Generalizing Naive and Stable Semantics in Argumentation Frameworks with Necessities and Preferences

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Abstract. In [4] [5], the classical acceptability semantics are generalized by preferences. The extensions under a given semantics correspond to maximal elements of a relation encoding this semantics and defined on subsets of arguments. Furthermore, a set of postulates is proposed to provide a full characterization of any relation encoding the generalized stable semantics. In this paper, we adapt this approach to preference-based argumentation frameworks with necessities. We propose a full characterization of stable and naive semantics in this new context by new sets of adapted postulates and we present a practical method to compute them by using a classical Dung argumentation framework.

Keywords: abstract argumentation, acceptability semantics, necessities, preferences.

1 Introduction

Dung's abstract argumentation frameworks (AFs) [12] are a very influential model which has been widely studied and extended in different directions. Moreover, some works (see for example [2], [14]) started recently to bridge the gap between this model and the logical-based model [7] in which arguments are constructed from (possibly inconsistent) logical knowledge bases as couples of the form (support, claim). Among the various extensions of Dung model we are interested in this paper on two of them : adding information about preferences and representing support relations between arguments.

We need to handle preferences in argumentation theory because in real contexts, arguments may have different strengths. In Dung style systems, adding preferences may lead to the so-called "critical attacks" that arise when a less preferred argument attacks a more preferred one. Most of the first proposals on preference-based argumentation like [1] [6] [16] suggest to simply remove the critical attacks but the drawback of these approaches is to possibly tolerate non conflict-free extensions. A new approach proposed in [3] [4] [5] encodes acceptability semantics for preference-based argumentation frameworks (PAFs) as a relation on the powerset of the set of arguments and the extensions under a

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given semantics as the maximal elements of this relation. This approach avoids the drawback mentioned above and ensures the recovering of the classical acceptability semantics when no critical attack is present. Moreover, [4] [5] give a set of postulates that must be verified by any relation encoding the stable semantics in preference-based frameworks.

At a different level, different approaches have been proposed to enrich Dung model by information expressing supports between arguments. The bipolar argumentation frameworks (BAFs) [10] [11] add an explicit support relation to Dung AFs and define new acceptability semantics. Their main drawback is that admissibility of extensions is no more guaranteed. [8] introduces the so-called deductive supports and proposes to use a meta Dung AF to obtain the extensions. Abstract dialectical frameworks [9] represent a powerful generalization of Dung AFs in which the acceptability conditions of an argument are more sophisticated. The acceptability semantics are redefined by adapting the Gelfond/Lifshitz reduct used in answer set programming (ASP). The Argumentation Frameworks with Necessities (AFNs) [17] are a kind of bipolar AFs where the support relation has the meaning of "necessity". The acceptability semantics are extended in a natural way that ensures admissibility without borrowing techniques from LPs or making use of a Meta Dung model. The aim of this paper is threefold:

- Adapting the approach proposed in [4] [5] for stable semantics to the case of preference-based AFNs (PAFNs). We show that by introducing some suitable notions, we obtain new postulates that are very similar to the original ones.
- Giving a full characterization of naive semantics in the context of PAFNs. Since the naive semantics is the counterpart of justified Łukaszewicz extensions [15] in default logic and of ι-answer sets [13] in logic programming (see [18]), this characterization allows a better understanding of these approaches.
- Computing a Dung AF whose naive and stable extensions correspond exactly to the generalized naive and stable extensions of the input PAFN.

The rest of the paper is organized as follows. In section 2 we recall some basics of AFs with necessities and/or preferences as well as the approach generalizing stable semantics by preferences. In section 3 we present some further notions that are useful to define the new postulates for PAFNs. Section 4 presents the stable and naive semantics in AFNs seen as dominance relations. Section 5 discusses the generalization of these semantics by preferences. In section 6 we characterize PAFNs by classical Dung AFs. Finally, section 7 concludes the paper.

2 Background

2.1 Argumentation Frameworks, Necessities and Preferences

A Dung AF [12] is a pair $F = \langle A, R \rangle$ where A is a set of arguments and R is a binary attack relation over A. A set $S \subseteq A$ attacks an argument b iff $(\exists a \in S)$ *s.t* $a\ R\ b$. S is *conflict-free* iff $(\nexists a, b \in S)$ *s.t.* $a\ R\ b$. Many *acceptability semantics* have been proposed to define how sets of collectively acceptable arguments may be derived from an attack network. Among them we will focus in this paper on the *naive* and the *stable semantics*.

Definition 1. Let $F = \langle A, R \rangle$ be an AF and $S \subseteq A$. S is a naive extension of F iff S is a $\subseteq$-maximal conflict-free set. S is a stable extension of F iff S is conflict-free and $(\forall b \in A \setminus S)(\exists a \in S)$ *s.t.* $a\ R\ b$.

For an AF $F = \langle A, R \rangle$, we use the notation $Ext^N(F)$ (resp. $Ext^S(F)$) to denote the set of naive (resp. stable) extensions of F.

Argumentation frameworks with necessities (AFNs) (see [17] for details) represent a kind of bipolar extension of Dung AFs where the support relation is a *necessity* that captures situations in which one argument is necessary for another. Formally, an AFN is defined by $\Gamma = \langle A, R, N \rangle$ where A is a set of arguments, $R \subseteq A \times A$ is a binary *attack* relation over A and $N \subseteq A \times A$ is a *necessity* relation over A. For $a, b \in A$, $a\ N\ b$ means that the acceptance of a is necessary for the acceptance of b. We suppose that there are no cycles of necessities, i.e., $\nexists a_1, \ldots a_k$ for $k \geq 1$ such that $a_1 = a_k = a$ and $a_1\ N\ a_2 \ldots N\ a_k$.

