

Wide-Angle Lens Distortion Correction Using Division Models

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Abstract. In this paper we propose a new method to automatically correct wide-angle lens distortion from the distorted lines generated by the projection on the image of 3D straight lines. We have to deal with two major problems: on the one hand, wide-angle lenses produce a strong distortion, which makes the detection of distorted lines a particularly difficult task. On the other hand, the usual single parameter polynomial lens distortion models is not able to manage such a strong distortion. We propose an extension of the Hough transform by adding a distortion parameter to detect the distorted lines, and division lens distortion models to manage wide-angle lens distortion. We present some experiments on synthetic and real images to show the ability of the proposed approach to automatically correct this type of distortion. A comparison with a state-of-the-art method is also included to show the benefits of our method.

Keywords: lens distortion, wide-angle lens, Hough transform, line detection.

1 Introduction

Wide-angle lenses are specially suited for some computer vision tasks, such as real-time tracking, surveillance, close range photogrammetry or even for simple aesthetic purposes. The main advantage these lenses offer is that they provide a wide view up to 180 degrees. However, the strong distortion produced by these lenses may cause severe problems, not only visually, but also for further processing in applications such as object detection, recognition and classification.

To model the lens distortion, we consider radial distortion models given by the expression:

$$\begin{pmatrix} \hat{x} - x_c \\ \hat{y} - y_c \end{pmatrix} = L(r) \begin{pmatrix} x - x_c \\ y - y_c \end{pmatrix}, \quad (1)$$

where (x, y) is the original (distorted) point, (\hat{x}, \hat{y}) is the corrected (undistorted) point, (x_c, y_c) is the center of the camera distortion model, $L(r)$ is the function which defines the shape of the distortion model and $r = \sqrt{(x - x_c)^2 + (y - y_c)^2}$.

According to the choice of function $L(r)$, there exist two widely accepted types of lens distortion models: the polynomial model and the division model.

The polynomial model, or simple radial distortion model [10], is formulated as:

$$L(r) = 1 + k_1 r^2 + k_2 r^4 + \dots, \quad (2)$$

where the set $\mathbf{k} = (k_1, \dots, k_{N_k})^T$ contains the distortion parameters estimated from image measurements, usually by means of non-linear optimization techniques. The two-parameter model is the usual approach, due to its simplicity and accuracy [12], [1]. Alvarez, Gomez and Sendra [1] proposed an algebraic method suitable for correcting significant radial distortion which is highly efficient in terms of computational cost. An on-line demo of the implementation of this algebraic method can be found in [2].

Camera calibration is a topic of interest in Computer Vision which, in order to be efficient, requires including the distortion into the camera model. Most calibration techniques rely on the linear pinhole camera and use a calibration pattern to establish a point-to-point correspondence between 2D and 3D points (see a review on camera calibration in [14]). In this applications, the polynomial model with only one distortion parameter, k_1 (*one-parameter model*), achieves an accuracy around 0.1 pixels in image space using lenses exhibiting large distortion [7], [8]. However, [7] also indicates that for cases of strong radial distortion, the one-parameter model is not recommended.

The division model has initially been proposed by [13], but it has received special attention after the more recent research by Fitzgibbon [9]. It is formulated as:

$$L(r) = \frac{1}{1 + k_1 r^2 + k_2 r^4 + \dots}. \quad (3)$$

The main advantage of the division model is the requirement of fewer terms than the polynomial model for the case of severe distortion. Therefore, the division model seems to be more adequate for wide-angle lenses (see a recent review on distortion models for wide-angle lenses in [11]). Additionally, when using only one distortion parameter, its inversion is simpler, since it requires finding the roots of a second degree polynomial instead of a third degree polynomial. In fact, a single parameter version of the division model is normally used.

For both models, $L(r)$ can be estimated by considering that 3D lines in the image must be projected onto 2D straight lines, and minimizing the distortion error, which is given by the sum of the squares of the distances from the points to the lines [7].

Once a lens distortion model has been selected, we must decide how to apply it. Some methods rely on the human-supervised identification of some known straight lines in one or more images [3], [4], [15]. As a consequence of the human intervention, these methods are robust, independent of the camera parameters, and require no calibration patterns. However, for the same reason, these methods are slow and tedious for the case of dealing with large sets of images.

