Blind Separation of Convolutive Mixtures of Non-stationary and Temporally Uncorrelated Sources Based on Joint Diagonalization

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Abstract. In this paper, we propose a new method for blindly separating convolutive mixtures of non-stationary and temporally uncorrelated sources. It estimates each source and its delayed versions up to a scale factor by Jointly Diagonalizing a set of covariance matrices in the frequency domain, contrary to most existing second-order methods which require a Block Joint Diagonalization algorithm followed by a blind deconvolution to achieve the same result. Consequently, our method is much faster than these classical methods especially for higer-order mixing filters and may lead to better performance as confirmed by our simulation results.

1 Introduction

In this paper, we propose a new method for blindly separating convolutive mixtures of non-stationary and temporally uncorrelated signals. Consider M mixtures $x_i(n)$ of N discrete-time sources $s_j(n)$ and suppose the mixing filters are FIR (Finite Impulse Response). Denoting by $A_{ij}(z) = \sum_{k=0}^{K} a_{ij}(k)z^{-k}$ the transfer function of each mixing filter where K is the order of the longest filter, we can write

$$x_i(n) = \sum_{j=1}^{N} \sum_{k=0}^{K} a_{ij}(k)s_j(n-k), \quad i = 1, ..., M. \tag{1}$$

This convolutive mixture may be rewritten as an instantaneous mixture [1–4] in the following manner. Considering delayed versions of the mixtures, i.e. $x_i(n-l)\,(l = 0, 1, ..., L-1)$, Eq. (1) reads

$$x_i(n-l) = \sum_{j=1}^{N} \sum_{k=0}^{K} a_{ij}(k)s_j(n-(k+l)), \;\; (i,l) \in [1,M] \times [0, L-1]. \tag{2}$$

These ML **generalized observations** $x_{il}(n) = x_i(n-l), (i,l) \in [1,M]\times[0,L-1]$ can be then considered as instantaneous mixtures of $N(K+L)$ **generalized sources** $s_{jr}(n) = s_j(n-r) = s_j(n-(k+l)), (j,r) \in [1,N] \times [0, K+L-1]$.

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This mixture is (over-)determined if $ML \geqslant N(K+L)$. It is clear that this condition may be satisfied only if $M > N$ i.e. if the original convolutive mixture is strictly over-determined. In this case, by choosing the integer number L so that $L \geqslant \frac{NK}{M-N}$, the reformulated instantaneous mixture (2) is (over-)determined. To represent the reformulated mixture in vector form, we define

$$\tilde{\mathbf{x}}(n) = \left[x_{10}(n), x_{11}(n), ..., x_{1(L-1)}(n), ..., x_{M0}(n), x_{M1}(n), ..., x_{M(L-1)}(n)\right]^T,$$
$$\tilde{\mathbf{s}}(n) = [s_{10}(n), s_{11}(n), ..., s_{1(K+L-1)}(n), ..., s_{N0}(n), s_{N1}(n), ..., s_{N(K+L-1)}(n)]^T,$$

which yield using (2) :

$$\tilde{\mathbf{x}}(n) = \tilde{\mathbf{A}}\tilde{\mathbf{s}}(n), \tag{3}$$

where $\tilde{\mathbf{A}} = \begin{pmatrix} \mathbf{A}_{11} & \dots & \mathbf{A}_{1N} \\ \vdots & \ddots & \vdots \\ \mathbf{A}_{M1} & \dots & \mathbf{A}_{MN} \end{pmatrix}$ and $\mathbf{A}_{ij} = \begin{pmatrix} a_{ij}(0) & \dots & a_{ij}(K) & 0 & \dots & 0 \\ & \ddots & & & \ddots & \\ & & & \ddots & & \ddots \\ 0 & \dots & 0 & a_{ij}(0) & \dots & a_{ij}(K) \end{pmatrix}$,

each block $\mathbf{A}_{ij}$ being a matrix of dimension $L \times (K+L)$.

