Combined Feedforward/Model Predictive Tracking Control Design for Nonlinear Diffusion-Convection-Reaction-Systems

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Abstract. The tracking control design for setpoint transitions of a quasi-linear diffusion-convection-reaction system with boundary control is considered. For this a suitable model-based feedforward control is determined that relies on the flatness-based parametrization of the control input. A receding horizon feedback control is added within a two-degrees-of-freedom control scheme to account for disturbances, model inaccuracies, and input constraints. The tracking performance of this control scheme is shown by means of simulation studies.

A large class of chemical reactors with an interaction of diffusive, convective, and reactive effects leads to infinite-dimensional mathematical models in the form of nonlinear boundary-controlled parabolic partial differential equations (PDEs) [6]. The control design for setpoint transitions of chemical reactors, e. g., for ignition, extinction, or grade-transitions constitutes a challenging problem. In this contribution, the well-known two-degrees-of-freedom (2DOF) control scheme is applied in order to tackle this control task. The basic idea consists in first designing a feedforward control to steer the system along prescribed trajectories. The trajectory planning and feedforward control are complemented with a state feedback tracking control stabilizing the system about the desired trajectories.

In the literature, there exists a variety of concepts for the design of both feedforward and feedback tracking controllers. For the feedforward control design, approaches using the flatness concept [2] have found widespread attention. The flatness property allows for a parametrization of the state and input in terms of a so-called flat output and its time derivatives and therefore provides a systematic approach for feedforward control design. Originally proposed for finite-dimensional systems, generalizations of the flatness concept have been successfully carried over to certain classes of PDEs, see, e. g., [7,10,12]. In these so-called late lumping approaches the parametrization is directly solved for the underlying PDE. In contrast, the early lumping approach to control design is based on a

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finite-dimensional approximation of the system. Using suitable finite difference schemes to approximate the spatial derivatives results in a semi-discretization which is differentially flat, and the equivalence of the respective feedforward controls has been shown under certain conditions for some types of PDEs, see, e. g., [14,19].

In practice, the feedforward part has to be amended by an additional feedback tracking control in order to compensate for model uncertainties or disturbances. For the design of such stabilizing feedback controllers for infinite-dimensional systems, receding horizon optimal control, e. g., [9,15], constitutes a promising tool. This method is particularly attractive since it provides, in contrast to most state-of-the-art tracking control design methods for DCRSs, see, e. g., [12,13], the possibility to systematically include constraints as they are frequently encountered in technical systems.

The paper is structured as follows: In Section 1, the diffusion-convection-reaction system (DCRS) and the control task is introduced. Section 2 is devoted to the control design based on a semi-discretization of the considered infinite-dimensional system. The paper provides simulation results in Section 3 and conclusions are drawn in Section 4.

1 Problem Formulation

The quasilinear, scalar DCRS, described by the (suitably scaled) parabolic PDE

$$p(\theta(z,t))\partial_t\theta(z,t) = \partial_z(q(\theta(z,t))\partial_z\theta(z,t)) - \nu\partial_z\theta(z,t) + r(\theta(z,t))\theta(z,t) \ , \quad (1)$$

$z \in (0,1)$, $t > 0$ is considered. The storage coefficient $p(\theta(z,t)) = p_0 + p_1\theta(z,t)$, the diffusion coefficient $q(\theta(z,t)) = q_0 + q_1\theta(z,t)$, and the reaction coefficient $r(\theta(z,t)) = r_0 + r_1\theta(z,t)$ depend on the state $\theta(z,t)$ in an affine way, whereas the convection parameter $\nu \geq 0$ is constant. The boundary conditions

$$\partial_z\theta(z,t)|_{z=0} = d(t) \ , \quad (2a)$$

$$q(\theta(1,t))\ \partial_z\theta(z,t)|_{z=1} = \tilde{q}(u(t) - \theta(1,t)) \ , \quad (2b)$$