New approaches have recently appeared to eliminate human intervention. In [6] and [5], an automatic radial estimation method is discussed. This method works on a single image and no human intervention or special calibration pattern are required. The method applies the one-parameter Fitzgibbon's division model to estimate the distortion from a set of automatically detected non-overlapping circular arcs within the image. The main limitation of the method is that each circular arc has to be a collection of contiguous points in the image and, therefore, the method fails if there are no such arcs.

In this paper, we propose a new unsupervised method which makes use of the one-parameter division model to correct, from a single image, the radial distortion caused by a wide-angle lens. We first automatically detect the distorted lines within the image by adapting the usual Hough transform to our problem. The adaptation consists in embedding the radial distortion parameter into the Hough parametric space to tackle the detection of the longest arcs (*distorted lines*) within the image. From the improved Hough transform, we obtain a collection of distorted lines and an initial value for the distortion parameter k_1 . Next, we optimize this parameter by minimizing the distance of the corrected line points to straight lines.

2 A Hough Space Including a Division Lens Distortion Parameter

In order to correct the distortion, we need to estimate the magnitude and sign of the distortion parameter and, to this aim, we can rely on the information provided by line primitives. Line primitives are searched in the edge image which is computed using any edge detector. One of the most commonly used techniques to extract lines in an edge image is the Hough transform, which searches for the most reliable candidates within a certain space. This space is usually a two-dimensional space which considers the possible values for the orientation and the distance to the origin of the candidate lines. Each edge point votes for those lines which could contain this point, and the lines which receive the highest scores are considered the most reliable ones.

However, this technique does not consider the influence of the distortion in the alignment of the edge points, in such a way that straight lines are split into different segments due to the effect of the distortion. For this reason, we propose to include a new dimension in the Hough space, namely the distortion parameter. For practical reasons, instead of considering the distortion parameter value itself in the Hough space, we make use of the percentage of correction obtained with that value, which is given by:

$$p = (\tilde{r}_{max} - r_{max})/r_{max}, \quad (4)$$

where r_{max} is the distance from the center of distortion to the furthest point in the original image, and \tilde{r}_{max} is the same distance, but after applying the distortion model. This way, the parameter p is easier to interpret than the distortion parameter itself. Another advantage of using p as an extra parameter in

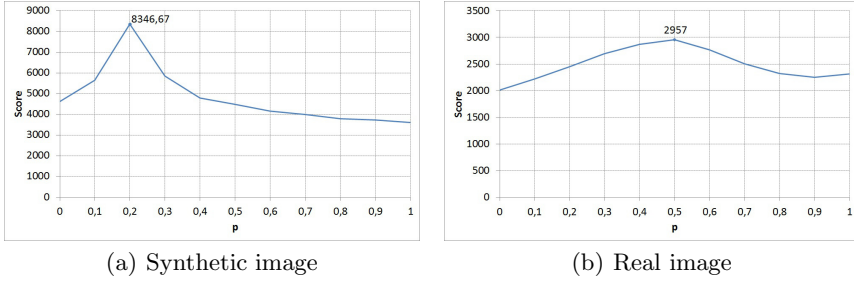


Fig. 1. Values of the maximum in the voting space with respect to the percentage of correction for the images in (a) the synthetic image in Fig. 2 and (b) the real image in Fig. 3 using the modified Hough transform and division lens distortion model

the Hough space is that it does not depend on the image resolution. When we use single parameter division models the relation between parameter p and k_1 is straightforward and it is given by the expression :

$$k_1 = \frac{-p}{(1 + p)r_{max}^2}. \tag{5}$$

To reduce the number of points which vote and the number of lines that each edge point votes for, we first estimate the magnitude and orientation of the edge for every edge point. Only those points where the magnitude of the gradient is higher than a certain threshold are considered. Afterward, we select, for every value of p and every edge point, those lines which, after being corrected according to the distortion model associated to this value of p , are close enough to the point and present an orientation which is similar to the orientation of the edge in that point. Furthermore, the vote of a point for a line depends on how close they are, and is given by $v = 1/(1 + d)$, where d is the distance from the point to the line.

In the Hough space, the different lines may have different orientations and distances to the origin. Nevertheless, they should all have the same value of the distortion parameter (i.e. the same value of p), since it is a single value for the whole image. This means that we must not search for the best candidates individually, but for the value of p which concentrates the largest number of significant lines.

