Chapter 4
Effective Theories and Elementary Particle Masses

Abstract The concepts of effective theory have a rich history in particle physics. The early days of effective theories have many examples, including Fermi's theory of nucleon decay and chiral lagrangian dynamics for pion scattering. These examples are touched upon briefly before going to the most pressing issue of today, which is the origin of elementary particle masses. The problem of mass generation is first described, where it is shown that simply writing down mass terms manifestly breaks cherished symmetries. It is then shown that spontaneous symmetry breaking cures this problem. The influence of effective field theory is then addressed, where it is shown that the smallness of neutrino masses nicely conforms with our intuition, but the weak-scale value of the Higgs boson mass is confusing. The chapter concludes with an essay describing this mystery and what the resolutions might be.

4.1 Introduction

Effective theories play a central role in particle physics. Perhaps the most famous effective theory of them all is Fermi's four-fermion interaction theory that described nucleon decay and muon decay. The theory is a "V-A theory" (vector minus axial vector interaction) and has the form:

$$\mathscr{L}_{V-A} = -\frac{G_F}{\sqrt{2}}\,\bar{\nu}_\ell \gamma^\mu (1-\gamma^5) f_\ell \; \bar{f}_q \gamma_\mu (1-\gamma^5) f_{q'} \tag{4.1}$$

where $G_F = 1.15\times 10^{-6}\,\mathrm{GeV}^{-2}$ is the Fermi constant determined by experiment. These operators can then induce β decays of the neutron via the constituent quark decays $d \to ue\bar{\nu}$, and can also induce muon decay through $\mu \to e\nu_\mu \bar{\nu}_e$. The history behind determining the precise nature of this interaction is a fascinating one that required painstaking experiment and insightful theory (Renton 1990).

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We know now that the Fermi theory is just the low-energy limit of the electroweak theory of the Standard Model.[1] The Fermi constant G_F that gives the strength of the four-fermion interaction is the low-energy limit of a W-boson propagator multiplied by its couplings to the two bilinear currents:

$$\lim_{p^2 \to 0} \frac{-g^2}{p^2 - M_W^2} \implies \frac{g^2}{M_W^2} \equiv \frac{G_F}{\sqrt{2}} \tag{4.2}$$

where g is the $SU(2)_L$ gauge coupling of the Standard Model. Thus the propagator of the W-boson at very low energies compared with the W mass contracts to a point and makes an effective four-fermion interaction term governed by the Fermi Effective Theory coupling constant G_F.

Another place where Effective Theories are put to good use is in low-energy pion scattering theory. Pions are the lightest strongly interacting hadrons known in nature. The pions will interact with a very large number of other hadrons in the theory to mediate and alter even pure pion-pion scattering. Computing all of these interactions with the multitude of other intermediate hadrons is a daunting prospect to say the least. However, the effective lagrangian approach allows one to simplify these complicated dynamics of higher mass particles interactions into a few low-energy parameters of a chiral lagrangian. This technique is described well in many places (Donoghue et al. 1992).

Yet another manifestation of the power of effective theories is Wilson's discovery of the renormalization group (Wilson and Kogut 1974; Peskin and Schroeder 1995). There it was understood in a general way that at low energies all modes can be "integrated out" to form an effective lagrangian with renormalization group improved parameters. This integration-out procedure was not just hiding the effects of heavier particles into non-dynamical lagrangian mass scales, but also resuming all the higher momentum mode contributions above a cut-off scale. This technique has been extremely powerful in particle physics as both a technically useful tool to resum large quantum logarithms, but also as a conceptual tool to understand the energy flow of a theory. All modern quantum field theory textbooks, including the one listed in Wilson and Kogut (1974); Peskin and Schroeder (1995), have very thorough treatments of this most important issue.

There are numerous other examples of effective theories being employed in the particle physics context. All of the theories of physics beyond the Standard Model also utilize the concepts in one form or another. The language of effective theory concepts is so deeply ingrained in the minds of practitioners now there is rarely need to explicit point out or argue for its utility.

There is, however, one area of particle physics where the notions of effective theories are hard to mesh with reality. This is regarding the structure of the vacuum. For one, effective theory concepts would tell us that the cosmological constant is many orders of magnitude beyond what we observe today. This is usually just ignored in the field, with hope that some other quantum-gravity solution as yet not understood

[1] The electroweak theory of the Standard Model will be discussed in more detail in Sect. 4.3.

will come to save the cosmological constant. I will not talk about this. The second place where our experimental understanding of the vacuum may be at odds with effective theories is in the generation of elementary particle masses. That will be the focus of this chapter. I will first outline the challenges to giving mass to elementary particles in chiral theories and then I will give a brief introduction to the Standard Model electroweak theory. After that I will describe the effective theory issues for the masses of leptons and quarks, neutrinos, and the Higgs boson. The Higgs boson is especially interesting since it has likely been discovered lately (Aad et al. 2012; Chatrchyan et al. 2012), and the theoretical controversies surrounding why its mass is light are very hot today.

4.2 The Problem of Mass in Chiral Gauge Theories

The fermions of the Standard Model and some of the gauge bosons have mass. This is a troublesome statement since gauge invariance appears to allow neither. Let us review the situation for gauge bosons and chiral fermions and introduce the Higgs mechanism that solves it. First, we illustrate the concepts with a massive $U(1)$ theory—spontaneously broken QED.

