A Method for Reducing the Cardinality of the Pareto Front

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Abstract. Multi-objective problems are characterised by the presence of a set of optimal trade-off solutions –a Pareto front–, from which a solution has to be selected by a decision maker. However, selecting a solution from a Pareto front depends on large quantities of solutions to select from and dimensional complexity due to many involved objectives, among others. Commonly, the selection of a solution is based on preferences specified by a decision maker. Nevertheless a decision maker may have not preferences at all. Thus, an informed decision making process has to be done, which is difficult to achieve. In this paper, selecting a solution from a Pareto front is addressed as a multi-objective problem using two utility functions and operating in the objective space. A quantitative comparison of stereo correspondence algorithms performance is used as an application domain.

Keywords: Multi-objective problems, Pareto front, decision making, computer vision, stereo correspondence, quantitative evaluation.

1 Introduction

A Multi-objective Optimisation Problem (MOP) involves several conflicting and incommensurable objectives [1]. A MOP can be addressed using a Multi-objective Evolutionary Algorithm (MOEA), such as the non-dominated sorting genetic algorithm [2], the strength Pareto approach [3], and the Pareto archived evolutionary strategy [4], among others. In a MOP, a single solution simultaneously optimising all objectives may not exist [5]. Thus, a MOEA solves a MOP by computing an approximation to the Pareto front, which is a set of mathematically equally good compromise solutions. As part of a decision making process, a solution from the Pareto front has to be selected in order to solve the problem being optimised. This selection is a responsibility of a decision maker (DM). However, in most of cases, a Pareto front may overload the judging capabilities of a DM, due to factors such as its large cardinality [1], the multidimensional complexity of the problem being solved [6], plus inherent limitations of a DM for effectively handling large amounts of data and more than several factors at once [7], among others. Although a visualisation of the Pareto front may assist a DM, a visualisation becomes complex with several solutions and

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three objectives or more, as well as visualised information for making a decision may become difficult to use [8]. Difficulties in a decision making process may be alleviated by introducing preferences [9]. Preferences can be seen as knowledge and/or expectations about a problem solution. They can be used as a mechanism to decide if a specific solution is preferable than other solutions [10]. Nevertheless, in some cases a DM may lack of information for selecting a solution and/or has not preferences among all objectives. In the absence of preferences, it is generally assumed that the most preferable solution correspond to a region in the maximum convex bulge of a Pareto curve/surface, termed as the *knee* region [8]. However, identifying the *knee* region of a Pareto front requires solving a non-linear optimisation problem, as well as some a priori knowledge on a Pareto front. In addition, determining the *knee* region(s) may become prohibitively complex as the dimensionality of a problem increases [11].

In this paper, without loss of generalisation, a quantitative comparison of stereo correspondence algorithms (SCAs) performance [12, 13] is used as an application domain. An SCA takes as input a stereo image pair, estimates projections from points in 3D space into image plains, and produces as output a disparity map. A quantitative comparison of SCAs should be conducted following an evaluation methodology, which is composed by a set of evaluation elements and methods interacting in an ordered sequence of steps, in order to produce evaluation results. A comparison of SCAs is addressed as a MOP in the $\mathbf{A}^{*}$ Groups evaluation methodology presented in [13]. This methodology computes a Pareto front using vectors of error measure scores, from which a solution, or solutions, should be selected as part of evaluation results interpretation. In this case a methodology user is acting in the role of decision maker. Thus, if preferences are introduced at this stage of an evaluation process, it may bias results interpretation.

In this context, a decision making scenario is addressed with the following characteristics:

- The decision making process is posterior to the search process.
- The DM lacks of preferences about the problem for selecting a solution.
- It is not possible to assign an importance to involved objective functions.
- Although at least one *knee* region should exist, the selection of a solution is not based on the proximity to that region.
- The problem on which a solution should be selected involves many objectives.