Then, Eq. (3) models an (over-)determined instantaneous mixture with $M' = ML$ observations $x_{il}(n)$ and $N' = N(K+L)$ sources $s_{jr}(n)$. The $M' \times N'$ mixing matrix $\tilde{\mathbf{A}}$ is supposed to admit a *pseudo-inverse* $\tilde{\mathbf{A}}^+$, called the separating matrix, that we want to estimate for retrieving the **generalized source vector** $\tilde{\mathbf{s}}(n)$.

Several second-order methods, initially developed for separating Linear Instantaneous Mixtures (LIM), have been reformulated in this manner and used to separate convolutive mixtures. For example, *SOBI* [5], *BGML* [6], and *TF-BSS* [7] are three well-known methods proposed for separating LIM of mutually uncorrelated sources. Since the covariance matrix of the source vector, $\mathcal{R}_{\mathbf{s}}(n,\tau)$, is diagonal $\forall\, n,\tau$ for mutually uncorrelated sources, these methods jointly diagonalize a set of such matrices to achieve source separation. The approaches proposed in [1], [2] and [3], called respectively *SOBI-C*, *BGML-C* and *TFBSS-C* in the following, result from the generalization of these three methods to convolutive mixtures using the above reformulation. However, after reformulation the diagonality property of the covariance matrix of the generalized source vector, $\mathcal{R}_{\tilde{\mathbf{s}}}(n,\tau)$, is no longer met $\forall\, n,\tau$, but $\mathcal{R}_{\tilde{\mathbf{s}}}(n,\tau)$ is **block-diagonal**, whatever the nature of the original sources $s_j(n)$. As a result, the convolutive methods *SOBI-C*, *BGML-C* and *TFBSS-C* are based on **Joint Block-Diagonalization** (**JBD**) of a set of covariance matrices. The JBD algorithm provides several filtered versions of each initial source. Then, a blind deconvolution algorithm [3] may be used to estimate each of the generalized sources $s_{jr}(n)$, and in particular each of the initial sources $s_j(n)$, up to a scale factor.

In [4], we recently proposed a frequency-domain second-order approach for separating convolutive mixtures of non-stationary sources, also based on the reformulation of the mixture as an LIM and on the JBD. Contrary to the three methods mentioned above [1–3], our approach [4] requires neither global stationarity (supposed in [1]) nor piecewise stationarity (supposed in [2]) of the sources nor their sparseness (supposed in [3]). Our simulation results in [4] using speech sources (i.e. non-stationary and temporally correlated signals) confirmed the better performance of this approach [4] compared to the other methods [1–3]. Nevertheless, the main drawback of all above four methods [1–4] is the high computational cost, especially for high-order mixing filters. This cost is mainly due to the JBD algorithm. That's why we propose in this paper another algorithm which avoids JBD and blind deconvolution. We show that when the sources are non-stationary and temporally uncorrelated, it is possible to directly estimate each of the generalized sources $s_{jr}(n)$ up to a scale factor just by jointly diagonalizing a set of covariance matrices in the frequency domain.

2 Proposed Approach

In [8], we proposed a new method for separating LIM of non-stationary and temporally uncorrelated signals based on the joint diagonalization of covariance matrices in the frequency domain. The approach proposed in the current paper is an extension of that method to convolutive mixtures and uses the same joint diagonalization algorithm as in [8]. Like in the initial method [8], we suppose that the initial sources $s_j(n)$ are

(H1) : real and non-stationary,
(H2) : zero-mean and temporally uncorrelated, i.e.$\forall j, \forall n \neq m, E\left[s_j(n)s_j(m)\right] = 0$,
(H3) : mutually uncorrelated, i.e. $\forall j \neq k, \forall n, m, E\left[s_j(n)s_k(m)\right] = 0$.