$t > 0$, $\tilde{q} > 0$, and the initial condition

$$\theta(z,0) = \theta_{\text{init}}(z) \ , \quad (3)$$

$z \in [0,1]$, complete the infinite-dimensional system. In order for (1) to be parabolic it has to be assured that $q(\theta(z,t))$ is positive in the considered range of the state variable. The exogenous input variable $d(t)$ in (2a) represents an additional sink or source term that will be considered as an unknown disturbance and is assumed to be zero for the feedforward controller design. Additionally, the control input $u(t)$ entering via the boundary condition (2b) is supposed to be subject to so-called box constraints, i. e.,

$$u(t) \in [u^-, u^+] \ . \quad (4)$$

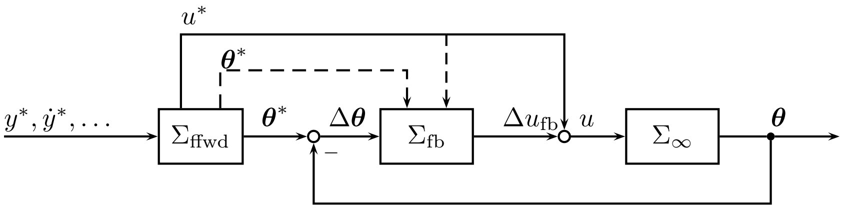

Fig. 1. 2DOF control scheme consisting of the infinite-dimensional system Σ_∞, a trajectory planning and feedforward control $\Sigma_{\rm ffwd}$, which provides nominal state and control input trajectories $\boldsymbol{\theta}^*$ and u^* based on desired output trajectories y^*, and a tracking controller $\Sigma_{\rm fb}$, which provides a correction to the nominal control input trajectories $\Delta u_{\rm fb}$ based on the state tracking error $\Delta\boldsymbol{\theta}$

Denoting the state evaluated at $z = 0$ as the output of the system, i. e. $y(t) = \theta(0,t)$, the control task consists in stably and robustly carrying out transitions between setpoint values and the associated steady-state profiles of (1)–(3), within the transition time $T = 1$. The 2DOF control scheme used to accomplish these transitions is schematically depicted in Figure 1.

2 Tracking Control Based on Finite Difference Semi-Discretization

The design of both the flatness-based feedforward control and the receding horizon tracking control depends on a semi-discretization of the infinite-dimensional system (1)–(3), see also [17]. The methodology to obtain the semi-discretized system pursued in this contribution is to discretize the spatial coordinate z using finite differences on an equidistant grid with N grid elements and the nodes $z_0 = 0, z_1 = \Delta z, \ldots, z_k = k\Delta z, \ldots, z_N = 1$ where $\Delta z = 1/N$. Applying the central finite difference schemes

$$\partial_z \theta_k = \frac{1}{2\Delta z}\left(\theta_{k+1} - \theta_{k-1}\right) + \mathcal{O}(\Delta z^2)\ , \tag{5a}$$

$$\partial_z^2 \theta_k = \frac{1}{\Delta z^2}\left(\theta_{k+1} - 2\theta_k + \theta_{k-1}\right) + \mathcal{O}(\Delta z^2)\ , \tag{5b}$$

$$(\partial_z \theta_k)^2 = \frac{1}{\Delta z^2}\big(\theta_{k+1} - \theta_k\big)\big(\theta_k - \theta_{k-1}\big) + \mathcal{O}(\Delta z^2) \tag{5c}$$

to the PDE (1) and the boundary conditions (2), leads to the following system of $N+1$ ODEs for the discretized states[1] $\theta_k(t) = \theta(z_k, t)$:

$$(p_0 + p_1\theta_0)\,\dot{\theta}_0 = (q_0 + q_1\theta_0)\,\frac{2(\theta_1 - \theta_0 - \Delta z d)}{\Delta z^2} + q_1 d^2 - \nu d + (r_0 + r_1\theta_0)\theta_0\ , \tag{6a}$$

[1] For the sake of readability, time-dependencies as in $\theta_k(t)$ are omitted whenever they are clear from the context.

$$(p_0 + p_1\theta_k)\,\dot{\theta}_k = (q_0 + q_1\theta_k)\,\frac{\theta_{k+1} - 2\theta_k + \theta_{k-1}}{\Delta z^2} + q_1\frac{(\theta_{k+1} - \theta_k)(\theta_k - \theta_{k-1})}{\Delta z^2}$$
$$- \nu\frac{\theta_{k+1} - \theta_{k-1}}{2\Delta z} + (r_0 + r_1\theta_k)\theta_k\ , \quad k = 1, \ldots, N-1\ , \tag{6b}$$
$$(p_0 + p_1\theta_N)\,\dot{\theta}_N = \frac{2(q_0 + q_1\theta_N)}{\Delta z^2}\left(\frac{\Delta z\tilde{q}}{q_0 + q_1\theta_N}(u - \theta_N) - \theta_N + \theta_{N-1}\right)$$
$$+ \frac{q_1}{\Delta z^2}\left(\frac{2\Delta z\tilde{q}}{q_0 + q_1\theta_N}(u - \theta_N) + \theta_{N-1} - \theta_N\right)(\theta_N - \theta_{N-1})$$
$$- \frac{\nu\tilde{q}}{q_0 + q_1\theta_N}(u - \theta_N) + (r_0 + r_1\theta_N)\theta_N\ . \tag{6c}$$