Figure 1 illustrates how the maximum of the voting score varies within the Hough space according to the percentage of correction determined by the distortion parameter.

Once we have searched for the best value of p within the three-dimensional Hough space, we refine it to obtain a more accurate approximation. To this aim, by using standard optimization techniques (gradient descent method) we minimize the following error function:

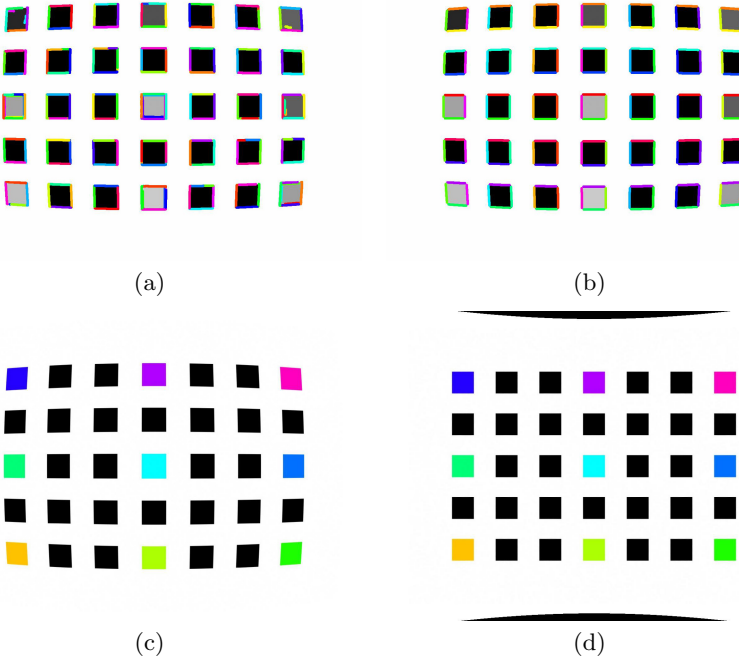


Fig. 2. Lens distortion correction for a test image: (a) lines detected using the Bukhari-Dailey method, (b) lines detected using the proposed method, (c) undistorted image using the Bukhari-Dailey method, and (d) undistorted image using the proposed method.

$$E(p) = \sum_j^{Nl} \sum_i^{Np(j)} \text{dist}(\bar{x}_{ji}, \text{line}_j)^2 \quad (6)$$

Nl is the number of lines, $Np(j)$ is the number of points of the j^{th} line and \bar{x}_{ji} are the points associated to line_j . This error measures how distant the points are from their respective lines, so that the lower this value, the better the matching.

3 Experimental Results

We have tested our model in some images showing wide-angle lens distortion and we have compared the results with those obtained using the Bukhari-Dailey method [5]. We have used the code available on F. Bukhari's web page¹.

Figure 2 (1024 × 683 pixels) presents the results for a synthetic image. It consists of a calibration pattern in which the radial distortion has been simulated using a division model. The magnitude of such distortion is 20% ($p = 0.2$). Figure 2(a) shows the arcs detected using the Bukhari-Dailey method, whereas

