tau_s}\right],$$
$$D^{++++} = \frac{1}{4\pi N_0\tau^2}\left[\frac{1}{D\vec{q}^2+\omega_n}+\frac{1}{D\vec{q}^2+\omega_n+4/3\tau_s}\right],$$

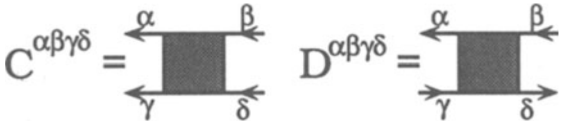

Fig.3 (a) cooperon (b) diffuson

$$D^{+-+-} = \frac{1}{4\pi N_0 \tau^2}\left[\frac{1}{D\vec{q}^2+\omega_n} - \frac{1}{D\vec{q}^2+\omega_n+4/3\tau_s}\right],$$
$$D^{++--} = \frac{1}{2\pi N_0 \tau^2}\frac{1}{D\vec{q}^2+\omega_n+4/3\tau_s}. \tag{6}$$

There exists a reversal symmetry with respect to spin index; for example $C^{++++} = C^{----}$.

It should be noticed that only the singlet diffuson is without mass and the others are massive. Therefore, the process including this singlet diffuson will be most effective in the presence of strong spin scattering limit, $\tau_s \to 0$. This conclusion is not affected even if the Zeemann energy is taken into account because the singlet diffuson is independent of it.

Using these results, we can evaluate the fluctuation of the Josephson current as follows:

$$\begin{aligned} <\delta I^2(\varphi)> = 8e^2T^2 \sum_{\epsilon_n,\epsilon'_n>0} |\Delta^l(\epsilon_n)||\Delta^r(\epsilon_n)||\Delta^l(\epsilon'_n)||\Delta^r(\epsilon'_n)| \int d\vec{r}_{1\perp} d\vec{r}_{2\perp} d\vec{r}'_{1\perp} d\vec{r}'_{2\perp} \\ \times\, \sigma^x_{\alpha\gamma}\sigma^x_{\beta\delta}\sigma^x_{\alpha'\gamma'}\sigma^x_{\beta'\delta'}\big[< X^{\alpha\beta\gamma\delta}(\vec{r}_1,\vec{r}_2;\epsilon_n) X^{\beta'\alpha'\delta'\gamma'}(\vec{r}'_2,\vec{r}'_1,\epsilon'_n) > \\ - < X^{\alpha\beta\gamma\delta}(\vec{r}_1,\vec{r}_2;\epsilon_n) X^{\alpha'\beta'\gamma'\delta'}(\vec{r}'_1,\vec{r}'_2;\epsilon'_n) > \cos 2\varphi\big], \end{aligned} \tag{7}$$

where $X^{\alpha\beta\gamma\delta}$ means $G_{\alpha\beta}G_{\gamma\delta}$ and the Pauli matrix σ^x projects out singlet components. In eq.(7), the first and the second terms, except for the factor $\cos 2\varphi$, are generally different, since the first term includes diffuson processes besides cooperon processes, whereas the second term has only cooperon processes. Hence eq.(7) does not vanish even if $\varphi = 0$, *i.e.* the fluctuation persists even if the phase difference between the two superconductors, φ, is zero in contrast to the normal impurity case. Especially, in the strong depairing limit, *i.e.* $\tau_s \to 0$, the average is obviously vanishing but the fluctuation survives and is independent of the phase difference because only the first term of the r.h.s of eq.(7) survives. In this limit, processes given in Fig.4 determine the magnitude of fluctuations.

If $L > \xi_D$, these processes gives the result that the relative intensity of the fluctuation to the critical current in the case of $\tau_s^{-1} = 0$ is given by $\sqrt{c_d}/(N_0 S v_F)$, where $N_0 S v_F$ corresponds to the channel number, N_c, and the c_d is defined above. Hence

$$\frac{\sqrt{<\delta I^2(\varphi)>_{\tau_s\to 0}}}{\sqrt{<\delta I^2(\varphi)>_{\tau_s^{-1}=0}}} \simeq \frac{N_{c,eff}}{N_c} \simeq \frac{\ell}{L}, \tag{8}$$

where ℓ is the mean free path and $\ell \ll L$ in the present diffusive regime. The reason why the fluctuation is suppressed in this limit compared to the case without the spin scattering is that the cooperon among the same system does not contribute to the fluctuation.