Note that the scheme (5c) for the squared first derivative with respect to z is only applied to nodes with $k > 0$, which is advantageous for the parametrization considered in the subsequent section. The initial conditions obtained by evaluating (3) at the nodes

$$\theta_k(0) = \theta_{\text{init}}(z_k)\ , \tag{7}$$

$k = 0, \ldots, N$ complete the finite-dimensional semi-discretized approximation of the infinite-dimensional system (1)–(3).

2.1 Flatness-Based State and Input Parametrization

The semi-discretization (6) has the property of being flat, which is shown in the following. The k-th equation of (6a) and (6b) is affine in $\theta_{k+1}(t)$, $k = 0, \ldots, N-1$ and (6c) is affine in the control input $u(t)$. Thus, solving these equations for $\theta_{k+1}(t)$ and $u(t)$, respectively, yields

$$\theta_1 =: \Psi_0(\theta_0, \dot{\theta}_0)\ , \tag{8a}$$
$$\theta_{k+1} =: \Psi_k(\theta_k, \dot{\theta}_k, \theta_{k-1})\ , \quad k = 1, \ldots, N-1, \tag{8b}$$
$$u =: \Psi_N(\theta_N, \dot{\theta}_N, \theta_{N-1})\ . \tag{8c}$$

Considering $y(t) = \theta_0(t)$ it follows from (8a) that $\theta_1(t)$ can be parametrized in terms of $y(t)$ and $\dot{y}(t)$. Differentiating (8a) with respect to time

$$\dot{\theta}_1 = \frac{\partial\Psi_0}{\partial\theta_0}\dot{\theta}_0 + \frac{\partial\Psi_0}{\partial\dot{\theta}_0}\ddot{\theta}_0 \tag{9}$$

and inserting this result into (8b) for $k = 1$ directly yields a parametrization of $\theta_2(t)$ by $y(t)$, $\dot{y}(t)$ and $\ddot{y}(t)$. Obviously, this procedure can be analogously continued for $k = 2, \ldots, N-1$ with

$$\dot{\theta}_{k+1} = \frac{\partial\Psi_k}{\partial\theta_k}\dot{\theta}_k + \frac{\partial\Psi_k}{\partial\dot{\theta}_k}\ddot{\theta}_k + \frac{\partial\Psi_k}{\partial\theta_{k-1}}\dot{\theta}_{k-1}\ , \tag{10}$$

such that every state $\theta_k(t)$, $k = 1, \ldots, N$ as well as the control input $u(t)$ are recursively parametrized by $y(t)$ and its first $N+1$ time derivatives. Hence $y(t)$ constitutes a flat output.

As a consequence, nominal state trajectories $\theta_k^*(t)$ and a control input $u^*(t)$ can be calculated by recursively evaluating (8) using a sufficiently smooth trajectory for the flat output $y^*(t)$, such that the output of (6)–(7) for an exact model and in the absence of disturbances exactly tracks $y^*(t)$. Under certain conditions on the growth of the time-derivatives of the prescribed flat output $y^*(t)$ (defined by its so-called Gevrey-class) and given a suitable set of system parameters p_0, p_1, q_0, q_1, ν, r_0, and r_1, it can be shown that the control input $u^*(t)$ converges for $N \to \infty$ to a suitable control input for the DCRS (1)–(3), see, e. g., [10,19].

2.2 Receding Horizon Tracking Control

In the case of model uncertainties or in order to account for disturbances or control constraints, the feedforward control Σ_{ffwd} in Figure 1 has to be extended by a feedback controller Σ_{fb}. The receding horizon control strategy thereby is formulated for the same semi-discretization that is used for the flatness-based parametrization.