Gauge Boson Mass

The lagrangian of QED is

$$\mathscr{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi \tag{4.3}$$

where

$$D_{\mu} = \partial_{\mu} + ieA_{\mu} \tag{4.4}$$

and $Q = -1$ is the charge of the electron. This lagrangian respects the $U(1)$ gauge symmetry

$$\psi \to e^{-i\alpha(x)}\psi \tag{4.5}$$

$$A_{\mu} \to A_{\mu} + \frac{1}{e}\partial_{\mu}\alpha(x). \tag{4.6}$$

Since QED is a vector-like theory—left-handed electrons have the same charge as right-handed electrons—an explicit mass term for the electron does not violate gauge invariance.

If we wish to give the photon a mass we may add to the lagrangian the mass term

$$\mathscr{L}_{mass} = \frac{m_A^2}{2}A_{\mu}A^{\mu}. \tag{4.7}$$

However, this term is not gauge invariant since under a transformation $A_\mu A^\mu$ becomes

$$A_\mu A^\mu \to A_\mu A^\mu + \frac{2}{e} A^\mu \partial_\mu \alpha + \frac{1}{e^2} \partial_\mu \alpha \partial^\mu \alpha \tag{4.8}$$

This is not the right way to proceed if we wish to continue respecting the gauge symmetry. There is a satisfactory way to give mass to the photon while retaining the gauge symmetry. This is the Higgs mechanism, and the simplest way to implement it is via an elementary complex scalar particle that is charged under the symmetry and has a vacuum expectation value (vev) that is constant throughout all space and time. This is the Higgs boson field Φ.

Let us suppose that the photon in QED has a mass. To see how the Higgs boson implements the Higgs mechanism in a gauge invariant manner, we introduce the field Φ with charge q to the lagrangian:

$$\mathscr{L} = \mathscr{L}_{QED} + (D_\mu \Phi)^* (D^\mu \Phi) - V(\Phi) \tag{4.9}$$

where

$$V(\Phi) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4 \tag{4.10}$$

where it is assumed that $\lambda > 0$ and $\mu^2 < 0$.

Since Φ is a complex field we have the freedom to parametrize it as

$$\Phi = \frac{1}{\sqrt{2}} \phi(x) e^{i\xi(x)}, \tag{4.11}$$

where $\phi(x)$ and $\xi(x)$ are real scalar fields. The scalar potential with this choice simplifies to

$$V(\Phi) \to V(\phi) = \frac{\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4. \tag{4.12}$$

Minimizing the scalar potential one finds

$$\left. \frac{dV}{d\phi} \right|_{\phi=\phi_0} = \mu^2 \phi_0 + \lambda \phi_0^3 = 0 \implies \phi_0 = \sqrt{\frac{-\mu^2}{\lambda}}. \tag{4.13}$$

This vacuum expectation value of ϕ enables us to normalize the ξ field by ξ/ϕ_0 such that its kinetic term is canonical at leading order of small fluctuation, legitimizing the parametrization of Eq. (4.11). We can now choose the unitary gauge transformation, $\alpha(x) = -\xi(x)/\phi_0$, to make Φ real-valued everywhere. One finds that the complex scalar kinetic terms expand to

$$(D_\mu \Phi)^* (D^\mu \Phi) \to \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} e^2 q^2 \phi^2 A_\mu A^\mu \tag{4.14}$$

At the minimum of the potential $\langle\phi\rangle = \phi_0$, so one can expand the field ϕ about its vev, $\phi = \phi_0 + h$, and identify the fluctuating degree of freedom h with a propagating real scalar boson.

The Higgs boson mass and self-interactions are obtained by expanding the lagrangian about ϕ_0. The result is

$$-\mathcal{L}_{Higgs} = \frac{m_h^2}{2}h^2 + \frac{\mu'}{3!}h^3 + \frac{\eta}{4!}h^4 \tag{4.15}$$

where

$$m_h^2 = 2\lambda\phi_0^2, \quad \mu' = \frac{3m_h^2}{\phi_0}, \quad \eta = 6\lambda = 3\frac{m_h^2}{\phi_0^2}. \tag{4.16}$$

The mass of the Higgs boson is not dictated by gauge couplings here, but rather by its self-interaction coupling λ and the vev.

The complex Higgs boson kinetic terms can be expanded to yield

$$\Delta\mathcal{L} = \frac{1}{2}e^2q^2\phi_0^2A_\mu A^\mu + e^2q^2hA_\mu A^\mu + \frac{1}{2}e^2q^2h^2A_\mu A^\mu. \tag{4.17}$$

The first term is the mass of the photon, $m_A^2 = e^2q^2\phi_0^2$. A massive vector boson has a longitudinal degree of freedom, in addition to its two transverse degrees of freedom, which accounts for the degree of freedom lost by virtue of gauging away $\xi(x)$. The second and third terms of Eq. 4.17 set the strength of interaction of a single Higgs boson and two Higgs bosons to a pair of photons:

$$hA_\mu A_\nu \text{ Feynman rule}: \ i2e^2q^2\phi_0 g_{\mu\nu} = i2\frac{m_A^2}{\phi_0} \tag{4.18}$$

$$hhA_\mu A_\nu \text{ Feynman rule}: \ i2e^2q^2 g_{\mu\nu} = i2\frac{m_A^2}{\phi_0^2} \tag{4.19}$$

after appropriate symmetry factors are included.