Similar phenomena can be observed in persistent currents in disordered metallic rings with spin glass ordering. Namely, the persistent current fluctuates even if there

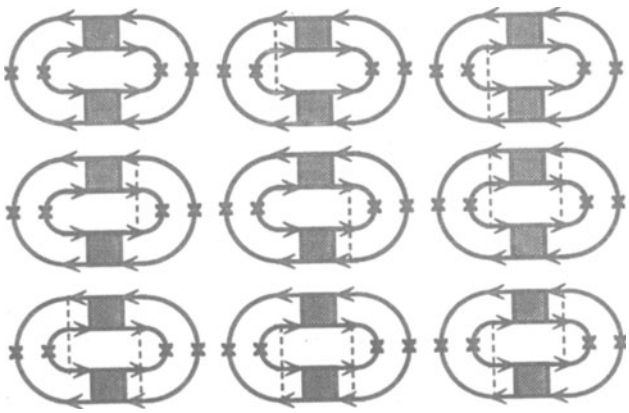

Fig.4 Diagrams in the lowest order of $1/\epsilon_F\tau$ in strong depairing limit.

is no AB flux and the intensity is independent of the flux in the strong spin scattering limit[15].

Such a remarkable phenomenon of the finite fluctuation with the vanishing average has been noticed in RKKY interaction in disordered media[16].

In calculation of cooperons and diffusons, we did not take into account the effect of the magnetic field on spin glass systems. In general, spin glass systems have a finite susceptibility, so we need to consider the effect of the spin polarization due to the applied magnetic field[15], and we find that the result in eq.(8) is not modified except for a numerical factor.

3. AB Ring Geometry

The Josephson current of an SNS system with the N region of the AB ring structure as shown in Fig.5 is of particular interest, since the transport current through the AB ring is known to exhibit AB oscillation.

We have investigated this problem[17] and the Josephson current was shown to be given as follows in the case, $L_1 + L_2 + R\theta, L_1 + L_2 + R(2\pi - \theta) > \xi$ with $\xi = \xi_D$ in the dirty case and $\xi = \xi_C \equiv v_F/2\pi T$ in the clean case[18]:

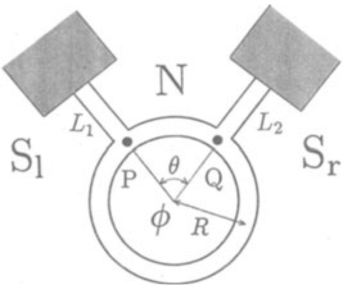

Fig.5 Superconductor - Normal Metal - Superconductor junction with an AB ring in the N region.

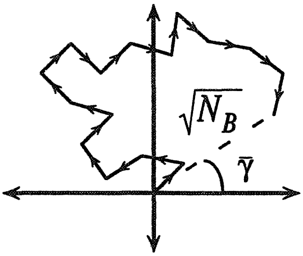

Fig.6
Definition of $\bar{\gamma}$.

$$I = I_d \exp[-\frac{L_1 + L_2}{\xi}]|\Delta^l(\pi T)||\Delta^r(\pi T)|$$
$$\left[A_1 \sin(\Psi - 4\pi\phi/\phi_0) + A_2 \sin\Psi + 2\sqrt{A_1 A_2/N_B}\sin(\Psi - 2\pi\phi/\phi_0)\cos\bar{\gamma}\right], \qquad (9)$$

where $A_1 = \exp[-R(2\pi - \theta)/\xi]$, $A_2 = \exp[-R\theta/\xi]$, Ψ and ϕ_0 are the gauge invariant phase difference between S_l and S_r[19] and the unit flux, hc/e, respectively, whereas I_d is a constant depending on the spatial dimension and the temperature[20]. In eq.(9) N_B is the effective number of Cooper pairs contributing to the proximity effect. The first and the second terms in eq.(9) are due to processes that two electrons forming a single Cooper pair go through the same path (process 1), while the third term results from the process that the two electrons go through the different path separately (process 2). The impurity scattering or boundary roughness results in the random phase shifts of electron waves. In process 1, these phase shifts can be canceled because two electrons forming a single Cooper pair can be scattered by the same impurity. However, such a cancellation cannot occur in process 2. The factor $\cos\bar{\gamma}$ in the third term results from the summation of these random phase shifts and is defined in Fig.6. Therefore, contributions from processes 1 and 2 are of the order of N_B and $\sqrt{N_B}$, respectively. This is the reason why the factor N_B appears.