To give the new definitions of acceptability semantics in AFNs, we need to introduce the key notions of coherence and strong coherence. The latter plays in some way the same role of conflict-freeness in Dung AFs.

Definition 2. Let $\Gamma = \langle A, R, N \rangle$ be an AFN and $S \subseteq A$. S is :

- *coherent* iff S is closed under N^{-1}, i.e. $(\forall a \in S)(\forall b \in A)$ if $b\ N\ a$ then $b \in S$.
- *strongly coherent* iff S is coherent and conflict-free (w.r.t. R).
- *a naive extension* of Γ iff S is a $\subseteq$-maximal strongly coherent set.
- *a stable extension* of Γ iff S is strongly coherent and $(\forall b \in A \setminus S)$ *either* $(\exists a \in S)$ *s.t.* $a\ R\ b$ *or* $(\exists a \in A \setminus S)$ *s.t.* $a\ N\ b$.

For an AFN $\Gamma = \langle A, R, N \rangle$, we use the notation $Ext^N(\Gamma)$ (resp. $Ext^S(\Gamma)$) to denote the set of naive (resp. stable) extensions of Γ.

The main properties of naive and stable extensions in AFs continue to hold for AFNs, namely : naive extensions always exist; naive extensions do not depend on the attacks directions; an AFN may have zero, one or several stable extensions and each stable extension is a naive extension but the inverse is not true. Besides, for an AFN $\Gamma = \langle A, R, N \rangle$ where $N = \emptyset$, strong coherence coincides with classical conflict-freeness and naive and stable extensions correspond to naive and stable extensions of the simple AF $\langle A, R \rangle$ respectively (in the sense of definition 1.).

Finally, a preference-based AF (PAF) (resp. a preference-based AFN (PAFN)) is defined by $\Lambda = \langle A, R, \geq \rangle$ (resp. $\Sigma = \langle A, R, N, \geq \rangle$) where $\langle A, R \rangle$ is an AF (resp. $\langle A, R, N \rangle$ is an AFN) and the additional element $\geq$ is a (partial or total) preorder on A. $a \geq b$ means that a it at least as strong as b.

2.2 Stable Semantics as a Dominance Relation in PAFs

The idea of representing acceptability semantics in a PAF as dominance relations has been first developed in [3] for grounded, stable and preferred semantics and a full characterization of stable semantics by a set of postulates has been proposed in [4] [5]. This approach encodes acceptability semantics, by taking into account the possible preferences, as a relation $\succeq$ on the powerset 2^A of the set of arguments : for $\mathcal{E}, \mathcal{E}' \in 2^A$, $\mathcal{E} \succeq \mathcal{E}'$ means that $\mathcal{E}$ is at least as good as $\mathcal{E}'$. $\succ$ denotes the strict version of $\succeq$. For a PAF $\Lambda = \langle A, R, \geq \rangle$, $\mathcal{E} \subseteq A$ is an extension under $\succeq$ iff $\mathcal{E}$ is a maximal element wrt $\succeq$, i.e., for each $\mathcal{E}' \subseteq A$, $\mathcal{E} \succeq \mathcal{E}'$. The set of extensions of Λ under $\succeq$ is denoted by $Ext_{\succeq}(\Lambda)$. The set of conflict-free sets of a PAF Λ (resp. an AF F) is denoted by $CF(\Lambda)$ (resp. $CF(F)$).

In [4] [5] the authors give a full characterisation of any dominance relation encoding stable semantics in PAFs (called pref-stable semantics) by means of the four postulates below. Let $\Lambda = \langle A, R, \geq \rangle$ be a PAF and $\mathcal{E}, \mathcal{E}' \in 2^A$:

Postulate 1. : for $\mathcal{E}, \mathcal{E}' \in 2^A$, $\frac{\mathcal{E} \in CF(\Lambda) \qquad \mathcal{E}' \notin CF(\Lambda)}{\mathcal{E} \succ \mathcal{E}'}$

Postulate 2. : for $\mathcal{E}, \mathcal{E}' \in CF(A)$, $\frac{\mathcal{E} \succeq \mathcal{E}'}{\mathcal{E} \setminus \mathcal{E}' \succeq \mathcal{E}' \setminus \mathcal{E}} \qquad \frac{\mathcal{E} \setminus \mathcal{E}' \succeq \mathcal{E}' \setminus \mathcal{E}}{\mathcal{E} \succeq \mathcal{E}'}$

Postulate 3. : for $\mathcal{E}, \mathcal{E}' \in CF(A)$ $s.t.$ $\mathcal{E} \cap \mathcal{E}' = \emptyset$,

$$\frac{(\exists x' \in \mathcal{E}')(\forall x \in \mathcal{E})\ \neg(x\ R\ x'\ \wedge\ \neg(x' > x))\ \wedge\ \neg(x > x')}{\neg(\mathcal{E} \succeq \mathcal{E}')}$$

Postulate 4. : for $\mathcal{E}, \mathcal{E}' \in CF(A)$ $s.t.$ $\mathcal{E} \cap \mathcal{E}' = \emptyset$,

$$\frac{(\forall x' \in \mathcal{E}')(\exists x \in \mathcal{E})\ s.t\ (x\ R\ x'\ \wedge\ \neg(x' > x))\ \vee\ (x'\ R\ x\ \wedge\ x > x')}{\mathcal{E} \succeq \mathcal{E}'}$$

In other words, a dominance relation encodes a pref-stable semantics iff it satisfies the previous postulates. Such a relation is called a pref-stable relation. Postulate 1 ensures the conflict-freeness of extensions. Postulate 2 ensures that the comparison of two conflict-free sets depends entirely on their distinct elements. Postulates 3 and 4 compare distinct conflict-free sets and state when a set is considered as preferred or not to another one. It has been shown that the extensions under any pref-stable relation are the same and that pref-stable semantics generalizes classical stable semantics in the sense that for a PAF $\Lambda = \langle A, R, \geq \rangle$, if preferences do not conflict with attacks, then pref-stable extensions coincide with the stable extensions of the simple AF $\langle A, R \rangle$.