The general principles to retain from this discussion are first that massive gauge bosons can be accomplished in a gauge-invariant way through the Higgs mechanism. The Higgs boson that gets a vev breaks whatever symmetries it is charged under—the Higgs vev carries charge into the vacuum. And finally, the Higgs boson that gives mass to the gauge boson couples to it proportional to the gauge boson mass.

Chiral Fermion Masses

In quantum field theory a four-component fermion can be written in its chiral basis as

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{4.20}$$

where $\psi_{L,R}$ are two-component chiral projection fermions. A mass term in quantum field theory is equivalent to an interaction between the ψ_L and ψ_R components

$$m\bar{\psi}\psi = m\psi_L^\dagger\psi_R + m\psi_R^\dagger\psi_L. \tag{4.21}$$

In vectorlike QED, the ψ_L and ψ_R components have the same charge and a mass term can simply be written down. However, let us now suppose that in our toy $U(1)$ model, there exists a set of chiral fermions where the $P_L\psi = \psi_L$ chiral projection carries a different gauge charge than the $P_R\psi = \psi_R$ chiral projection. In that case, we cannot write down a simple mass term without explicitly breaking the gauge symmetry.

The resolution to this conundrum of masses for chiral fermions resides in the Higgs sector. If the Higgs boson has just the right charge, it can be utilized to give mass to the chiral fermions. For example, if the charges[2] are $Q[\psi_L] = 1$, $Q[\psi_R] = 1 - q$ and $Q[\Phi] = q$ we can form the gauge invariant combination

$$\mathscr{L}_f = y_\psi\, \psi_L^\dagger \Phi \psi_R + c.c. \tag{4.22}$$

where y_f is a dimensionless Yukawa coupling. Now expand the Higgs boson about its vev, $\Psi = (\phi_0 + h)/\sqrt{2}$, and we find

$$\mathscr{L}_f = m_\psi\, \psi_L^\dagger\psi_R + \left(\frac{m_\psi}{\phi_0}\right) h\psi_L^\dagger\psi_R + c.c. \tag{4.23}$$

where $m_\psi = y_\psi\phi_0/\sqrt{2}$.

We have successfully generated a mass by virtue of the Yukawa interaction with the Higgs boson. That same Yukawa interaction gives rise to an interaction between the physical Higgs boson and the fermions:

$$h\bar{\psi}\psi \text{ (Feynman rule) } : \; i\frac{m_\psi}{\phi_0}. \tag{4.24}$$

Just as was the case with the gauge bosons, the generation of fermion masses by the Higgs boson leads to an interaction of the physical Higgs bosons with the fermion proportional to the fermion mass. As we will see in the Standard Model, this rigid connection between mass and interaction is what enables us to anticipate Higgs boson phenomenology with great precision once the mass is precisely known.

4.3 Standard Model Electroweak Theory

The bosonic electroweak lagrangian is an $SU(2)_L \times U(1)_Y$ gauge invariant theory

$$\mathscr{L}_{bos} = |D_\mu\Phi|^2 - \mu^2|\Phi|^2 - \lambda|\Phi|^4 - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu} \tag{4.25}$$

[2] We ignore the additional fields that would be needed in order to make the spectrum gauge anomaly free. Doing so is straightforward and would not change the message of this example.

where Φ is an electroweak doublet with Standard Model charges of $(\mathbf{2}, 1/2)$ under $SU(2)_L \times U(1)_Y$ $(Y = +1/2)$. In our normalization electric charge is $Q = T^3 + \frac{Y}{2}$, and the doublet field Φ can be written as two complex scalar component fields ϕ^+ and ϕ^0:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \tag{4.26}$$

The covariant derivative and field strength tensors are

$$D_\mu \Phi = \left(\partial_\mu + ig\frac{\tau^a}{2} W_\mu^a + ig'\frac{Y}{2} B_\mu \right) \Phi \tag{4.27}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{4.28}$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - gf^{abc} W_\mu^b W_\nu^c \tag{4.29}$$

The minimum of the potential does not occur at $\Phi = 0$ if $\mu^2 < 0$. Instead, one finds that the minimum occurs at a non-zero value of Φ—its vacuum expectation value (vev)—which via a gauge transformation can always be written as

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{where} \quad v \equiv \sqrt{\frac{-\mu^2}{\lambda}}. \tag{4.30}$$

This vev carries hypercharge and weak gauge charge into the vacuum, and what is left unbroken is electric charge. This result we anticipated in Eq. (4.26) by defining a charge Q in terms of hypercharge and an eigenvalue of the $SU(2)$ generator T^3, and then writing the field Φ in terms of ϕ^0 and ϕ^+ of zero and positive $+1$ definite electric charge.