From this formula, we find the critical current[18];

$$I_M = I_d \exp[-\frac{L_1 + L_2}{\xi}]F(2\pi\phi/\phi_0), \qquad (10)$$
$$F(x) = \left[A_1^2 + A_2^2 + 4A_1A_2\cos^2\bar{\gamma}/N_B + 2A_1A_2\cos 2x + 4(A_1 + A_2)\sqrt{A_1A_2/N_B}\cos\bar{\gamma}\cos x\right]^{1/2}. \qquad (11)$$

It is to be noted that there exists ϕ_0 oscillation in addition to the well-known $\phi_0/2$ oscillation. This ϕ_0 oscillation results from the interference between two electron waves forming a single Cooper pair in process 2 and then is the effect of mesoscopic fluctuation. This oscillation is one example indicating the fact that phases of mesoscopic systems and those in superconductors are entangled[17]. The relative intensity of the former to the latter is given by $2(\sqrt{A_1/A_2} + \sqrt{A_2/A_1})/\sqrt{N_B}$ besides a random factor $\cos\bar{\gamma}$ representing the fact that this is the mesoscopic fluctuation effect and depends on the geometry and the effective number N_B. It is important to estimate the actual value of N_B, which is a new concept, in order to assess the observability of the ϕ_0 oscillation.

However, the relative intensity of the ϕ_0 and $\phi_0/2$ oscillation discussed above is nothing but that of the fluctuation to the average. Therefore, neglecting the factor from ring geometry, the value, $1/\sqrt{N_B}$, will be of the same order as that in the wire system studied in sec.2. In contrast to the simple wire geometry studied in sec.2.1 in which mesoscopic fluctuation will not easily be observed on the top of Fraunhofer pattern, this ϕ_0 oscillation can be clearly observed as the Fourier power spectrum in the critical current.

4. Summary and Discussion

In this paper we have studied the effects of mesoscopic fluctuations on Josephson current in various SNS systems. In wire systems with normal impurities, the relative intensity of the fluctuation to the average, $\delta I / < I >$, is of the same order as $1/ < g >$ in the high temperature regime, $L > \xi_D$, with $< g >$ being the averaged conductance in units of e^2/h. We also examined effects of pair breaking on the fluctuation. In the presence of paramagnetic impurities, magnetic scattering affects the fluctuation in a similar way as the average as a pair breaking mechanism. However, in the spin glass ordering, the fluctuation does not vanish even if the average is greatly reduced. Especially in the strong depairing limit, the fluctuation is suppressed but remains finite and it is independent of the difference of the phases of the superconducting order parameter. All the results are due to the fact that the singlet diffuson among the different systems is massless. In Aharonov - Bohm ring geometry, we have found hc/e oscillation in critical currents in addition to the well-known $hc/2e$ oscillation. This oscillation results from the interference between the electrons forming a single Cooper pair and is characteristic in mesoscopic systems.

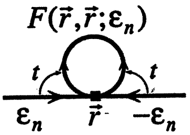

Fig.7
Schematic description of the model we treat.

Our method, which is introduced by Kresin, corresponds to the model where an incident electron is reflected into a hole by an effect of transfer t between N and S region as in the study by Aslamazov *et al.*[21], which is schematically shown in Fig.7, where C, defined in sec.2, is given by t^2/v_{Fs} with v_{Fs} being the Fermi velocity in the S region. Our treatment is different from that by Al'tshuler *et al.* and Beenakker in which they considered a model where an incident electron is reflected into a hole by Andreev reflection.

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