Different pref-stable relations exist. In [4] [5] the authors show the most general pref-stable relation ($\succeq_g$) and the most specific one ($\succeq_s$). The first (resp. the second) returns exactly the facts $\mathcal{E} \succeq_g \mathcal{E}'$ that can be proved by the postulates 1-4 (resp. whose negation cannot be proved by the postulates 1-4). For any pref-stable relation $\succeq$ we have : if $\mathcal{E} \succeq_g \mathcal{E}'$ then $\mathcal{E} \succeq \mathcal{E}'$ and if $\mathcal{E} \succeq \mathcal{E}'$ then $\mathcal{E} \succeq_s \mathcal{E}'$.

3 Emerging Necessities, Attacks and Preferences

In this section we introduce new notions representing *hidden* forms of necessities, attacks and preferences. These new notions are of great importance since they allow to extend in an easy and natural way the approach presented in [4] [5] for PAFs to the case of PAFNs.

The first notion is that of extended necessity relation : If a is necessary for b and b is necessary for c then we can deduce that a is indirectly necessary for c.

Definition 3. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN. An extended necessity between a and b is denoted by $a\ N^+\ b$. It holds if there is a sequence $a_1, \ldots, a_k$ for $k \geq 2$ such that $a = a_1\ N\ a_2 \ldots N\ a_k = b$. $N^+(a)$ denotes the set of all the arguments that are related to a by an extended necessity, i.e., $N^+(a) = \{b \in A | b\ N^+\ a\}$. Moreover, we use the notation $a\ N^*\ b$ for any a such that $a\ N^+\ b$ or $a = b$ and we put $N^*(a) = N^+(a) \cup \{a\}$.

The interaction between attacks and necessities results in further implicit attacks that we call the extended attacks. In general, an extended attack between two arguments a and b holds whenever an element of $N^*(a)$ attacks (directly) an elements of $N^*(b)$. Indeed, if we accept a then we must accept a' which excludes b' (since a' attacks b') and this excludes in turn b.

Definition 4. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN and $\Gamma = \langle A, R, N \rangle$ be its corresponding simple AFN. An extended attack from an argument a to an argument b is denoted by $a\ R^+\ b$. It holds iff $(\exists b' \in N^*(b))(\exists a' \in N^*(a))\ s.t\ a'\ R\ b'$.

$CF^+(\Sigma)$ (and $CF^+(\Gamma)$) denotes the set of conflict-free subsets of A wrt R^+ and $SC(\Sigma)$ (and $SC(\Gamma)$) the set of strongly coherent subsets of A. It turns out that strong coherence is stronger than conflict freeness wrt R^+ :

Property 1. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN and $S \subseteq A$. If S is strongly coherent then $S \in CF^+(\Sigma)$. The inverse is not true.

The different preference-based argumentation approaches more or less agree that the relevant problem to solve in handling preferences in AFs is that of critical attacks (a critical attack arises when an argument a attacks an argument b while b is better than a). It turns out that the interaction between preferences and necessities does not lead to a similar problem since a necessity between two arguments does not necessarily contradict the fact that one of these two arguments is better (or worse) than the second. To understand how preferences interact with necessities, we have to look at the very meaning of necessity. Indeed, accepting an argument requires the acceptance of all its (direct and indirect) necessary arguments. Thus, the input preference assigned to an argument a represents solely a rough preference. Its effective preference depends on that of all the elements of the set $N^*(a)$. To induce the effective preference of the arguments from their input (rough) preference we use the usual democratic relation .

Definition 5. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN and $a, b \in A$. The *effective preference* relation between arguments is denoted by $\trianglerighteq \subseteq A \times A$ and defined as follows : $(\forall a, b \in A)\ a \trianglerighteq b$ iff $(\forall b' \in N^*(b) \setminus N^*(a))(\exists a' \in N^*(a) \setminus N^*(b))\ s.t\ a' \geq b'$. $\triangleright$ is the strict version of $\trianglerighteq$,i.e., $a \triangleright b$ iff $a \trianglerighteq b$ *and not* $b \trianglerighteq a$.

Finally we assume in the rest of the paper that the set A of arguments is finite and that there is no $a \in a$ s.t. $a\ R^+\ a$.