In view of the 2DOF control structure in Figure 1, the feedback controller Σ_{fb} is designed to stabilize the system Σ_∞ along the reference trajectories $\theta_k^*(t)$, $k = 0, \dots, N$ provided by the flatness-based trajectory planning Σ_{ffwd}. This means that the tracking errors

$$\Delta\theta_k(t) = \theta(z_k, t) - \theta_k^*(t) \ ,$$

$k = 0, \dots, N$ have to be suppressed by the control action $\Delta u_{\mathrm{fb}}(t)$, which corrects the feedforward control $u^*(t)$, see Figure 1. Using $\boldsymbol{\theta}(t) = [\theta_0(t), \dots, \theta_N(t)]^\mathsf{T} \in \mathbb{R}^{N+1}$ to summarize the differential equations (6) in the form

$$\dot{\boldsymbol{\theta}}(t) = \mathbf{f}(\boldsymbol{\theta}(t), u(t)) \ ,$$

the tracking error $\Delta\boldsymbol{\theta}(t) = [\Delta\theta_0(t), \dots, \Delta\theta_N(t)]^\mathsf{T} \in \mathbb{R}^{N+1}$ satisfies the error dynamics

$$\Delta\dot{\boldsymbol{\theta}}(t) = \mathbf{f}\Big(\boldsymbol{\theta}^*(t) + \Delta\boldsymbol{\theta}(t), u^*(t) + \Delta u_{\mathrm{fb}}(t)\Big) - \dot{\boldsymbol{\theta}}^*(t) =: \mathbf{F}(\Delta\boldsymbol{\theta}(t), \Delta u_{\mathrm{fb}}(t), t) \ . \quad (11)$$

The receding horizon controller design accounts for the nonlinear and time-varying error dynamics (11) by solving the following optimal control problem (OCP) in a discrete-time fashion for each instant of time $t_i = i\Delta t$ with the given sampling time Δt:

$$\min_{\Delta u(\cdot)} J(\Delta u(\cdot), \Delta\boldsymbol{\theta}^i) = ||\Delta\boldsymbol{\theta}(t_{i,\mathrm{f}})||_P^2 + \int_{t_i}^{t_{i,\mathrm{f}}} ||\Delta\boldsymbol{\theta}(t)||_Q^2 + R\Delta u(t)^2 \,\mathrm{d}t \quad (12a)$$

$$\text{s.t.} \quad \Delta\dot{\boldsymbol{\theta}}(t) = \mathbf{F}(\Delta\boldsymbol{\theta}(t), \Delta u(t), t) \ , \quad \Delta\boldsymbol{\theta}(t_i) = \Delta\boldsymbol{\theta}^i \quad (12b)$$

$$\Delta u(t) \in [\Delta u^-(t), \Delta u^+(t)] \ , \quad t \in [t_i, t_{i,\mathrm{f}}] \cdot \quad (12c)$$

Starting from the tracking error $\Delta\boldsymbol{\theta}^i = \boldsymbol{\theta}(t_i) - \boldsymbol{\theta}^*(t_i)$ at time t_i, the error dynamics in (12b) are used to predict the error trajectory $\Delta\boldsymbol{\theta}(t)$ over a finite prediction horizon $t \in [t_i, t_{i,\mathrm{f}}]$ with the final time $t_{i,\mathrm{f}} = t_i + t_\mathrm{f}$, where $t_\mathrm{f} \geq \Delta t$ denotes the (constant) horizon length. The cost functional (12a) to be minimized penalizes the tracking error $\Delta\boldsymbol{\theta}(t)$ as well as the feedback control action $\Delta u(t)$ with respect to the positive definite matrices[2] P, Q and the positive scalar R. The time-varying input constraints (12c) follow from the original constraints (4) of the feedforward trajectory $u^*(t)$ in the form

$$\Delta u^\pm(t) := u^\pm - u^*(t)\,, \quad t \in [t_i, t_{i,\mathrm{f}}]\,. \tag{13}$$

In the following, it is assumed that the OCP (12) possesses an optimal solution $\Delta\bar{u}(t;\Delta\boldsymbol{\theta}^i)$, $\Delta\bar{\boldsymbol{\theta}}(t;\Delta\boldsymbol{\theta}^i)$, $t \in [t_i, t_{i,\mathrm{f}}]$. Note that this assumption is not very restrictive due to the absence of terminal and state constraints.

