Intuitionistic Multi Fuzzy Soft Set and its Application in Decision Making

Sujit Das[1] and Samarjit Kar[2]

[1] Dept. of CSE, Dr. B.C. Roy Engineering College, Durgapur-713206, India
{sujit_cse,kar_s_k}@yahoo.com
[2] Dept. of Mathematics, National Institute of Technology Durgapur-713209, India

Abstract. Soft set theory initiated by Molodtsov in 1999 has been emerging as a generic mathematical tool for dealing with uncertainty. A noticeable progress is found concerning the practical use of soft set in decision making problems. This paper introduces the concept of intuitionistic multi fuzzy soft set (IMFSS) by combining the intuitionistic multi fuzzy set (IMFS) and soft set models. Then an algorithmic approach is presented by using induced fuzzy soft set and level soft set for dealing with decision making problem based on IMFSS. Finally the proposed algorithm has also been illustrated through a numerical example.

Keywords: Soft sets, intuitionistic fuzzy soft sets, intuitionistic multi fuzzy soft sets, decision making.

1 Introduction

In recent years a number of theories have been proposed to deal with uncertainty, imprecision and vagueness. Theory of probability, fuzzy set theory [1], intuitionistic fuzzy sets [2,3], vague sets [4], theory of interval mathematics [3,5], rough set theory [6] etc. are consistently being utilised as efficient tools for dealing with diverse types of uncertainties and imprecision embedded in a system. However, each of these theories has its inherent difficulties as pointed out by Molodtsov [7]. The reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theories. Molodtsov initiated a novel concept of soft set theory for modelling vagueness and uncertainty which is free from the difficulties affecting existing methods [7]. This theory has proven useful in many different fields such as decision making [8], data analysis [9], forecasting [10] and simulation [11].

Combining soft set models with other mathematical models has attracted the attention of many researchers. Maji et al. presented the concept of fuzzy soft set [12] which is based on a combination of the fuzzy set and soft set models. Yang et al. introduced the concept of the interval-valued fuzzy soft set [13] by combining the interval-valued fuzzy set and soft set. Feng et al. explored a relation between rough sets and soft sets, as also fuzzy sets and soft sets [14, 15]. By combining the trapezoidal fuzzy number and soft set, Xiao et al. proposed the concept of the trapezoidal fuzzy soft set [16]. Concept of multi fuzzy set [17] was proposed by Sebastian et al. which is a more general fuzzy set. Recently Shinoj el al. used the concept of intuitionistic

P. Maji et al. (Eds.): PReMI 2013, LNCS 8251, pp. 587–592, 2013.

multi fuzzy set for decision making purpose [18]. The purpose of this paper is to combine the intuitionistic multi fuzzy set and soft set, from which we can obtain a new soft set model named intuitionistic multi fuzzy soft set. To facilitate our discussion, we first review some preliminaries on soft set in section 2. In section 3, the concept of intuitionistic multi fuzzy soft set is presented. In section 4, intuitionistic multi fuzzy soft set is used to analyze a medical decision making problem using an algorithmic approach. Finally conclusions are pointed in section 5.

2 Preliminaries

Throughout this paper, U refers to an initial universe set, E is a set of parameters, P(U) is the power set of U and $A \subset E$.

Definition 1 [7]. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a mapping from parameters to P(U), and it is not only a set, but a parameterized family of subsets of U. For any parameter $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2 [12]. Let $\tilde{P}(U)$ denote the set of all fuzzy subsets of U, A pair $\left(\tilde{F}, A\right)$ is called a fuzzy soft set (FSS) over U, where $\tilde{F}$ is a mapping given by $\tilde{F} : A \to \tilde{P}(U)$.