Example 1. Consider the PAFN $\Sigma = \langle A, R, N, \geq \rangle$ where $A = \{a, b, c, d\}$, $R = \{(c, a), (b, d)\}$, $N = \{(a, b), (c, d)\}$ and $a \geq c,\ a \geq d,\ d \geq b$. The corresponding AFN $\langle A, R, N \rangle$ is depicted in Fig. 1 :

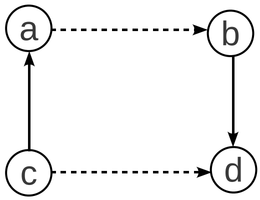

Fig. 1. The AFN corresponding to the PAFN Σ where $a \geq c,\ a \geq d,\ d \geq b$

By applying the previous definitions on this example we obtain :

$N^+ = N$ and $N^* = \{(a, a), (b, b), (c, c), (d, d), (a, b), (c, d)\}$.
$R^+ = \{(b, d), (c, a), (c, b), (d, a), (d, b)\}$.
$SC(\Sigma) = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{c, d\}\}$.
$CF^+(\Sigma) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{c, d\}\}$.
$\trianglerighteq = \{(a, a), (b, b), (c, c), (d, d), (a, c), (a, d), (b, a), (b, c), (b, d), (d, c)\}$.
$\triangleright = \{(a, c), (a, d), (b, a), (b, c), (b, d), (d, c)\}$.

4 Generalizing Naive and Stable Semantics in AFNs

In this section we give a characterization of any relation $\succeq \subseteq A \times A$ that encodes naive or stable semantics in a simple AFN without preferences. The idea is that one must recover this characterization in the particular case of a PAFN where no conflict with attacks and necessities is caused by the presence of additional information about preferences. The characterization given here adapts the original one introduced in [4] [5] for the relations encoding stable semantics in simple AFs to the case where a necessity relation is present and give a new version dealing with naive semantics.

The following theorem[1] states the requirements that any relation encoding naive or stable semantics must fulfil :

Theorem 1. Let $\Gamma = \langle A, R, N \rangle$ be an AFN and $\succeq \subseteq 2^A \times 2^A$ then,

– $(\forall \mathcal{E} \in 2^A)$ $(\mathcal{E} \in Ext^N(\Gamma) \Leftrightarrow (\mathcal{E}$ *is maximal wrt* $\succeq^N))$ iff :
  1. $(\forall \mathcal{E} \in 2^A)$ if $\mathcal{E} \notin SC(\Gamma)$ then $(\exists \mathcal{E}' \in SC(\Gamma))$ s.t. $\neg(\mathcal{E} \succeq^N \mathcal{E}')$.
  2. if $\mathcal{E} \in SC(\Gamma)$ and $(\forall a' \notin \mathcal{E})(\exists a \in \mathcal{E})$ s.t. $(a\ R^+\ a'$ or $a'\ R^+\ a)$ then $(\forall \mathcal{E}' \in 2^A)\ \mathcal{E} \succeq^N \mathcal{E}'$.
  3. if $\mathcal{E} \in SC(\Gamma)$ and $(\exists a' \notin \mathcal{E})$ s.t. $(\nexists a \in \mathcal{E})$ and $(a\ R^+\ a'$ or $a'\ R^+\ a)$ then $(\exists \mathcal{E}' \in 2^A)$ s.t. $\neg(\mathcal{E} \succeq^N \mathcal{E}')$.

[1] Because of space limitation, the proofs of theorems and properties are not included in the paper.

- $(\forall \mathcal{E} \in 2^A)$ $(\mathcal{E} \in Ext^S(\Gamma) \Leftrightarrow (\mathcal{E}\ is\ maximal\ wrt\ \succeq^S))$ iff :
  1. $(\forall \mathcal{E} \in 2^A)$ if $\mathcal{E} \notin SC(\Gamma)$ then $(\exists \mathcal{E}' \in SC(\Gamma))$ s.t. $\neg(\mathcal{E} \succeq^S \mathcal{E}')$.
  2. if $\mathcal{E} \in SC(\Gamma)$ and $(\forall a' \notin \mathcal{E})(\exists a \in \mathcal{E})$ s.t. $(a\ R^+\ a')$ then $(\forall \mathcal{E}' \in 2^A)$ $\mathcal{E} \succeq^S \mathcal{E}'$.
  3. if $\mathcal{E} \in SC(\Gamma)$ and $(\exists a' \notin \mathcal{E})$ s.t. $(\nexists a \in \mathcal{E})$ and $(a\ R^+\ a')$ then $(\exists \mathcal{E}' \in 2^A)$ s.t. $\neg(\mathcal{E} \succeq^S \mathcal{E}')$.

Let $\Gamma = \langle A, R, N \rangle$ be an AFN and $\mathcal{E}, \mathcal{E}' \in 2^A$. The relation $\succeq_1^N$. (resp. $\succeq_1^S$) below is an example of a relation encoding naive (resp. stable) semantics in Γ :

$\mathcal{E} \succeq_1^N \mathcal{E}'$ (resp. $\mathcal{E} \succeq_1^S \mathcal{E}'$)iff:

- $\mathcal{E} \in SC(\Gamma)$ and $\mathcal{E}' \notin SC(\Gamma)$, or
- $\mathcal{E}, \mathcal{E}' \in SC(\Gamma)$ and $(\forall a' \in \mathcal{E}' \setminus \mathcal{E})(\exists a \in \mathcal{E} \setminus \mathcal{E}')$ $s.t$ $(a\ R^+\ a'$ or $a'\ R^+\ a)$ (resp. $\mathcal{E}, \mathcal{E}' \in SC(\Gamma)$ and $(\forall a' \in \mathcal{E}' \setminus \mathcal{E})(\exists a \in \mathcal{E} \setminus \mathcal{E}')$ $s.t$ $a\ R^+\ a')$

Example 1 (Cont.). Let us consider again the PAFN of example 1 and take the corresponding simple AFN $\Gamma = \langle A, R, N \rangle$. It is easy to check that the maximal sets wrt $\succeq_1^N$ are $\{a, b\}$ and $\{c, d\}$ and only $\{c, d\}$ is a maximal set wrt $\succeq_1^S$. Notice that the same results are obtained by applying definition 2.