The selection of a solution from the Pareto front is seen as a MOP, based on two utility functions computed over the objective space. The paper is structured as follows. Related works are presented in Section 2. The proposed method is introduced in Section 3. Experimental validation is included and discussed in Section 4. Final remarks are stated in Section 5.

2 Related Works

Preferences can be specified by a DM in three ways: *a priori*, *interactive* and *a posteriori*. In the *a priori* way, preferences are specified before the beginning of search process by the aggregation of objective function into lexicographic order or into a linear/nonlinear combination, among others [9]. A deep knowledge of the problem and a clear understanding of the search space are required. In the *interactive* way, preferences are specified during the search, based on a progressively and interactively acquired knowledge of the problem [1]. An intensive effort of a DM is required, since, he/she is asked to give preference information at each algorithm's iteration, commonly consisting in specifying aspiration levels for each objective function, classifying objective functions according to their relevance, or introducing references points, among others. However, a DM may have large optimistic or pessimistic aspiration levels. In addition, when there are two or more DMs may arise disagrees about preferences. Preferences specified in *a priori* or *interactive* way have an impact on search results. In the *a posteriori* way, the search is executed first, and after that, a decision method is applied into the Pareto front [14]. In this case, a DM has too many choices to select from, and a fair comparison among them is not an easy task to achieve due to the inherent dimensional complexity. There are two main approaches, to perform *a posteriori* multi-criteria decision making: utility functions and outranking methods [5]. Utility functions assign a numerical value to each solution. Outranking methods are based on pairwise comparisons of all solutions, in order to establish if there exists preference, indifference or incomparability. However, commonly used methods under these two approaches rely on weights that should be specified by a DM [14]. Consequently, these methods cannot be used in the problem context specified in this paper. Methods such as the average rank, the maximum rank, and the favour rank do not require weights [10]. The average and the maximum rank can be seen as utility functions. The average rank uses multiple ranks considering each objective independently and a final rank is calculated as the average of previously assigned ranks, whilst the maximum rank takes the best rank as the global rank for each solution. In the favour rank, a solution *x* is preferred over a solution *y*, only if *x* outperforms *y* on more objectives than those on which *y* outperforms *x*. However, the maximum rank method tends to favour solutions with high performance in some of the objectives, but with a poor overall performance. In addition, the average rank and the favour rank may produce even ranks, or indifferences, respectively, very often. Moreover, none of them considers the magnitude on which a solution outperforms another according to the involved objective functions.

3 A Method for Reducing the Cardinality of the Pareto Front

Without loss of generalisation, a MOP consists in finding the vector of decision variables $x = (x_1, x_2, \dots, x_n)^T$ that optimises the following equation.

$$Min_x\, f(x) = \left(f_1(x), f_2(x), \dots, f_K(x)\right)^T, \tag{1}$$

subject to:

$$g_i(x) \leq 0 \qquad i = 1, \dots, P\ , \tag{2}$$

$$h_j(x) = 0 \qquad j = 1, \dots, Q\ , \tag{3}$$

where $f_k\colon \mathbb{R}^n \to \mathbb{R}\ (k = 1, \dots, K)$ are the objective functions, and g_i and $h_j\colon \mathbb{R}^n \to \mathbb{R}\ (\ i = 1, \dots, P;\ \ j = 1, \dots, Q)$ are the constraints of the problem.

In addition, some key definitions are presented for the sake of completeness.