Our symmetry breaking pattern is then simply $SU(2)_L \times U(1)_Y \to U(1)_Q$. The original group, $SU(2)_L \times U(1)_Y$, has a total of four generators and $U(1)_Q$ has one generator. Thus, three generators are 'broken'. Goldstone's theorem (Goldstone et al. 1962) tells us that for every broken generator of a symmetry there must correspond a massless field. These three massless Goldstone bosons we can call $\phi_{1,2,3}$. We now can rewrite the full Higgs field Φ as

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ h + i\phi_3 \end{pmatrix} \tag{4.31}$$

The fourth degree of freedom of Φ is the Standard Model Higgs boson h. It is a propagating degree of freedom. The other three states $\phi_{1,2,3}$ can all be absorbed as longitudinal components of three massive vector gauge bosons Z, $W^\pm$ which are defined by

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^{(1)} \mp iW_\mu^{(2)} \right) \tag{4.32}$$

$$B_\mu = \frac{-g' Z_\mu + g A_\mu}{\sqrt{g^2 + g'^2}} \tag{4.33}$$

$$W_\mu^{(3)} = \frac{g Z_\mu + g' A_\mu}{\sqrt{g^2 + g'^2}}. \tag{4.34}$$

It is convenient to define $\tan\theta_W = g'/g$. By measuring interactions of the gauge bosons with fermions it has been determined experimentally that $g = 0.65$ and $g' = 0.35$, and therefore $\sin^2\theta_W = 0.23$.

After performing the redefinitions of the fields above, the kinetic terms for the $W_\mu^\pm$, Z_μ, A_μ will all be canonical. Expanding the Higgs field about the vacuum, the contributions to the lagrangian involving Higgs boson interaction terms are

$$\mathscr{L}_{h\,int} = \left[m_W^2 W_\mu^+ W^{-,\mu} + \frac{m_Z^2}{2} Z_\mu Z^\mu \right] \left(1 + \frac{h}{v}\right)^2 \tag{4.35}$$

$$- \frac{m_h^2}{2} h^2 - \frac{\xi}{3!} h^3 - \frac{\eta}{4!} h^4 \tag{4.36}$$

where

$$m_W^2 = \frac{1}{4} g^2 v^2, \quad m_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2 \implies \frac{m_W^2}{m_Z^2} = 1 - \sin^2\theta_W \tag{4.37}$$

$$m_h^2 = 2\lambda v^2, \quad \xi = \frac{3 m_h^2}{v}, \quad \eta = 6\lambda = \frac{3 m_h^2}{v^2}. \tag{4.38}$$

From our knowledge of the gauge couplings, the value of the vev v can be determined from the masses of the gauge bosons: $v \simeq 246\,\mathrm{GeV}$.

The Feynman rules for Higgs boson interactions are

$$hhh : -\frac{i 3 m_h^2}{v} \tag{4.39}$$

$$hhhh : -i \frac{3 m_h^2}{v^2} \tag{4.40}$$

$$h W_\mu^+ W_\nu^- : i2 \frac{m_W^2}{v} g^{\mu\nu} \tag{4.41}$$

$$h Z_\mu Z_\nu : i2 \frac{m_Z^2}{v} g_{\mu\nu} \tag{4.42}$$

$$hh W_\mu^+ W_\nu^- : i2 \frac{m_W^2}{v^2} g_{\mu\nu} \tag{4.43}$$

$$hh Z_\mu Z_\nu : i2 \frac{m_Z^2}{v^2} g_{\mu\nu} \tag{4.44}$$

Fermion masses are also generated in the Standard Model through the Higgs boson vev, which in turn induces an interaction between the physical Higgs boson and the fermions. Let us start by looking at b quark interactions. The relevant lagrangian for couplings with the Higgs boson is

$$\Delta\mathscr{L} = y_b Q_L^\dagger \Phi b_R + c.c. \quad \text{where} \quad Q_L^\dagger = (t_L^\dagger \; b_L^\dagger) \tag{4.45}$$

where y_b is the Yukawa coupling. The Higgs boson, after a suitable gauge transformation, can be written simply as

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} \tag{4.46}$$

and the interaction lagrangian can be expanded to

$$\Delta\mathscr{L} = y_b Q_L^\dagger \Phi b_R + c.c. = \frac{y_b}{\sqrt{2}} (t_L^\dagger \; b_L^\dagger) \begin{pmatrix} 0 \\ v + h \end{pmatrix} b_R + h.c. \tag{4.47}$$

$$= m_b (b_R^\dagger b_L + b_L^\dagger b_R) \left(1 + \frac{h}{v}\right) = m_b \bar{b} b \left(1 + \frac{h}{v}\right) \tag{4.48}$$

where $m_b = y_{bv}/\sqrt{2}$ is the mass of the b quark.

The quantum numbers work out perfectly to allow this mass term. See Table 4.1 for the quantum numbers of the various fields under the Standard Model symmetries. Under $SU(2)$ the interaction $Q_L^\dagger \Phi b_R$ is invariant because $\mathbf{2} \times \mathbf{2} \times \mathbf{1} \in \mathbf{1}$ contains a singlet. And under $U(1)_Y$ hypercharge the interaction is invariant because $Y_{Q_L^\dagger} + Y_\Phi + Y_{b_R} = -\frac{1}{6} + \frac{1}{2} - \frac{1}{3}$ sums to zero. Thus, the interaction is invariant under all gauge groups, and we have found a suitable way to give mass to the bottom quark.