5 Generalizing Naive and Stable Semantics in PAFNs

The objective of this section is twofold. On the one hand we extend the full characterisation of Pref-Stable-Semantics (the generalization of stable semantics to PAFs) to the case of PAFNs. We call the resulting semantics the N-pref-stable semantics. On the other hand we give an additional full characterization of what we call the N-pref-naive extension which generalizes the naive extensions in PAFNs. We show that the relationship between stable and naive semantics in AFNs remains valid for the new generalized semantics and that the original semantics of simple AFNs are recovered if preferences are not in conflict with extended attacks. We call N-pref-naive (resp. N-pref-stable) relation any relation that encodes a N-pref-naive (resp. N-pref-stable) semantics. If there is no ambiguity we use indifferently the symbol $\succeq$ to denote a N-pref-naive or a N-pref-stable relation, otherwise we use $\succeq^N$ to denote a N-pref-naive relation and $\succeq^S$ to denote a N-pref-stable relation.

Consider the PAFN $\Sigma = \langle A, R, N, \geq \rangle$ and the relation $\succeq \subseteq 2^A \times 2^A$. We denote by $Ext_{\succeq}(\Sigma)$ the maximal subsets of arguments wrt $\succeq$ [2]. Let us first present and discuss the set of postulates we will use in characterizing N-pref-naive and N-pref-stable relations :

Postulate 1' : Let $\mathcal{E}, \mathcal{E}' \in 2^A$. Then : $\frac{\mathcal{E} \in SC(\Sigma) \qquad \mathcal{E}' \notin SC(\Sigma)}{\mathcal{E} \succ \mathcal{E}'}$

Postulate 2' : Let $\mathcal{E}, \mathcal{E}' \in SC(\Sigma)$. Then : $\frac{\mathcal{E} \succeq \mathcal{E}'}{\mathcal{E} \setminus \mathcal{E}' \succeq \mathcal{E}' \setminus \mathcal{E}} \qquad \frac{\mathcal{E} \setminus \mathcal{E}' \succeq \mathcal{E}' \setminus \mathcal{E}}{\mathcal{E} \succeq \mathcal{E}'}$

[2] We recall that $\mathcal{E} \in Ext_{\succeq}(\Sigma)$ iff $(\forall \mathcal{E}' \in 2^A)$ $\mathcal{E} \succeq \mathcal{E}'$.

Postulate 3' : Let $\mathcal{E}, \mathcal{E}' \in CF^+(\Sigma)$ $s.t. \mathcal{E} \cap \mathcal{E}' = \emptyset$. Then :

$$\frac{(\exists x' \in \mathcal{E}')(\forall x \in \mathcal{E})\ \neg(x\ R^+\ x') \wedge \neg(x' R^+ x) \wedge \neg(x \rhd x') \wedge \neg(x' \rhd x)}{\neg(\mathcal{E} \succeq \mathcal{E}')}$$

Postulate 3" : Let $\mathcal{E}, \mathcal{E}' \in CF^+(\Sigma)$ $s.t. \mathcal{E} \cap \mathcal{E}' = \emptyset$. Then :

$$\frac{(\exists x' \in \mathcal{E}')(\forall x \in \mathcal{E})\ \neg(x\ R^+\ x' \wedge \neg(x' \rhd x)) \wedge \neg(x \rhd x')}{\neg(\mathcal{E} \succeq \mathcal{E}')}$$

Postulate 4' : Let $\mathcal{E}, \mathcal{E}' \in CF^+(\Sigma)$ $s.t. \mathcal{E} \cap \mathcal{E}' = \emptyset$. Then :

$$\frac{(\forall x' \in \mathcal{E}')(\exists x \in \mathcal{E})\ s.t\ (x\ R^+\ x') \vee (x' R^+ x)}{\mathcal{E} \succeq \mathcal{E}'}$$

Postulate 4" : Let $\mathcal{E}, \mathcal{E}' \in CF^+(\Sigma)$ $s.t. \mathcal{E} \cap \mathcal{E}' = \emptyset$. Then :

$$\frac{(\forall x' \in \mathcal{E}')(\exists x \in \mathcal{E})\ s.t\ (x\ R^+\ x' \wedge \neg(x' \rhd x)) \vee (x'\ R^+\ x \wedge x \rhd x')}{\mathcal{E} \succeq \mathcal{E}'}$$

Postulates (1') and (2') are similar to postulates (1) and (2). They just replace conflict-freeness by strong coherence. Postulate (1) ensures that maximal elements of any relation satisfying it are strong coherent sets. Postulate (2') states that comparing two strongly coherent sets depends only on their distinct elements. Notice that if $\mathcal{E}, \mathcal{E}' \in SC(\Sigma)$ then it is obvious that $\mathcal{E} \setminus \mathcal{E}' \in CF^+(\Sigma)$ but it is not necessarily the case that $\mathcal{E} \setminus \mathcal{E}' \in SC(\Sigma)$. This is why the rest of postulates are defined on elements of $CF^+(\Sigma)$.

Postulate (3') and (3") are adapted forms of postulate (3). They capture the case where a set must not be better than another for any N-pref-naive and any N-pref-stable relation respectively. For N-pref-stable relations (postulate (3")) the adaptation consists just in replacing the direct attack by the extended one and the input preference relation by the effective one. The information about necessities is incorporated to these two notions in order to keep the original form of this postulate : a set must not be better than another whenever the second contains an argument that is neither successfully attacked nor strictly less preferred than any element of the first. For N-pref-naive relations (postulate(3')), a form of symmetry is introduced so that the orientation of attacks and preferences are no more important. Thus, a set must not be better than another whenever the second contains an element which is neither involved in an attack wrt R^+ (whatever its direction) nor compared by $\unrhd$ with any element of the first set.