Definition 3 [19, 20]. Let $I\hat{P}(U)$ denotes the set of all intuitionistic fuzzy sets of U. A pair $(\hat{F}, A)$ is called an intuitionistic fuzzy soft set (IFSS) over U, where $\hat{F}$ is a mapping given by $\hat{F} : A \to I\hat{P}(U)$. For any parameter $e \in A, \hat{F}(e)$ is an intuitionistic fuzzy subset of U and is called intuitionistic fuzzy value set of parameter e. Clearly $\hat{F}(e)$ can be written as an intuitionistic fuzzy set such that $\hat{F}(e) = \{< x, \mu_{\hat{F}(e)}(x), \nu_{\hat{F}(e)}(x), \pi_{\hat{F}(e)}(x) > | \, x \in U\}$. Here $\mu_{\hat{F}(e)}(x)$, $\nu_{\hat{F}(e)}(x)$ and $\pi_{\hat{F}(e)}(x)$ are membership and non membership functions and hesitation margin respectively and $\forall x \in U, \mu_{\hat{F}(e)}(x) + \nu_{\hat{F}(e)}(x) + \pi_{\hat{F}(e)}(x) = 1.$

Example: Assume that U is set of five diseases {Viral fever, Malaria, Typhoid, Stomach problem, Chest problem} given by $U = \{d_1, d_2, d_3, d_4, d_5\}$ and E is the set of symptoms {Temperature, Headache, Stomach pain, Cough, Chest pain} given by $E = \{s_1, s_2, s_3, s_4, s_5\}$. Let $A = \{s_1, s_2, s_3\}$. Then intuitionistic fuzzy soft set $\hat{(F, A)}$ can describe the possibilities of the diseases under intuitionistic fuzzy circumstances.

$\hat{F}(s_1) = \{d_1/(0.7,0.2,0.1), d_2/(0.8,0.2,0.0), d_3/(0.2,0.4,0.4), d_4/(0.3,0.2,0.5), d_5/(0.4,0.2,0.4)\}$,

$\hat{F}(s_2) = \{d_1/(0.7,0.2,0.1), d_2/(0.3,0.2,0.5), d_3/(0.9,0.0,0.1), d_4/(0.4,0.2,0.4), d_5/(0.1,0.2,0.7)\}$,

$\hat{F}(s_3) = \{d_1/(0.1,0.5,0.4), d_2/(0.8,0.2,0.0), d_3/(0.3,0.5,0.2), d_4/(0.7,0.1,0.2), d_5/(0.3,0.1,0.6)\}$.

Definition 4 [17]. An intuitionistic multi fuzzy set $\tilde{A}$ is defined by

$\tilde{A} = \{x/(\mu_{\tilde{A}}^1(x), \mu_{\tilde{A}}^2(x), ..., \mu_{\tilde{A}}^k(x)), (\nu_{\tilde{A}}^1(x), \nu_{\tilde{A}}^2(x), ..., \nu_{\tilde{A}}^k(x)), (\pi_{\tilde{A}}^1(x), \pi_{\tilde{A}}^2(x), ..., \pi_{\tilde{A}}^k(x)) : x \in U\}$,

such that $0 \le \mu_{\tilde{A}}^i(x) + \nu_{\tilde{A}}^i(x) + \pi_{\tilde{A}}^i(x) \le 1$ for every $x \in U$ and $i = 1, 2, ..., k$ where k is a positive integer and known as the dimension of $\tilde{A}$.

3 Intuitionistic Multi Fuzzy Soft Set

Definition 5. Let $IMFS(U)$ denotes the set of all intuitionistic multi fuzzy sets of U. A pair $(\mathcal{F}, A)$ is called a intuitionistic multi-fuzzy soft set (IMFSS) of dimension k over U, where $\mathcal{F}$ is a mapping given by $\mathcal{F} : A \to IMFS^k(U)$. An intuitionistic multi-fuzzy soft set is a mapping from parameters A to $IMFS^k(U)$. It is a parameterized family of intuitionistic multi fuzzy subsets of U. For $e \in A, \mathcal{F}(e)$ may be considered as the set of e-approximate elements of the intuitionistic multi fuzzy soft set $(\mathcal{F}, A)$.