How does this work for giving mass to the top quark? Obviously, $Q_L^\dagger \Phi t_R$ is not invariant. However, we have the freedom to create the conjugate representation of Φ which still transforms as a $\mathbf{2}$ under $SU(2)$ but switches sign under hypercharge: $\Phi^c = i\sigma^2 \Phi^*$. This implies that $Y_{\Phi^c} = -\frac{1}{2}$ and

$$\Phi^c = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h \\ 0 \end{pmatrix} \tag{4.49}$$

when restricted to just the real physical Higgs field expansion about the vev. Therefore, it becomes clear that $y_t Q_L^\dagger \Phi^c t_R + c.c.$ is now invariant since the $SU(2)$ invariance remains $\mathbf{2} \times \mathbf{2} \times \mathbf{1} \in \mathbf{1}$ and $U(1)_Y$ invariance follows from $Y_{Q_L^\dagger} + Y_{\Phi^c} + Y_{t_R} = -\frac{1}{6} - \frac{1}{2} + \frac{2}{3} = 0$. Similar to the b quark one obtains an expression for the mass and Higgs boson interaction:

Table 4.1 Charges of standard model fields

Field	$SU(3)$	$SU(2)_L$	T^3	$\frac{Y}{2}$	$Q = T^3 + \frac{Y}{2}$
g^a_μ (gluons)	**8**	**1**	0	0	0
$(W^\pm_\mu, W^0_\mu)$	**1**	**3**	$(\pm 1, 0)$	0	$(\pm 1, 0)$
B^0_μ	**1**	**1**	0	0	0
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	**3**	**2**	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
u_R	**3**	**1**	0	$\frac{2}{3}$	$\frac{2}{3}$
d_R	**3**	**1**	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$E_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	**1**	**2**	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
e_R	**1**	**1**	0	-1	-1
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	**1**	**2**	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\Phi^c = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$	**1**	**2**	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$$\Delta\mathscr{L} = y_t Q_L^\dagger \Phi^c t_R + c.c. = \frac{y_t}{\sqrt{2}}(t_L^\dagger \; b_L^\dagger) \begin{pmatrix} v+h \\ 0 \end{pmatrix} t_R + c.c. \tag{4.50}$$

$$= m_t(t_R^\dagger t_L + t_L^\dagger t_R)\left(1 + \frac{h}{v}\right) = m_t \bar{t}t \left(1 + \frac{h}{v}\right) \tag{4.51}$$

where $m_t = y_t v/\sqrt{2}$ is the mass of the t quark.

The mass of the charged leptons follows in the same manner, $y_e E_L^\dagger \Phi e_R + c.c.$, and interactions with the Higgs boson result. In all cased the Feynman diagram for Higgs boson interactions with the fermions at leading order is

$$h\bar{f}f \;:\; i\frac{m_f}{v}. \tag{4.52}$$

We see from this discussion several important points. First, the single Higgs boson of the Standard Model can give mass to all Standard Model states, even to the neutrinos as we will see in the next section. It did not have to be that way. It could have been that quantum numbers of the fermions did not enable just one Higgs boson to give mass to everything. This is the Higgs boson miracle of the Standard Model. The second thing to keep in mind is that there is a direct connection between the Higgs boson giving mass to a particle and it interacting with that particle. We have seen that all interactions are directly proportional to a mass factor. This is why Higgs boson phenomenology is completely determined in the Standard Model just from the Higgs boson mass.

4.4 The Special Case of Neutrino Masses

For many years it was thought that neutrinos might be exactly massless. Although recent experiments have shown that this is not the case, the masses of neutrinos are extraordinarily light compared to other Standard Model fermions. In this section we discuss the basics of neutrino masses (Grossman 2003; De Gouvea 2004; Mohapatra 2004; Altarelli 2007), with emphasis on how the Higgs boson plays a role.

Some physicists define the Standard Model without a right-handed neutrino. Thus, there is no opportunity to write down a Yukawa interaction of the left and right-handed neutrinos with the Higgs boson that gives neutrinos a mass. A higher-dimensional operator is needed,

$$\mathscr{O}_\nu = \frac{\lambda_{ij}}{\Lambda}(E^\dagger_{iL}H^c)^\dagger(E^\dagger_{jL}H^c) \tag{4.53}$$

where $E_L = (\nu_L\ e_L)$ is the $SU(2)$ doublet of left-handed neutrino and electron. Taking into account the various flavors $i = 1, 2, 3$ results in a 3×3 mass matrix for neutrino masses

$$(m_\nu)_{ij} = \lambda_{ij}\frac{v^2}{\Lambda}. \tag{4.54}$$

Λ can be considered the cutoff of the Standard Model effective theory (see Sect. 4.5), and the operator given by Eq. (4.53) is the only gauge-invariant, Lorentz-invariant operator that one can write down at the next higher dimension ($d = 5$) in the theory. Thus, it is a satisfactory approach to neutrino physics, leading to an indication of new physics beyond the Standard Model at the scale Λ. For this reason, many view the existence of neutrino masses as a signal for physics beyond the Standard Model.