Like postulate (4), postulates (4') and (4") capture the case where a set must be better than another for any N-pref-naive (resp. N-pref-stable) relation. For N-pref-stable relations (postulate 4") the adaptation consists again to just replacing the direct attack by the extended one and the input preference relation by the effective one. A set must be better than another whenever for each argument b of the second set there is an argument a in the first set such that either a attacks b (wrt. R^+) and b is not strictly better than a (wrt. $\unrhd$) or b attacks a but a is strictly better than b. For N-pref-naive relations (postulate(4')), a set must be considered as better than another whenever for each argument of the second set there is an argument in the first set which is in conflict with it (wrt. R^+). The N-pref-naive and N-pref-stable semantics are then defined as follows.

Definition 6. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN and le us consider a relation $\succeq \subseteq 2^A \times 2^A$. $\succeq$ encodes N-pref-naive semantics iff it verifies the postulates 1',2',3',4' and $\succeq$ encodes N-pref-stable semantics iff it verifies the postulates 1',2',3",4".

All the results obtained for pref-stable semantics in PAFs (in absence of necessities) are easily generalized to the case of N-pref-stable semantics. Moreover, the relationship between naive and stable semantics is kept in the generalized semantics. First we have that N-pref-naive and N-pref-stable extensions are strongly coherent.

Property 2. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN and $\succeq$ be a N-pref-naive or a N-pref-stable relation. If $\mathcal{E} \in Ext_{\succeq}(\Sigma)$ then $\mathcal{E} \in SC(\Sigma)$.

It turns out that postulate (3') is stronger than postulate (3") and postulate (4") is stronger than postulate (4'). This means that N-pref-naive relations derive more positive facts (of the form $\mathcal{E} \succeq \mathcal{E}'$) and allow less negative facts (of the form $\neg(\mathcal{E} \succeq \mathcal{E}')$) than N-pref-stable relations:

Theorem 2. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN and $\succeq \subseteq A \times A$. if $\succeq$ satisfies postulate 3' then it satisfies postulate 3" and if it satisfies postulate 4" then it satisfies postulate 4'.

It is not difficult to check that one consequence of this result is that any N-pref-stable extension is also a N-pref-naive extension.

Corollary 1. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN and $\succeq^N$ be a N-pref-naive relation and $\succeq^S$ a N-pref-stable relation. If $\mathcal{E} \in Ext_{\succeq^S}(\Sigma)$ then $\mathcal{E} \in Ext_{\succeq^N}(\Sigma)$.

As in the case of pref-stable semantics, all the N-pref-naive (resp. N-pref-stable) relations share the same maximal elements.

Theorem 3. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN. For any pair of N-pref-naive (or N-pref-stable relations) $\succeq, \succeq' \subseteq A \times A$ we have : $Ext_{\succeq}(\Sigma) = Ext_{\succeq'}(\Sigma)$.

Although some N-pref-naive relations depend on the preferences (for example the most specific relation discussed later), the extensions themselves are independent from the preferences. This is true since there is always a N-pref-naive relation that is independent from the preferences (for example, the most general one discussed later in this section) and according to the previous theorem, any other N-pref-naive relation leads to the same extensions.

Theorem 4. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN, $\Gamma = \langle A, R, N \rangle$ be its corresponding simple AFN and $\succeq^N$ be a N-pref-naive relation then : $Ext_{\succeq^N}(\Sigma) = Ext^N(\Gamma)$ and $\succeq^N$ satisfies conditions 1,2,3 of theorem 1 for naive extensions.

The following theorem states that N-pref-stable semantics generalizes stable semantics for AFNs : if there is no conflict between extended attacks and effective preferences, the stable extensions of the corresponding AFN are recovered.

Theorem 5. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN and $\Gamma = \langle A, R, N \rangle$ the corresponding AFN. If $\succeq^S$ is a N-Pref-Stable relation and $(\nexists a, b \in A)$ $s.t.$ $a\ R^+ b$ and $b \rhd a$, then : $Ext_{\succeq^S}(\Sigma) = Ext^S(\Gamma)$ and $\succeq^S$ satisfies conditions 1,2,3 of theorem 1 for stable extensions.

Since N-pref-naive extensions are independent from preferences and each N-pref-stale extension is a N-pref-naive extension one can conclude the exact role of adding or updating preferences in an AFN :

Corollary 2. In a PAFN, preferences have no impact on naive extensions but they affect the selection function of stable extensions among naive extensions.

N-pref-stable semantics also generalizes pref-stable semantics. Indeed when the necessity relation is empty the N-pref-stable extensions coinside with the pref-stable extensions of the corresponding PAF.

Theorem 6. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN with $N = \emptyset$, $\Lambda = \langle A, R, \geq \rangle$ be its corresponding simple PAF. If $\succeq^S$ is a N-pref-stable relation then : $Ext_{\succeq^S}(\Sigma) = Ext_{\succeq^S}(\Lambda)$ and $\succeq^S$ satisfies the postulates 1,2,3 and 4 characterizing pref-stable extensions of a PAF.