¹ <http://www.cs.ait.ac.th/vgl/faisal/downloads.html>

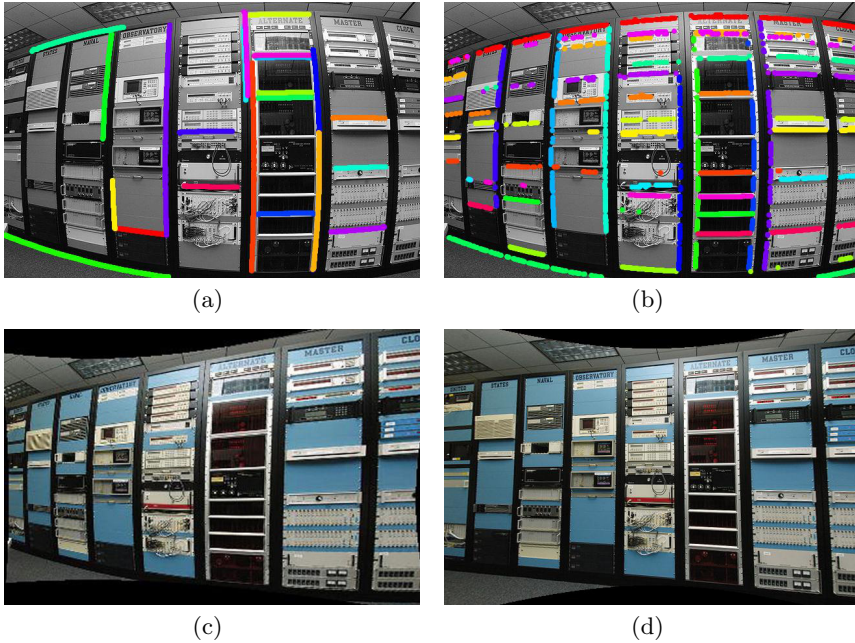


Fig. 3. Lens distortion correction for a real image: (a) lines detected using the Bukhari-Dailey method, (b) lines detected using the proposed method, (c) undistorted image using the Bukhari-Dailey method, and (d) undistorted image using the proposed method

the lines detected using the proposed method (modified Hough transform and division model) are shown in Fig. 2(b). We have represented each line using a different color to identify them. In both cases, from the detected arcs or distorted lines, the distortion is estimated and the images are corrected. Figure 2(c) illustrates the result using the Bukhari-Dailey method, whereas Fig. 2(d) presents the corrected image using the proposed method. As observed, the Bukhari-Dailey method splits those lines where points are not contiguous, while the proposed method is able to identify a single line from different disconnected segments (see, for instance, how the edges of the squares in the same row or column are not associated using the Bukhari-Dailey method, but are properly linked using our method). Since longer lines provide more useful information than shorter ones, this results in a better distortion estimation for the proposed method.

Figure 3 (640×425 pixels)² illustrates the same experiment on a real image with a strong distortion. Figure 3(a) shows the arcs detected using the Bukhari-Dailey method. As observed, when different segments of the same line are visible, this method is not able to associate them (see for instance the lower green line, which is not continued on the right side of the image), but the proposed method associates them into the same line (see Fig. 3(b)). For this case, the corrected

² US Air Force CC0 <http://commons.wikimedia.org/wiki/File:Usno-amc.jpg>

Table 1. Number of lines, number of points, CPU time and percentage of correction for Fig. 2 and 3 using the Bukhari-Dailey method and the proposed method

Figure	Measure	Bukhari-Dailey	Our method
Figure 2 (synthetic image)	No. of arcs	306	24
	No. of points	11,255	9,033
	CPU time (sec.)	79.611	7.844
	% correction	0	19.9555
Figure 3 (real image)	No. of arcs	22	22
	No. of points	2,894	3,651
	CPU time (sec.)	57.41	3.209
	% correction	63.3116	49.9186

image using the proposed method is also better than that obtained by means of the Bukhari-Dailey method (compare Fig. 3(c) and Fig. 3(d)).

Table 1 shows some quantitative results. If we analyze the results for the calibration pattern, we can observe two important advantages of our method. First, the number of lines which have been identified is 24, which is exactly the number of lines within the image. Nevertheless, the Bukhari-Dailey method extracts a higher number of lines, since each one of them has been split in many segments. Second, the percentage of correction obtained with our method is very close to the real value (20%). In this case the Bukhari-Dailey method does not provide a good result (0% of correction), probably because the obtained segments are too small to properly estimate the distortion model. Concerning the total amount of points of the arcs obtained by both methods, the Bukhari-Dailey method obtains more points (11,255 points in all) than our method (9,033 points) probably due to the spurious arcs extracted by the Bukhari-Dailey method.

For the real image, both methods have identified the same number of lines, but those obtained by our method are longer (3,651 points in all) and they have not been split. Regarding the computational cost, in the experiments presented, our method is about 10 times faster than the one proposed by Bukhari-Dailey.

4 Conclusions

In this paper we propose a new method to automatically correct wide-angle lens distortion. The main novelty of the paper is the combination of an improved 3D Hough space, which includes the distortion parameter to detect distorted lines, and the division distortion model which is able to manage the strong distortion produced by wide-angle lenses. We present some experiments which show that the proposed method properly corrects the lens distortion in the case of wide-angle lenses and outperforms the results obtained in [5] specially in the case where the distorted lines are not contiguous arcs in the image.

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