In multi fuzzy set [17], membership grades by the experts are recorded at different time instants to evaluate the disease status. Intuitionistic multi fuzzy set provides the situation where each expert can prescribe his/her opinion in more accurate way using membership, non membership and hesitation margins. IMFSS $(\mathcal{F}, A)$ can describe the possibilities of the diseases using intuitionistic multi fuzzy sets. An IMFSS $(\mathcal{F}, A)$ can be easily represented by an Intuitionistic multi fuzzy soft matrix (IMFSM).

Definition 6. Using the definition of intuitionistic fuzzy soft matrix (IFSM) [21] we present the definition of intuitionistic multi fuzzy soft matrix to describe IMFSS.

Suppose $\mathcal{F} : E \to IMFS^k(U)$, $U = \{u_1, u_2, \cdots, u_m\}, E = \{e_1, e_2, \cdots, e_n\}$, the intuitionistic multi fuzzy soft matrix could be defined as $\mathcal{F} = (f_{ij})_{m \times n}$ where $f_{ij} = (\mu_{\mathcal{F}(e_j)}^l(x_i), \nu_{\mathcal{F}(e_j)}^l(x_i), \pi_{\mathcal{F}(e_j)}^l(x_i)), i = 1, 2, ..., m, j = 1, 2, ..., n, l = 1, 2, ..., k$,

$[f_{ij}]_{m \times n} = \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ f_{21} & f_{22} & \cdots & f_{2n} \\ \vdots & \vdots & & \vdots \\ f_{m1} & f_{m2} & \cdots & f_{mn} \end{bmatrix}$, where k is the dimension of IMFS

4 Application of IMFSS in Decision Making

4.1 Algorithmic Approach

Step 1. An intuitionistic multi fuzzy soft set $(\mathcal{F}, A)$ and relative weight $\omega(e_i)$ of parameter e_i is taken as input.

Step 2. Induced fuzzy soft set [22] $\Delta_{\mathcal{F}} = (\mathcal{F}, A)$ is computed.

Suppose $\omega(e) = (\omega_1, \omega_2, ..., \omega_n)^T$ where $\sum_{i=1}^{n} \omega_i = 1$ be the relative weight of parameter e, then induced fuzzy set can be defined as

$$[If_{ij}]_{m\times n} = \begin{pmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ f_{21} & f_{22} & \cdots & f_{2n} \\ \vdots & \vdots & & \vdots \\ f_{m1} & f_{m2} & \cdots & f_{mn} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_k \end{pmatrix} = \begin{pmatrix} \Sigma_{l=1}^{k} \{\omega(u_i).\mu^{l}_{\mathcal{F}(e_j)}(x_i) + \omega(v_i).\nu^{l}_{\mathcal{F}(e_j)}(x_i) + \omega(\pi_i).\pi^{l}_{\mathcal{F}(e_j)}(x_i) \\ \Sigma_{l=1}^{k} \{\omega(u_i).\mu^{l}_{\mathcal{F}(e_j)}(x_i) + \omega(v_i).\nu^{l}_{\mathcal{F}(e_j)}(x_i) + \omega(\pi_i).\pi^{l}_{\mathcal{F}(e_j)}(x_i) \\ \vdots \\ \Sigma_{l=1}^{k} \{\omega(u_i).\mu^{l}_{\mathcal{F}(e_j)}(x_i) + \omega(v_i).\nu^{l}_{\mathcal{F}(e_j)}(x_i) + \omega(\pi_i).\pi^{l}_{\mathcal{F}(e_j)}(x_i) \end{pmatrix}$$

Step 3. A threshold fuzzy set $\lambda : A \to [0,1]$ which is the mid level decision rule is chosen for decision making.

Step 4. The mid-level soft set $L(\Delta_{\mathcal{F}} : mid)$ is computed and presented in tabular form.

Step 5. The choice value c_i of u_i are calculated $\forall i$.

Step 6. The optimal decision is to select u_j if $c_j = \max(c_i) \forall i$.

Step 7. If j has more than one value, then any one of u_j might be chosen.