The most general N-pref-naive relation $\succeq_g^N$ of a PAFN $\Sigma = \langle A, R, N, \geq \rangle$ coincides with the relation $\succeq_1^N$ (see section 3) which does not depend on the preference relation. The most specific relation $\succeq_s^N$ is defined as follows[3]:

$$(\mathcal{E} \succeq_g^N \mathcal{E}') \text{ iff } (\mathcal{E}' \notin SC(\Sigma)) \text{ or } (\mathcal{E}, \mathcal{E}' \in SC(\Sigma) \text{ and}$$
$$(\forall a' \in \mathcal{E}' \setminus \mathcal{E})(\exists a \in \mathcal{E} \setminus \mathcal{E}')\ s.t.\ (a\ R^+\ a') \vee (a'\ R^+\ a) \vee (a \rhd a') \vee (a' \rhd a))$$

The relation $\succeq_g^N$ and $\succeq_s^N$ verify the postulates of a N-pref-naive relation and any other N-pref-naive relation is stronger than $\succeq_g^N$ and weaker than $\succeq_s^N$.

Theorem 7. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN. The relations $\succeq_g^N$ and $\succeq_s^N$ are N-pref-naive relations and for any N-pref-naive relation $\succeq^N$ we have : if $\mathcal{E} \succeq_g^N \mathcal{E}'$ then $\mathcal{E} \succeq^N \mathcal{E}'$ and if $\mathcal{E} \succeq^N \mathcal{E}'$ then $\mathcal{E} \succeq_s^N \mathcal{E}'$.

[3] We recall that the most general (resp. the most specific) N-pref-naive relation returns $\mathcal{E} \succeq_g^N \mathcal{E}'$ (resp. $\mathcal{E} \succeq_s^N \mathcal{E}'$) iff it can be proved (resp. it cannot be proved) with the postulates 1' to 4' that $\mathcal{E}$ is better than $\mathcal{E}$'(resp.$\mathcal{E}$' is better than $\mathcal{E}$). Similar definitions are used for N-pref-stable relations.

For a PAFN $\Sigma = \langle A, R, N, \geq \rangle$, the most general and the most specific N-pref-stable relations $\succeq_g^S$ and $\succeq_s^S$ respectively, are defined as follows:

$$(\mathcal{E} \succeq_g^S \mathcal{E}') \text{ iff } (\mathcal{E} \in SC(\Sigma) \textit{ and } (\mathcal{E}' \notin SC(\Sigma) \text{ or } \mathcal{E}, \mathcal{E}' \in SC(\Sigma) \text{ and }$$
$$(\forall a' \in \mathcal{E}' \setminus \mathcal{E})(\exists a \in \mathcal{E} \setminus \mathcal{E}') \; s.t. \; ((a \; R^+ \; a') \wedge \neg(a' \rhd a)) \vee ((a' \; R^+ \; a) \wedge (a \rhd a')))$$

$$(\mathcal{E} \succeq_s^S \mathcal{E}') \text{ iff } (\mathcal{E}' \notin SC(\Sigma) \text{ or } (\mathcal{E}, \mathcal{E}' \in SC(\Sigma) \text{ and }$$
$$(\forall a' \in \mathcal{E}' \setminus \mathcal{E})(\exists a \in \mathcal{E} \setminus \mathcal{E}') \text{ s.t. } ((a \; R^+ \; a') \wedge \neg(a' \rhd a)) \vee (a \rhd a'))$$

A similar result of that given by theorem 7 is valid for N-pref-stable relations.

Theorem 8. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN. The relations $\succeq_g^S$ and $\succeq_s^S$ are N-pref-stable relations and for any N-pref-stable relation $\succeq^S$ we have : if $\mathcal{E} \succeq_g^S \mathcal{E}'$ then $\mathcal{E} \succeq^S \mathcal{E}'$ and if $\mathcal{E} \succeq^S \mathcal{E}'$ then $\mathcal{E} \succeq_s^S \mathcal{E}'$.

Example 1 (cont.). Consider again the PAFN of example 1 and its corresponding simple AFN $\Gamma = \langle A, R, N \rangle$. We can easily check that $Ext_{\succeq^N}(\Sigma) = Ext^N(\Gamma) = \{\{a, b\}, \{c, d\}\}$ and $Ext_{\succeq^S}(\Sigma) = \{\{a, b\}\}$. We can remark that as expected, N-pref-naive extensions are not sensible to preferences contrarily to N-pref-stable extensions. Notice that for this particular example, the relations $\succeq_g^N$ and $\succeq_s^N$ (resp. $\succeq_g^S$ and $\succeq_s^S$) agree on the comparison between any two sets $\mathcal{E}, \mathcal{E}' \in CF(\Sigma)$ and between a set $\mathcal{E} \in CF(\Sigma)$ and a set $\mathcal{E}' \notin CF(\Sigma)$. But for $\succeq_s^N$ (resp. $\succeq_s^S$), we have also $\mathcal{E} \succeq_s^N \mathcal{E}'$ (resp. $\mathcal{E} \succeq_s^N \mathcal{E}'$) for any $\mathcal{E}, \mathcal{E}' \notin CF(\Sigma)$. In general, $\succeq_s^N$ (resp. $\succeq_s^S$) may have strictly more elements than $\succeq_g^N$ (resp. $\succeq_g^S$).

6 Characterisation in a Dung AF

In this section we give a characterization of N-pref-naive and N-pref-stable semantics in terms of classical naive and stable semantics of a Dung AF.

Under N-pref-naive semantics, $\mathcal{E}$ is an extension iff it is in conflict with each element outside it. Notice that this condition is independent from any preference.