The OCP (12) is solved in each time step t_i of the receding horizon scheme and only the first part of the control trajectory $\Delta\bar{u}(t;\Delta\boldsymbol{\theta}^i)$ is used as the feedback control in Figure 1, i. e.

$$\Delta u_\mathrm{fb}(t) = \Delta\bar{u}(t;\Delta\boldsymbol{\theta}^i)\,, \qquad t \in [t_i, t_i + \Delta t)\,, \qquad i \in \mathbb{N}_0^+\,. \tag{14}$$

In the next time step t_{i+1}, the OCP (12) is solved again with respect to the new tracking error $\Delta\boldsymbol{\theta}^{i+1}$ that is used as initial condition in (12b).

If the system exactly follows the reference trajectory at time t_i, i. e., if $\Delta\boldsymbol{\theta}^i = \mathbf{0}$, and in the absence of disturbances and model inaccuracies the optimal solution of the OCP (12) is

$$\Delta\bar{u}(t;\Delta\boldsymbol{\theta}^i) = 0\,, \qquad \Delta\bar{\boldsymbol{\theta}}(t;\Delta\boldsymbol{\theta}^i) = \mathbf{0}\,, \qquad t \in [t_i, t_{i,\mathrm{f}}] \tag{15}$$

with $J(\Delta\bar{u}(\cdot), \Delta\boldsymbol{\theta}^i) = 0$. Hence, if the system exactly tracks the nominal trajectories the control action of the feedback controller Σ_fb is zero, see Figure 1.

Important design parameters of the receding horizon control scheme are the choice of the weighting matrices $P \in \mathbb{R}^{(N+1)\times(N+1)}$ and $Q \in \mathbb{R}^{(N+1)\times(N+1)}$, of the scalar weight R, and the horizon length t_f. Receding horizon formulations in model predictive control often use terminal set or equality constraints to achieve stability. In the case of a free end point formulation as it is the case in (12), stability can be shown, e. g., if the terminal cost function $||\Delta\boldsymbol{\theta}(t_{i,\mathrm{f}})||_P^2$ represents a (local) control Lyapunov function [1,11,8] or if the horizon length t_f is sufficiently large [5]. For the error dynamics (12b), which is time-dependent due to the feedforward trajectories, the rigorous proof of stability [3] as well as the consistency of this finite-dimensional control with the original infinite-dimensional system is subject of current research. In this contribution, the stability and performance of the receding horizon tracking controller are demonstrated by means of simulation studies in the following section.

3 Simulation Example

With the flat output $y(t) = \theta_0(t)$, the control task under consideration consists in realizing the finite-time transition between two setpoints $y(0) = y_0$ and $y(1) =$

[2] Here $||\mathbf{x}||_S^2 = \mathbf{x}^\mathsf{T} S\mathbf{x}$ denotes the weighted Euclidean norm of the vector $\mathbf{x} \in \mathbb{R}^n$ with the matrix $S \in \mathbb{R}^{n\times n}$ being positive definite.

y_1, which correspond to the (infinite-dimensional) steady-state profiles $\theta(z, 0)$ and $\theta(z, 1)$. A suitable reference trajectory $y^*(t)$ for the trajectory planning has to comply with the desired steady-state output values at the beginning ($t = 0$) and at the end ($t = 1$) of the setpoint transition, i. e.

$$y^*(0) = y_0 \ , \qquad y^*(1) = y_1 \ , \qquad \left.\frac{\mathrm{d}^l}{\mathrm{d}t^l} y(t)\right|_{t=0} = \left.\frac{\mathrm{d}^l}{\mathrm{d}t^l} y(t)\right|_{t=1} = 0 \tag{16}$$

$l = 1, \ldots, N + 1$. The temporal path of the transition can be provided either by a polynomial of suitable order or, especially with regard to the continuous limit of the parametrization, by any smooth function of an appropriate Gevrey-class, see, e. g., [10].