The absolute value of neutrino masses has not been measured but the differences of mass squareds between various neutrino masses have been measured and range from about 10^{-5} to $10^{-2}\,\mathrm{eV}^2$ (Grossman 2003; De Gouvea 2004; Mohapatra 2004; Altarelli 2007; Kayser 2012). It is reasonable therefore to suppose that the largest neutrino mass in the theory should be around 0.1 eV. If we assume that this mass scale is obtained using the natural value of $\lambda \sim 1$ in Eq. (4.54) and a large mass scale Λ, this sets the scale of the cutoff Λ to be

$$\Lambda \simeq \frac{(246\,\mathrm{GeV})^2}{0.1\,\mathrm{eV}} \simeq 10^{15}\,\mathrm{GeV} \tag{4.55}$$

This is a very interesting scale, since it is within an order of magnitude of where the three gauge couplings of the Standard Model come closest to meeting, which may be an indication of grand unification. The scale Λ could then be connected to this Grand Unification scale.

Another approach to neutrino masses is to assume that there exists a right-handed neutrino ν_R. After all, there is no strong reason to banish this state, especially since there is an adequate right-handed partner state to all the other fermions. Furthermore, if the above considerations are pointing to a grand unified theory, right-handed neutrinos are generally present in acceptable versions, such as $SO(10)$ where all the fermions are in the **16** representation, including ν_R. Quantum number considerations indicate that ν_R is a pure singlet under the Standard Model gauge symmetries, and thus we have a complication in the neutrino mass sector beyond what we encountered for the other fermions of the theory. In particular, we are now able to add a Majorana mass term $\nu_R^T i\sigma^2 \nu_R$ that is invariant all by itself without the need of a Higgs boson. The full mass interactions available to the neutrino are now

$$\mathscr{L}_\nu = y_{ij} E_{iL}^\dagger \Phi^c \nu_{jR} + \frac{M_{ij}}{2} \nu_{iR}^T i\sigma^2 \nu_{jR} + c.c. \tag{4.56}$$

The resulting 6×6 mass matrix in the $\{\nu_L, \nu_R^c\}$ basis is

$$m_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \tag{4.57}$$

where M is the matrix of Majorana masses with values M_{ij} taken straight from Eq. (4.56), and m_D are the neutrino Dirac mass matrices taken from the Yukawa interaction with the Higgs boson

$$(m_D)_{ij} = \frac{y_{ij}}{\sqrt{2}} v. \tag{4.58}$$

Consistent with effective field theory ideas, there is no reason why the Majorana mass matrix entries should be tied to the weak scale. They should be of order the cutoff scale of when the Standard Model is no longer considered complete. Therefore, it is reasonable and expected to assume that M_{ij} entries are generically much greater than the weak scale. In that limit, the seesaw matrix of Eq. (4.57) has three heavy eigenvalues of $\mathscr{O}(M)$, and three light eigenvalues that, to leading order and good approximation, are eigenvalues of the 3×3 matrix

$$m_\nu^{\text{light}} = -m_D^T M^{-1} m_D \sim y^2 \frac{v^2}{M} \tag{4.59}$$

which is parametrically of the same form as Eq. (4.54). This is expected since the light eigenvalues can be evaluated from the operators left over after integrating out the heavy right-handed neutrinos in the effective theory. That operator is simply Eq. (4.53), where schematically Λ can be associated with the scale M, and λ can be associated with y^2.

We will emphasize in the next section that the story of neutrino masses conforms very nicely with our notions of effective field theories. It is for this reason that most physicists are not terribly alarmed about the smallness of neutrino masses, even though on the surface it would appear quite disturbing to know that neutrinos are orders of magnitude in mass below other particles that we measure very directly at colliders. They are 12 orders of magnitude below the top quark mass, for example. Nevertheless, there is no concern.

The role of effective theory becomes much more troublesome to understand in the context of Higgs boson physics, even though the Higgs boson mass is in the close neighborhood (i.e., less than an order of magnitude difference) of the W, Z, and top quark masses. The effective theory issues surrounding the peculiar spin zero Higgs boson, the main focus of this chapter that we have been building to, is something we come to now.

4.5 Natural Effective Theories, the Higgs Boson, and the Hierarchy Problem

The Standard Model with its postulated Higgs boson is an unsatisfactory theory for many reasons. There are several direct data-driven reasons why it is incomplete. The Standard Model has no explanation for the baryon asymmetry of the Universe. For some reason there are many more protons than anti-protons, and if the Universe is cooling from some primordial hot state with particles in thermal equilibrium, that is unexpected. Some mechanism that goes beyond the Standard Model dynamics must be at play. Similarly, there is plenty of astrophysical evidence for dark matter in the Universe. This dark matter helps to explain structure formation, details of the cosmic microwave background radiation, galactic rotation curves, etc. The problem is the Standard Model has no candidate explanation, and new physics must be invoked.

There are many other reasons to consider physics beyond the Standard Model. The three gauge forces could be unified and the matter unified within representations of a grand unified symmetry. The many different parameters of the flavor sector are hard to swallow without envisaging deeper principles that organize them. Furthermore, the integration of the Standard Model with quantum gravity is not obvious, and many think a deeper structure, such as that built from strings and branes, is needed for their coexistence.

So, there are many reasons to believe that there is physics beyond the Standard Model. But the issue that is front and center for us now, relevant to Higgs boson physics and electroweak explorations at the Large Hadron Collider, is the Hierarchy Problem. The Hierarchy Problem is often expressed as a question: Why is the weak scale ($\sim 10^2$ GeV) so much lighter than the Planck scale ($\sim 10^{18}$ GeV)? It is a bit uninspiring when phrased this way, since it begs the question of why we should be concerned at all about a big difference in scales. Blue whales are much bigger than nanoarchaeum equitans but we do not believe nature must reveal a dramatic new concept for us to understand it (Clauset 2012).