4.2 Experimental Analysis

Suppose a medical expert monitors the set of five symptoms {Temperature, Headache, Stomach pain, Cough, Chest pain} given by $E = \{s_1, s_2, s_3, s_4, s_5\}$ for a set of four patients $P = \{P_1, P_2, P_3, P_4\}$ to evaluate the most possible disease that a patient might suffer among a set of diseases {Viral fever, Malaria, Typhoid, Stomach problem, Chest problem} given by $U = \{d_1, d_2, d_3, d_4, d_5\}$. Observation of the expert is given in table 1. Let us assume that the medical system has imposed the following weights to the parameter set i.e. set of symptoms for the various possible diseases which are given in table 2. Table 3 shows the tabular representation of the induced fuzzy soft set for disease $d_1(\Delta_{\mathcal{F}d1} = (\mathcal{F}_{d1}, E)), \cdot d_2(\Delta_{\mathcal{F}d2} = (\mathcal{F}_{d2}, E))$ and $d_3(\Delta_{\mathcal{F}d3} = (\mathcal{F}_{d3}, E))$.

Mid-threshold of $\Delta_{\mathcal{F}_{d1}}$, $\Delta_{\mathcal{F}_{d2}}$ and $\Delta_{\mathcal{F}_{d3}}$ are fuzzy set given as follows.

$mid\,\Delta_{\mathcal{F}_{d1}} = \{(s_1, 1.07), (s_2, 1.05), (s_3, 0.93), (s_4, 1.07), (s_5, 1.11)\}$.

$mid\,\Delta_{\mathcal{F}_{d2}} = \{(s_1, 1.05), (s_2, 1.10), (s_3, 0.96), (s_4, 1.24), (s_5, 1.02)\}$.

Table 1. Expert's observation

Patients/ Symptoms	Temperature	Headache	Stomach pain	Cough	Chest pain
P_1	(0.3,0.7,0.5) (0.2,0.1,0.4) (0.5,0.2,0.1)	(0.4,0.3,0.4) (0.3,0.6,0.4) (0.3,0.1,0.2)	(0.1,0.2,0.0) (0.7,0.7,0.8) (0.2,0.1,0.2)	(0.5,0.6,0.7) (0.4,0.3,0.2) (0.1,0.1,0.1)	(0.4,0.3,0.4) (0.6,0.4,0.4) (0.0,0.3,0.2)
P_2	(0.4,0.3,0.5) (0.5,0.4,0.4) (0.1,0.3,0.1)	(0.7,0.6,0.8) (0.2,0.2,0.1) (0.1,0.2,0.1)	(0.6,0.5,0.4) (0.3,0.3,0.4) (0.1,0.2,0.2)	(0.3,0.6,0.2) (0.7,0.3,0.7) (0.0,0.1,0.1)	(0.4,0.7,0.5) (0.1,0.2,0.3) (0.5,0.1,0.2)
P_3	(0.1,0.2,0.1) (0.7,0.6,0.9) (0.2,0.2,0.0)	(0.3,0.2,0.1) (0.6,0.0,0.7) (0.1,0.8,0.2)	(0.8,0.7,0.8) (0.0,0.1,0.1) (0.2,0.2,0.1)	(0.3,0.2,0.2) (0.6,0.7,0.6) (0.1,0.1,0.2)	(0.4,0.3,0.2) (0.4,0.7,0.7) (0.2,0.0,0.1)
P_4	(0.5,0.4,0.5) (0.4,0.4,0.3) (0.1,0.2,0.2)	(0.4,0.3,0.4) (0.5,0.3,0.5) (0.1,0.4,0.1)	(0.2,0.1,0.0) (0.7,0.6,0.7) (0.1,0.3,0.3)	(0.5,0.6,0.3) (0.4,0.3,0.6) (0.1,0.1,0.1)	(0.4,0.5,0.4) (0.6,0.4,0.3) (0.0,0.1,0.3)