Definition 1 *(Pareto dominance relation).* Given two solutions $x, y \in \mathbb{R}^n$, x is said to dominate y, denoted as $x \prec y$, if and only if: $f_a(x) \leq f_a(y)\ \forall\, a \in \{1,..,K\}$ and $\exists\, b \in \{1,..,K\}$ where $f_b(x) < f_b(y)$.
Definition 2 *(Non-dominated solution).* A solution $x \in \mathbb{R}^n$ is said to be non-dominated if and only if there does not exist another solution $y \in \mathbb{R}^n$, such that $y \prec x$.
Definition 3 *(Pareto optimal solution).* A solution $x \in \mathrm{F} \subseteq \mathbb{R}^n$, where F is the decision space, is said to be Pareto optimal if it is non-dominated with respect to F.
Definition 4 *(Pareto optimal set).* Let P^* be the Pareto optimal set defined as $P^* = \{x \in \mathrm{F},\ x \text{ is Pareto optimal}\}$.
Definition 5 *(Pareto front).* Let PF^* be the Pareto front, defined as $PF^* = \{\, f(x) \in \mathbb{R}^K,\ x \in P^*\ \}$.

In the proposed method, the selection of a solution from the Pareto front is addressed as a MOP, based on two utility functions and the Pareto dominance relation. The utility functions are adapted from [15] in order to avoid the use of weights. They are computed over the vectors composing the Pareto front from which a solution should be selected. Thus, the proposed method consists in finding the vector $s = (f_1(x), f_2(x), \dots, f_K(x))^T$ that optimises the following equation:

$$Min_s\, u(s) = \left(u_1(s), u_2(s)\right)^T, \tag{4}$$

subject to:

$$s \in PF^*\ , \tag{5}$$

where $u_l\colon \mathbb{R}^K \to \mathbb{R}\ (l = 1, 2)$ are the objective functions.

Let u_1 be the sumof ranks assigned to $f_k\colon \mathbb{R}^n \to \mathbb{R}\ (k = 1, \dots, K)$ in the Pareto front:

$$u_1\,(s) = \sum_{k=1}^{K} Rank(f_k(x)). \tag{6}$$

Let u_2 be the sum of ratios of $f_k\colon \mathbb{R}^n \to \mathbb{R}\ (k = 1, \dots, K)$ in the Pareto front:

$$u_2\,(s) = \sum_{k=1}^{K} \frac{(f_k(x) - Min(f_k(x)))}{(Max(f_k(x) - Min(f_k(x)))}, \tag{7}$$

where $Min(f_k(x))$ and $Max(f_k(x))$, are the minimum and the maximum score of the k^{th} objective, respectively. The lowest sum of ranks is associated with the solution that, comparatively with other solutions in the Pareto front, minimises most of involved objectives, whilst the lowest sum of ratios is associated with the solution with the best objective function values. The selection of a final solution may be based on the above criteria, which are problem context independent. Thus, the set of possible solutions to select from is turned into a set of a small cardinality, or even into a single solution, depending on data, by the proposed method. The set that corresponds to a reduction of the original PF^* set is denoted as RPF^*. In addition, the reduction of cardinality allows the use of a parallel coordinates plotting diagram [6] as a visualisation tool for assisting a solution selection. Moreover, the values computed by the u_2 function can be used for plotting the diagram.

4 Experimental Validation

In the application domain context, the $\mathbf{A}^*$ Groups evaluation methodology [13] conceives the comparison of SCAs as a MOP. In the evaluation model of the $\mathbf{A}^*$ Groups methodology, the decision space is a discrete and finite set composed by the SCAs under comparison, whilst the objective space is a set composed by a set of vectors of error scores, calculated according to selected evaluation elements and methods. In this problem, a user of the methodology requires, not only the set of SCAs composing the Pareto front –denoted in the methodology as the A_1^* set–, but also an impartial interpretation of results and assistance for selecting a single solution (i.e. in an intra-technique comparison), or solutions (i.e. in an inter-technique comparison). Thus, the discussed evaluation scenario is mathematically equivalent to the scenario which arises using an *a posteriori* MOEA, for addressing a MOP. Three evaluation scenarios are considered for validating the proposed method. All of them use a test-bed of four images (the Tsukuba, the Venus, the Teddy and the Cones stereo image pairs) [12] and the SCAs repository available in [16], as well as a combination of different evaluation criteria and evaluation measures. In regard to evaluation criteria, the depth discontinuity areas –*disc*–, the non-occluded areas –*nonocc*–, and the entire image –*all*– are used as evaluation criteria. Three error measures are used as evaluation measures. The percentage of Bad Matched Pixels (BMP) measures the quantity of disparity estimation errors exceeding a threshold δ (equals to 1 pixel) [12]. The Sigma-Z-Error (SZE) measures the impact of estimation errors on a 3D reconstruction based on the distance between the real depth and the estimated depth, based on ground-truth disparity and estimated disparity, respectively [17]. The Mean Relative Error (MRE) is based on the ratio between the absolute difference of estimated disparity against ground-truth disparity, and ground-truth disparity [18].