In all simulation results shown in this contribution, the control design relies on a semi-discretization with $N = 10$ grid elements. The following set of system parameters $p_0 = 1.2$, $p_1 = 0.05$, $q_0 = 1$, $q_1 = -0.05$, $\tilde{q} = 1$, $\nu = 0.1$, $r_0 = 1$, and $r_1 = 0.2$ is used and the desired initial and final output values are $y_0 = 1$ and $y_1 = 2$, respectively. In addition, the values $u^{\pm} = \pm 4$ are used as box constraints (4) and a non-zero disturbance

$$d(t) = \begin{cases} -0.4 & \text{for } t \in [0.4, 0.6] \\ 0 & \text{else} \end{cases} \tag{17}$$

is considered in the simulations. The receding horizon control design is based on the time discretization $t_i = i\Delta t$ with the sampling time $\Delta t = 0.005$ and the horizon length $t_\mathrm{f} = 0.3$. In each time step t_i, the OCP (12) is numerically solved with a tailored gradient projection method [4], such that the OCP may be solved in a computationally very efficient way, see also [16]. The weighting matrix Q is set to a diagonal matrix with the diagonal element values interpolated between 200 for the first error state corresponding to the output and the value 2 for the last error state. The terminal weighting matrix P is set to $0.1Q$, while R is chosen as 0.3. The overall feedback controller Σ_fb is implemented as a Cmex function in MATLAB, and for the simulations, the standard MATLAB-solver `ode15s` is used to solve the semi-discretized system (6)–(7) on a grid with N_sim nodes, where $N_\mathrm{sim} \gg N$.

In Figure 2, the setpoint transition in the nominal case, i. e., without disturbance or input constraints, as well as a transition with disturbance (17) and a transition with input constraints (4) is shown. The time behaviour of the feedforward control $u^*(t)$ as well as the evolution of the cost $J(\Delta\bar{u}(\cdot), \Delta\boldsymbol{\theta}^i)$ show that the contribution of the tracking controller $\Delta u_\mathrm{fb}(t)$ vanishes in the nominal case. For the non-zero disturbance (17), this is of course no longer the case. However, it can be seen that after restoring the nominal conditions, the tracking error is reduced very fast, the transition is completed as prescribed and the simulation remains stable. At the same time, the cost $J(\Delta\bar{u}(\cdot), \Delta\boldsymbol{\theta}^i)$ decreases monotonically to zero. In the case of input constraints (4), deviations from the desired behaviour are also inevitable, since they are not considered in the feedforward control design. It can also be observed that due to the prediction horizon the

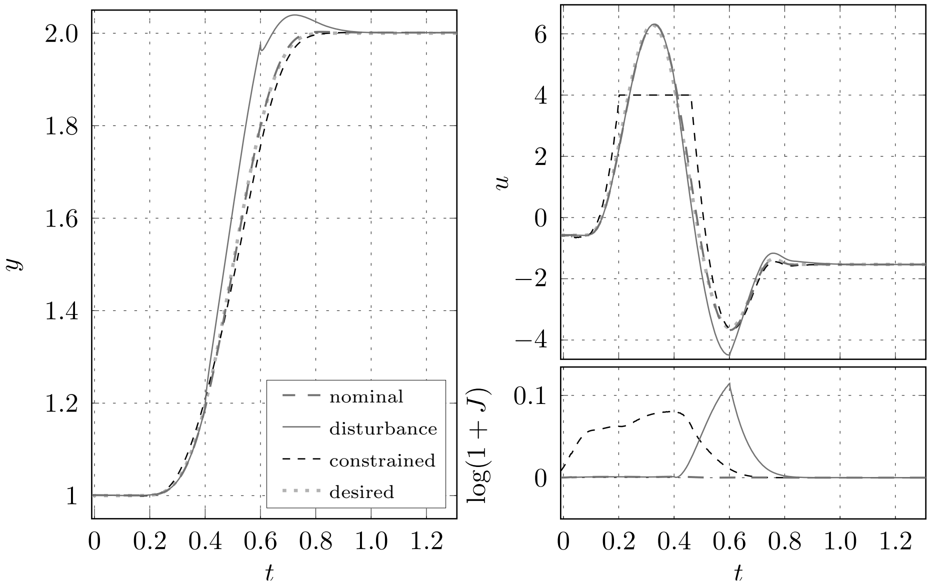

Fig. 2. Output $y(t)$, control input $u(t)$ and cost $J(\Delta\bar{u}(\cdot), \Delta\boldsymbol{\theta}^i)$ for the tracking control of the DCRS in the nominal case, in the case of non-zero disturbance, and under input constraints

feedback controller becomes aware of the impending violation of the constraints before it actually occurs. This results in a rise of the cost and subsequently in the pre-steering action visible in the control input time behaviour, see Figure 2.