Theorem 9. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN, R^+ be the corresponding extended attack relation, $\trianglerighteq$ be the corresponding effective preference relation and $\succeq^N$ be a N-pref-naive relation, then $\mathcal{E} \in Ext_{\succeq^N}(\Sigma)$ iff $\mathcal{E} \in SC(\Sigma)$ and $(\forall x' \in A \setminus \mathcal{E}')(\exists x \in \mathcal{E}) \; s.t \; (x \; R^+ \; x') \vee (x' \; R^+ \; x)$.

A set $\mathcal{E}$ is a N-pref-stable extension iff for each element b outside it there is an element a inside it such that either a attacks b (wrt R^+) and b is not strictly preferred to a (wrt. $\trianglerighteq$) or b attacks a but a is strictly preferred to b :

Theorem 10. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN, R^+ be the corresponding extended attack relation, $\trianglerighteq$ be the corresponding effective preference relation and $\succeq^S$ be a N-pref-stable relation, then $\mathcal{E} \in Ext_{\succeq^S}(\Sigma)$ iff $\mathcal{E} \in SC(\Sigma)$ and $(\forall x' \in A \setminus \mathcal{E}')(\exists x \in \mathcal{E}) \; s.t \; (x \; R^+ \; x' \wedge \neg(x' \rhd x)) \vee (x' \; R^+ \; x \wedge x \rhd x')$.

In Practice, The "structural" operations to perform on a PAF $\Sigma = \langle A, R, N, \geq \rangle$ to compute N-pref-naive and N-pref-stable extensions are the following :

- Compute the extended attack (R^+) and the effective preference ($\trianglerighteq$) relations.
- Compute the Dung AF $F = \langle A, Def \rangle$ where Def is a new attack relation obtained by inversing the direction of any attack of R^+ not in accordance with the preference relation $\trianglerighteq$. In other words, $Def = \{(a, b) \in A \times A | (a\ R^+\ b\ and\ \neg(b \triangleright a))\ or (b\ R^+\ a\ and\ a \triangleright b)\}$.
- Use the AF $F = \langle A, Def \rangle$ to compute as usual naive and stable extensions.

The N-pref-naive and the N-pref-stable extensions correspond respectively to the naive and stable extensions of the Dung AF whose set of arguments is that of the original PAFN and attack relation is R^+ after the inversion of any attack which is not in accordance with the effective preference relation $\trianglerighteq$.

Theorem 11. Let $\Sigma = \langle A, R, N, \geq \rangle$ be a PAFN, $\succeq^N$ be a N-pref-naive relation, $\succeq^S$ be a N-pref-stable relation, R^+ be the extended attack relation, $\trianglerighteq$ be the effective preference relation and F be the AF $\langle A, Def \rangle$ where Def is defined by $Def = \{(a, b) \in A \times A | (a\ R^+\ b\ and\ \neg(b \triangleright a))\ or(b\ R^+\ a\ and\ a \triangleright b)\}$ then: $Ext_{\succeq^N} = Ext^N(F)$ and $Ext_{\succeq^S} = Ext^S(F)$.

Example 1 (cont.). Consider again the PAFN of example 1. The Dung system with the extended attack R^+ is deicted in Fig. 2-(1). Inversing the directions of attacks wrt R^+ that are not compatible with the effective preference relation $\trianglerighteq$ (theses attacks are reperesented by thik arcs in Fig. 2-(1)) allows to compute the Dung AF $F = \langle A, Def \rangle$ with the new attack relation $Def = \{(a, c), (a, d), (b, c), (b, d)\}$ (see Fig. 2-(2)).

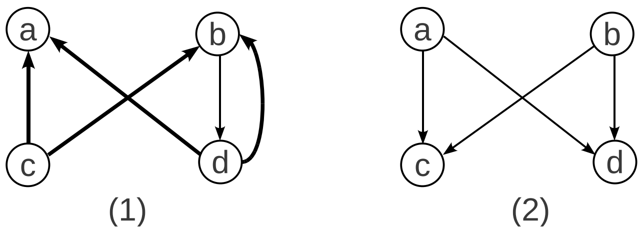

Fig. 2. (1) The Dung AF $\langle A, R^+ \rangle$ (before reparation), (2) The Dung AF $F = \langle A, Def \rangle$ (after reparation)

We can easily check that $Ext_{\succeq^N} = Ext^N(F) = \{\{a, b\}, \{c, d\}\}$ and $Ext_{\succeq^S} = Ext^S(F) = \{\{a, b\}\}$.

7 Conclusion

This paper builds upon the approach proposed in [4] [5] that introduces the interesting idea seeing acceptability semantics as a family of relations on the power set of the set of arguments. In our work we have extended this approach to the case of AFNs that represent a kind of bipolar argumentation frameworks where the support relation is a necessity. We have shown that a key point to perform this extension is to replace the input relations of necessities, attacks and preferences by the new relations of extended necessities, extended attacks and effective preferences. Thanks to these new notions, the obtained adapted form of stable semantics in preference-based AFNs is fully characterized by postualtes that are very similar to those proposed in [4] [5]. We have also extended the approach to the case of generalized naive semantics which represents the counterpart of justified extensions in default logics and ι-answer sets in logic programming. We have also shown how to represent any PAFN as a Dung AF so that a one to one correspondence is ensured between the (generalized) stable and naive extensions of the former and the (classical) stable and naive extensions of the second.

It has been shown in [17] that the argumentation frameworks we obtain by extending the necessity relation so that it can relate sets of arguments to single arguments allows to reach the same expressive power of arbitrary LPs. As a future work we want to use this idea to propose new argumentation-based approaches to handle preferences in logic programs.

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