4.1 Evaluation Scenario Suited for Semi-dense Disparity Maps

The first evaluation scenario is devised for selecting the best performance algorithm from the SCAs repository using the *disc* and the *nonocc* criteria. In addition, the BMP and the SZE, which are conceptually different measures, are used, for a total of 16 objectives. The cardinality of the Pareto front (A_1^*) and the reduced Pareto front (RPF^*) sets, for the first evaluation scenario, are shown in the first column of Table 1. It can be observed that the proposed method reduces in 96.0% the cardinality of the Pareto front. This reduction considerably alleviates the judging overload of a DM. The SCAs composing the RPF^* set and the utility function values are also shown in Table 1. It can be observed that, in this case, a decision should be taken between the *SubPixelDoubleBP* and the *GC+SegmBorder* algorithms [16]. However, the two solutions show similar sum of ratios values. This similarity may influence a DM in order to make a decision based on the sum of ranks. The parallel coordinates plotting diagram associated to the obtained RPF^* set is shown in Fig 1. It allows to a DM an analysis of the achieved trade-off in objective functions by solutions to finally select from.

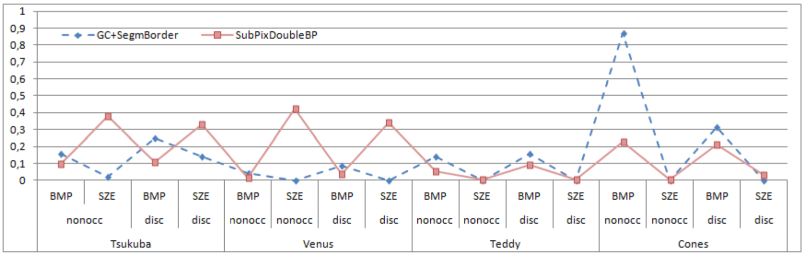

Fig. 1. Parallel coordinates plot of elements composing the $\boldsymbol{RPF}^*$ set in the first evaluation scenario

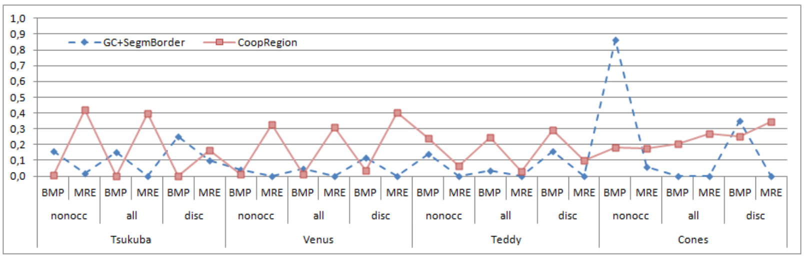

Fig. 2. Parallel coordinates plot of elements composing the $\boldsymbol{RPF}^*$ set in the second evaluation scenario