The use of a 2DOF control scheme offers some benefits for the realization of setpoint transition tracking control, which are pointed out in the following. In Figure 3 the same setpoint transition as before is considered with non-zero disturbance (17) and the input constraints (4). However, the flatness-based trajectory generation and feedforward control are replaced by a pure feedforward of the steady-state reference values of the state variables $\theta_k(t)$, $k = 0, \dots, N$ and of the input $u(t)$ at $t = 0.1$. Furthermore, a shorter prediction horizon $t_\mathrm{f} = 0.1$ is considered for the receding horizon controller. This could be motivated by the need to reduce the computational cost for the solution of the OCP (12). It can be observed on the one hand that with the prediction horizon $t_\mathrm{f} = 0.3$, the receding horizon control carries out the transition within a transition time comparable to the one observed for the 2DOF control scheme. However, this also results in significant control action especially at the beginning of the transition. On the other hand, the transition time is increased in the case of the short prediction horizon $t_\mathrm{f} = 0.1$ and the desired final output value is not reached within the simulation time. The largely different tracking behaviour seems comprehensible since the transition time is an important tuning parameter of the receding horizon control. This is in contrast to the simulation results obtained if the flatness-based trajectory generation and feedforward control are used in the

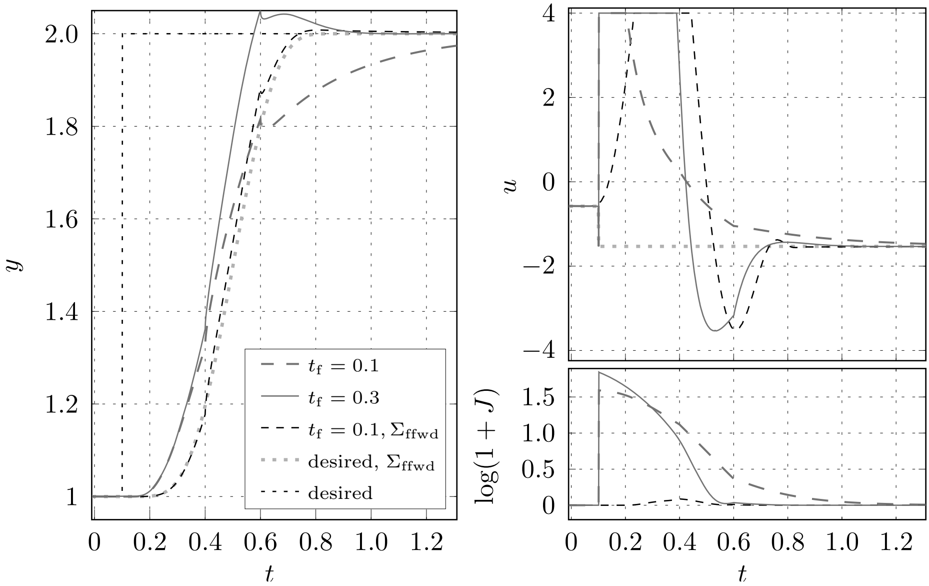

Fig. 3. Output $y(t)$, control input $u(t)$ and cost $J(\Delta\bar{u}(\cdot), \Delta\boldsymbol{\theta}^i)$ for the tracking control of the DCRS with non-zero disturbance and under input constraints both without flatness-based feedforward control and prediction horizon $t_f \in \{0.1, 0.3\}$, and with flatness-based feedforward control (denoted by Σ_{ffwd} in the figure legend) and prediction horizon $t_f = 0.1$

2DOF control scheme. A slight deterioration of the tracking behaviour can also be seen for a reduced prediction horizon. However, as the main control action for the transition is provided by the feedforward control, this deterioration remains comparatively small. This confirms the observation [18] that the disturbance rejection may be designed nearly independently from the setpoint transition in the 2DOF control scheme.

4 Conclusion

In this contribution, a 2DOF control scheme is presented for setpoint transition tracking control of a quasilinear scalar DCRS. A flatness-based feedforward controller and a receding horizon feedback tracking controller are designed in an early lumping approach using a finite-difference semi-discretization of the DCRS. Thereby, input constraints are systematically incorporated into the receding horizon control design. In the simulation studies, the 2DOF control scheme shows a good performance for both trajectory tracking and disturbance rejection. Furthermore, the 2DOF control scheme allows for a nearly independent tuning of the tracking performance and the disturbance rejection. Stability of the feedback tracking control scheme as well as the further decrease of the computational costs are subject to current research activities.

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