A knowing-just-enough-to-be-dangerous naive way to look at the Standard Model is that it is the "Theory of Particles", valid up to some out-of-reach scale where gravity might go strong, or some other violence is occurring that we do not care about. It is a renormalizable theory. I can compute everything at multiple quantum loop order, set counter terms, cancel infinities that are fake since they do not show up in observables, and then make predictions for observables that experiment agrees with. Quadratic divergences of the Higgs boson self-energy, which so many people make a fuss about, are not even there if I use dimensional regularization. The theory is happy, healthy, stable, and in no need of any fixes. New physics *near the electroweak scale* can still be justified (Wells 2003, 2005; Arkani-Hamed and Dimopoulos 2005; Giudice and Romanino 2004; Arkani-Hamed et al. 2005) after dismissing naturalness as impossibly imprecise to understand at this stage, but the urgency is certainly diminished for it being *at the electroweak scale*.

This viewpoint that the Standard Model is complete can be challenged right at the outset. It is simply not the "Theory of Particles"—it does break down. It is an effective theory, even if one thinks there is a way to argue it being valid to some very remote high scale where gravity goes strong, such as M_{Pl}. As an effective theory, all operators should have their dimensionality set by the cutoff of the theory (Polchinski 1992). If operator $\mathcal{O}^{(d)}$ has dimension d then its coefficient is $c\Lambda^{4-d}$, where Λ is the cutoff of the theory and c is expected to be $\sim$1 in value. Irrelevant operators with $d > 4$ cause no harm. Same goes for $d = 4$ marginal operators. The Standard Model is almost exclusively a theory of $d = 4$ marginal operators with its kinetic terms, gauge interaction terms, and Yukawa interaction terms. What is potentially problematic is the existence of any $d < 4$ relevant operators. In that case, the coefficients should be large, set by the cutoff of the theory.

Does the Standard Model have any gauge-invariant, Lorentz-invariant relevant $d < 4$ operators to worry about? Yes, two of them. The right-handed neutrino Majorana mass interaction terms $\nu_R^T i\sigma^2 \nu_R$, which is $d = 3$, and the Higgs boson mass operator $|H|^2$, which is $d = 2$. The expectations of effective field theories is that the scale of the coefficients of these operators should be set by high-scale cutoffs of the theory and disconnected from any other surviving mass scale in the infrared. As we saw in Sect. 4.4 this expectation is nicely met in the neutrino case, where we have actually measured the masses and see a self-consistent picture for large Majorana masses for the right-handed neutrinos, which serve as cutoff scale coefficients. These coefficients are tied to lepton number violation, for example, and not electroweak symmetry breaking, and therefore have naturally large values above the weak scale.

It did not have to be that way with neutrino physics. It could have been that the neutrino sector was shown experimentally to have independent left and right-handed components and the masses were of order the weak scale. This would have been in violation of effective field theory expectations, unless new symmetries tied to the weak scale were discovered to protect the right-handed neutrino from getting a large Majorana mass. The fact that the neutrino sector conforms with effective field theory expectations should be viewed as contributing evidence for these concepts.

In contrast to the neutrino operator, the $d = 2$ Higgs mass operator in the Standard Model is unwelcome if its coefficient is not set to the weak scale. From our effective field theory expectations, the Lagrangian operator should be

$$\Delta\mathscr{L}_{rel} = c\Lambda^2|H|^2. \tag{4.60}$$

This is a potential disaster for the theory, since from our previous work on the Higgs potential we stated that the Higgs mass must be $-\mu^2 \sim v^2$, where $v \simeq 246\,\text{GeV}$ is the Higgs boson vacuum expectation value needed to reproduce the W and Z masses. If we assume the Standard Model to be a valid theory to very high energies $E \gg v$, that implies the cutoff of the Standard Model effective theory is $\Lambda \gg v$, which "incorrectly" implies the coefficient of $|H|^2$ is $|\mu^2| = \Lambda^2 \gg v^2$. The effective theory would then need the coefficient c in Eq. (4.60) to be finetuned to an extraordinarily small and unnatural (Giudice 2004) value $c \sim v^2/\Lambda^2$ to make all the scales work out properly. The concern about how this can be so is the Hierarchy Problem.

The discussion is a bit abstract, but it bears fruit with direct computations. As one example out of an infinite number that would demonstrate the Hierarchy Problem, consider the possible existence of other scalar fields ϕ_i at higher energies. The assumption is that if there is a Higgs boson in the theory, then there is every reason to believe that there can be other scalars. They can have mass at the weak scale, intermediate scale, Planck scale, wherever. Let us suppose that we put one ϕ at the cutoff scale Λ of the theory. The operator $|\phi|^2|H|^2$ immediately gives a quantum correction to the Higgs mass operator coefficient of $\sim \Lambda^2/16\pi^2$. Although the $1/16\pi^2$ can help a little, if $\Lambda \gg 4\pi v$ there is serious problem, and the weak scale cannot exist naturally with such a hierarchy. For this reason, it is often assumed that naturalness of the Higgs boson sector of the Standard Model effective theory requires new physics to show up at some scale below $\Lambda \sim 4\pi v \sim$ few TeV.