Table 2. Medical knowledgebase table

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Temperature	(0.5,0.3,0.2)	(0.6,0.2,0.2)	(0.4,0.1,0.5)	(0.1,0.7,0.2)	(0.1,0.8,0.1)
Headache	(0.6,0.1,0.3)	(0.3,0.6,0.1)	(0.3,0.1,0.6)	(0.2,0.4,0.4)	(0.2,0.5,0.3)
Stomach pain	(0.4,0.2,0.4)	(0.3,0.3,0.4)	(0.2,0.4,0.4)	(0.6,0.2,0.2)	(0.3,0.5,0.2)
Cough	(0.2,0.5,0.3)	(0.5,0.4,0.1)	(0.4,0.3,0.3)	(0.2,0.7,0.1)	(0.4,0.5,0.1)
Chest pain	(0.3,0.5,0.2)	(0.4,0.3,0.3)	(0.1,0.5,0.4)	(0.2,0.7,0.1)	(0.6,0.1,0.3)

Table 3. Induced fuzzy set

	Induced fuzzy set for disease d_1					Induced fuzzy set for disease d_2					Induced fuzzy set for disease d_3				
	s_1	s_2	s_3	s_4	s_5	s_1	s_2	s_3	s_4	s_5	s_1	s_2	s_3	s_4	s_5
P_1	1.12	0.97	0.76	0.90	1.13	1.20	1.17	0.95	1.29	1.01	1.07	0.82	1.14	1.08	1.01
P_2	1.09	1.43	1.00	1.13	0.94	1.08	0.97	0.95	1.25	1.06	0.86	0.92	0.90	1.01	1.78
P_3	0.94	0.82	1.16	1.21	1.23	0.76	1.07	0.95	1.15	0.99	0.58	0.97	0.74	0.97	1.11
P_4	1.13	0.97	0.80	1.02	1.12	1.16	1.17	0.97	1.25	1.03	0.92	0.82	1.14	1.04	0.94

Table 4. Mid level soft set

	Mid level soft set for d_1						Mid level soft set for d_2						Mid level soft set for d_3					
	s_1	s_2	s_3	s_4	s_5	Ch	s_1	s_2	s_3	s_4	s_5	Ch	s_1	s_2	s_3	s_4	s_5	Ch
P_1	1	0	0	0	1	2	1	1	0	1	0	3	1	0	1	1	1	**4**
P_2	1	1	1	1	0	**4**	1	0	0	1	1	3	1	1	0	0	0	2
P_3	0	0	1	1	1	3	0	0	0	0	0	0	0	1	0	0	1	2
P_4	1	0	0	0	1	2	1	1	1	1	1	**5**	1	0	1	1	0	3

$mid\Delta_{F_{d_3}} = \{(s_1, 0.86), (s_2, 0.88), (s_3, 0.90), (s_4, 1.03), (s_5, 0.96)\}$

Then we find the mid-level soft set $L(\Delta_{F_{di}} : mid)$ of $\Delta_{F_{di}}$ where $i = 1, 2, ..., 5$. Table 4 shows the mid level soft set and obtained choice values for the first three diseases. From table 4 it is obvious that patient P_2 has a possibility to suffer from viral fever, patient P_4 has a possibility to suffer from malaria and patient p_1 has a possibilty to suffer from typhoid. Calculating mid level soft set of $\Delta_{F_{d4}}$ and $\Delta_{F_{d5}}$ we find that patient P_3 has a possibility to suffer from stomach problem and chest problem might occur for both patients P_3 and P_4.

5 Conclusion

In this paper we propose the concepts of intuitionistic multi fuzzy soft sets, which is a combination of intuitionistic multi fuzzy sets and soft sets. This paper also introduces intuitionistic multi fuzzy soft matrix which will bring much convenience for future

researches on soft sets. Using induced fuzzy sets and level soft sets this paper uses an algorithmic approach to explore application of intuitionistic multi fuzzy soft sets in decision making problems. Finally we give an illustrative example to show the validity of the proposed approach. In future researchers might enhance this study to many other decision making problems that contain such kinds of uncertainties.

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