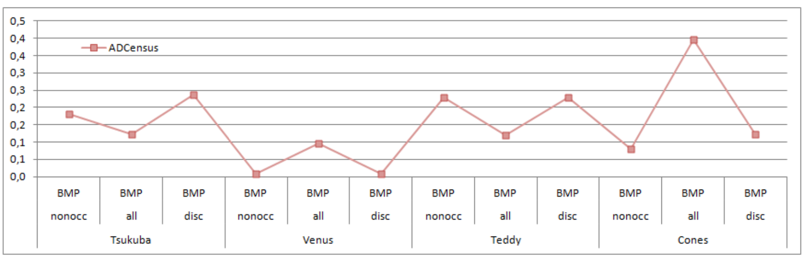

Fig. 3. Parallel coordinates plot of elements composing the $\boldsymbol{RPF}^*$ set in the third evaluation scenario

Table 1. Experimental validation data according to used evaluation scenarios

Evaluation Criteria	*disc & nonocc*		*all & disc & nonocc*		*all & disc & nonocc*	
Error Measures	BMP & SZE		BMP & MRE		BMP	
$\lvert A_1^* \rvert$	51		46		20	
$\lvert RPF^* \rvert$	2		2		1	
Reduction %	96.0		95.6		95.0	
Utility Functions / SCAs	u_1	u_2	u_1	u_2	u_1	u_2
SubPixDoubleBP	**184.0**	2.363				
GC+SegmBorder	214.5	**2.170**	251.0	**2.487**		
CoopRegion			**239.5**	4.494		
ADCensus					**59.0**	**1.826**

4.2 Evaluation Scenario Suited for Dense Disparity Maps

The second evaluation scenario is based on the *all*, the *disc*, and the *nonocc* evaluation criteria. The BMP and the MRE measures are used, for a total of 24 objectives. These measures may be conflicting since the MRE considers the inverse relation between depth and disparity. The cardinality of the A_1^* and the RPF^* sets, as well as the SCAs composing the RPF^* set and utility function values are shown in Table 1. It can be observed that the proposed method reduces in 95.6% the cardinality of the Pareto front. In this case, a decision should be taken between the *CoopRegion* and the *GC+SegmBorder* algorithms [16]. The parallel coordinates plotting diagram associated to the obtained RPF^* set is shown in Fig 2.

4.3 Evaluation Scenario of the Middlebury's Evaluation Methodology

The third evaluation scenario is devised in the same way that the one used in [16]. It considers the *all*, the *disc* and the *nonocc* evaluation criteria, and the BMP measure, for a total of 12 objectives. The cardinality of the A_1^*, and the RPF^* sets, as well as the single element composing the RPF^* set and utility function values are shown in the third column of Table 1. It can be observed that the proposed method reduces in a 95.0% the cardinality of the Pareto front. In this case, the proposed method reports a single solution, the *ADCensus* algorithm [16]. The parallel coordinates plotting diagram associated to the obtained RPF^* set is shown in Fig 3.

5 Final Remarks

In this paper, a challenging decision making scenario is addressed, in which decisions are taken *a posteriori*, and the DM lacks of preferences and information about the importance of many involved objectives. As innovative aspect, it addresses the selection of a solution from a Pareto front as a MOP, based on two utility functions and the Pareto dominance. The considered utility functions do not require weight specifications by a DM. A quantitative comparison of SCAs, using an evaluation model which produces as output a Pareto front, was used as an application domain. The experimental validation shows that the proposed method significantly reduces the cardinality of the Pareto front, even to a single solution, depending on data. Moreover, this cardinality reduction makes possible the use of conventional aids for making decision such as visualisation tools. The proposed method may alleviate the judging overload of a DM.