There are many different approaches to solving the Hierarchy Problem. One approach suggests that there is new physics at the TeV scale and the cutoff Λ in Eq. (4.60) is in the neighborhood of the weak scale. Supersymmetry (Martin 1997), little Higgs (Schmaltz and Smith 2005), conformal theories (Frampton and Vafa 1999), and extra dimensions (Sundrum 2005; Rattazzi 2006) can be employed in this approach. For example, supersymmetry accomplishes the task by a softly broken symmetry, where Λ is the supersymmetry breaking mass scale. All quadratic divergences to the Higgs boson mass operator cancel up to supersymmetry breaking terms. Extra dimensions accomplishes it by banishing all mass scales accessible to the Higgs boson above the TeV scale. Another approach suggests that fundamental scalars are banished from the theory that could form invariant $|\varphi|^2$ operators. For example, this is the approach of Technicolor (Lane and Martin 2009) and top-quark condensate theories (Hill 1991, 1995; Martin 1997; Chivukula et al. 1999) that try to reproduce the symmetry breaking of a Higgs boson with the condensate of a fermion bilinear operator. Higgsless theories and their variants are also in this category (Csaki et al. 2004a,b; Cui et al. 2009). These theories are obviously less interesting given the discovery of a Higgs-like boson, but it is extraordinarily difficult, and perhaps impossible, to resolve whether the Higgs boson is a fundamental scalar or merely a composite particle acting like a scalar. Also, theories with no true Higgs boson can have another particle—a dilation, for example—that acts like a Higgs boson.

Therefore, these theories still have life within them, and more data is required to gain confidence in these alternative explanations or rule them out.

Nevertheless, the least complicated thoughts suggest to us that a simple Higgs boson has been discovered with mass of approximately 126 GeV (Aad et al. 2012; Chatrchyan et al. 2012). Of course, there is no certainty that it is the SM Higgs boson. Indeed, such certainty is likely to never exist, but measurements at the LHC can likely give us confidence that its couplings are within 20 % of the values that the SM Higgs boson would have. Next-generation colliders, such as an e^+e^- linear collider, would be able to further refine this to percent level, or perhaps even show that there are small deviations from SM expectations. In any case, it is legitimate to call it "a Higgs boson" since it appears to be coupling to the vector boson and fermions according to their mass values, and that puts an added confidence that the particle is associated with mass generation. Again, metaphysical certainty into the nature of any particle will always be out of the question, but the evidence is accruing and the words "for all practical purposes" are just around the corner.

This has been a major achievement by humans. The historical theory development that culminated in a highly speculative prediction for a new Higgs boson that turned out to be there is just one aspect of this achievement. There is also the decades of work and expertise built up to invent and apply experimental techniques that discovered the boson. This is not to mention the impressive human resource management skills needed to herd all the people together in a collective effort to divide tasks and construct the coherent whole—the discovery.

The smugness we may feel for the discovery of the Higgs boson is to be tempered with the stark truth that nothing else has been found at the LHC at this time. If it continues this way it means that many predictions, influenced by concepts of effective theories, were wrong that insisted that the Higgs boson needed an entourage of other particles very close by in mass to tame its quantum instabilities. Maybe they were only wrong quantitatively, and new particles and dynamics are around the corner to vindicate effective theories.

Or perhaps there is yet another factor that is overriding our effective theory intuitions. Perhaps there is a multiverse where the solution to the Hierarchy Problem suggests that large statistics of finetuned solutions dominate over the fewer number of non-tuned solutions in the landscape, leading to a higher probability of our Universe landing in a highly tuned solution ($c \ll 1$). Thus, guided by concerns over the cosmological constant problem, it has been suggested that this statistical, stringy naturalness over the landscape may take precedence over normal naturalness envisioned from effective field theories (Douglas 2007; Kumar 2006). Although not directly related to external particle physics interactions, the cosmological constant can be considered as the coefficient of yet another gauge-invariant, Lorentz-invariant operator—the operator being merely a constant: $-\mathscr{L}_{cc} = \Lambda_{cc}^4$. The tiny value of this coefficient, $\Lambda_{cc}^4 \simeq (10^{-3}\,\mathrm{eV})^4$, is well below any conceivable theory expectation. It is the elephant in the room for effective field theories. However, it is an unexpressed article of faith among most particle physicists that the solution to the Cosmological Constant Problem lies in the details of mysterious quantum gravity, and that the new concepts buried in that unknown solution do not materially affect the natural solution to the Hierarchy Problem. Landscapists question that assumption. This is controversial with conflicting claims over unrealistic theories; nevertheless, it is an interesting idea that might one day be impactful.

Data keeps coming, and searches for new particles that would vindicate our most basic notions of effective field and naturalness continue. Many "good ideas" are now dead after years of data have found no evidences for them. There is no theorem that we will have full resolution to all the "good ideas" within our lifetimes, or that any of the colliders we are running or contemplating in the future will have enough energy or luminosity or precision to give a final say on the matter. Nevertheless, the field carries on and the tree of various interpretations of what has been seen and what has not been seen grows branches, flowers, and surely will bear fruit again.

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