References

1. Ben Said, L., Bechikn, S., Ghédira, K.: The r-Dominance: A New Dominance Relation for Interactive Evolution Multi-criteria Decision Making. IEEE Trans. on Evolutionary Computation 14(5), 801–818 (2010)
2. Deb, K., Pratap, P., Agarwal, S., Meyarivan, T.: A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II. IEEE Trans. Evolutionary Computation 6(2), 182–197 (2002)

3. Zitzler, E., Laumanns, L., Thiele, L.: SPEA 2: Improving the Strength Pareto Evolutionary Algorithm for Multiobjective Optimisation. In: Giannakoglou, K.C. (ed.) Evolutionary Methods for Design, Optimisation and Control with Application to Industrial Problems, pp. 95–100 (2002)
4. Knowles, J., Corne, D.: Approximating the Nondominated FrontUsing the Pareto Archived Evolutionary Strategy. In: Proc. IEEE Congress on Evolutionary Computation, vol. 8(2), pp. 173–195 (2000)
5. Cvetkovic, D., Coello, C.: Human Preferences and their Applications in Evolutionary Multi-objective Optimisation. In: Yaochun, J. (ed.) Knowledge Incorporation in Evolutionary Computation, pp. 479–502. Springer, Heidelberg (2005)
6. Brockhoff, D., Zitzler, E.: Are All Objectives Necessary? On Dimensionality Reduction in Evolutionary Multiobjective Optimization. In: Runarsson, T.P., Beyer, H.-G., Burke, E.K., Merelo-Guervós, J.J., Whitley, L.D., Yao, X. (eds.) PPSN 2006. LNCS, vol. 4193, pp. 533–542. Springer, Heidelberg (2006)
7. Cvetkovic, D., Parmee, I.: Preferences and their Application in Evolutionary Multiobjective Optimisation. IEEE Trans. Evolutionary Computation 6(1), 42–57 (2002)
8. Das, I.: On Characterizing the “knee” of the Pareto Curve Based on Normal-Boundary Intersection. Structural and Multidisciplinary Optimization 18(2), 107–115 (1999)
9. Rachmawati, L., Srinivasan, D.: Preference Incorporation in Multi-objective Evolutionary Algorithms: A Survey. In: Proc. IEEE Congress on Evolutionary Computation, pp. 962–968 (2006)
10. López, A., Coello, C.: Study of preference relations in many-objective optimization. In: Proc. Genetic and Evolutionary Computation Conference, pp. 611–618 (2009)
11. Rachmawati, L., Srinivasan, D.: Multiobjective Evolutionary Algorithm withControllable Focus on the Knees of the Pareto Front. IEEE Trans. Evolutionary Computation 13(4), 810–824 (2009)
12. Scharstein, D., Szeliski, R.: A Taxonomy and Evaluation of Dense Two-Frame Stereo Correspondence Algorithms. International Journal of Computer Vision 47, 7–42 (2002)
13. Cabezas, I., Trujillo, M., Florian, M.: An Evaluation Methodology for Stereo Correspondence Algorithms. In: Proc. Intl. Joint Conference on Computer Vision and Computer Graphics Theory and Applications, pp. 154–163 (2012)
14. Parreiras, R., Maciel, J., Vasconcelos, J.: The a posteriori decision in multiobjective optimization problems with smarts, promethee II, and a fuzzy algorithm. IEEE Trans. Magnetics 42(4), 1139–1142 (2006)
15. Bentley, P., Wakefield, J.: Finding Acceptable solutions in the Pareto-Optimal Range using Multiobjective Genetic Algorithms. In: Chawdhry, P., et al. (eds.) Soft Computing in Engineering Design and Manufacturing, pp. 231–240 (1997)
16. Scharstein, D., Szeliski, R.: Middlebury Stereo Evaluation - Version 2, `http://vision.middlebury.edu/stereo/eval/`
17. Cabezas, I., Padilla, V., Trujillo, M.: A Measure for Accuracy Disparity Maps Evaluation. In: San Martin, C., Kim, S.-W. (eds.) CIARP 2011. LNCS, vol. 7042, pp. 223–231. Springer, Heidelberg (2011)
18. Van der Mark, W., Gavrila, D.: Real Time Dense Stereo for Intelligent Vehicles. IEEE Trans. Intelligent Transportation Systems 7(1), 38–50 (2006)