Our spectral decorrelation method proposed in [8], which deals with frequency-domain sources $S_j(\omega)$ (the Fourier transforms of temporal sources $s_j(n)$), is based on the following principal properties[1]:

(P1) : *Uncorrelatedness* and *non-stationarity* in the time domain are transformed respectively into *wide-sense stationarity* and *autocorrelation* in the frequency domain. The **frequency-domain sources** $S_j(\omega)$ are then **wide-sense stationary** and **autocorrelated**.
(P2) : Since the temporal sources $s_j(n)$ are mutually uncorrelated, their Fourier transforms $S_j(\omega)$ are **mutually uncorrelated** too.

Thanks to the linearity of the Fourier transform, by mapping the initial time-domain LIM into the frequency domain, we obtain another LIM with the same mixing matrix, but with respect to the frequency-domain sources $S_j(\omega)$ which are wide-sense stationary and autocorrelated. Then, we can separate them using the classical BSS algorithms initially developed for separating mixtures of time-domain wide-sense stationary, time correlated signals like *SOBI* [5]. The main advantage of our approach [8] is that thanks to the wide-sense stationarity in the frequency domain, the expected values involved in the computation of covariance matrices can be rigorously estimated by frequency averages. In the following, we denote by *SOBI-F* the frequency-domain version of the *SOBI* algorithm.

[1] See [8], and in particular Theorem 4, for more details.

Note finally that the separating matrix may be estimated by jointly diagonalizing covariance matrices if and only if the following two conditions are satisfied [8]:

(C1) : the covariance matrix of the source vector[2] $\mathbf{s}(n)$, $\mathcal{R}_{\mathbf{s}}(n,\tau)$, is diagonal $\forall\, n,\tau$ (this condition is guaranteed by Hypothesis (H3)),
(C2) : the sources $s_j(n)$ have *different normalized variance profiles.*

In [8], we showed that for a given frequency shift ν_1, Condition (C2) is equivalent to the following identifiability condition:

$$\frac{E\left[S_i(\omega)S_i^*(\omega-\nu_1)\right]}{E\left[|S_i(\omega)|^2\right]} \neq \frac{E\left[S_j(\omega)S_j^*(\omega-\nu_1)\right]}{E\left[|S_j(\omega)|^2\right]}, \quad \forall\, i \neq j. \tag{4}$$

In the following, we present our extension of the above method to convolutive mixtures. As mentioned in Section 1, a convolutive mixture can be reformulated as an LIM mixture $\tilde{\mathbf{x}}(n) = \tilde{\mathbf{A}}\tilde{\mathbf{s}}(n)$. If we want to apply the above spectral decorrelation method (using a Joint Diagonalization algorithm) to this reformulated LIM for estimating the separating matrix $\tilde{\mathbf{A}}^+$, we must at first check the above two conditions (C1) and (C2). Nevertheless, we know that the matrix $\mathcal{R}_{\tilde{\mathbf{s}}}(n,\tau)$ is not diagonal $\forall\, n,\tau$, but only block-diagonal, whatever the nature of sources $s_j(n)$. However, using Hypothesis (H2) on the initial sources $s_j(n)$, we now show that this matrix is diagonal for $\tau = 0$. In fact, according to Hypothesis (H2), the generalized sources $s_{jr}(n)$ satisfy the following equation:

$$\forall\, j = k,\;\; \forall\, r \neq d,\;\; \forall\, n,\;\; E\left[s_{jr}(n)s_{kd}(n)\right] = E\left[s_j(n-r)s_j(n-d)\right] = 0, \tag{5}$$

and using Hypothesis (H3) we can write:

$$\forall\, j \neq k,\;\; \forall\, r,d,\;\; \forall\, n,\;\; E\left[s_{jr}(n)s_{kd}(n)\right] = E\left[s_j(n-r)s_k(n-d)\right] = 0. \tag{6}$$

Equations (5) and (6), together yield

$$\forall\, n,\;\; E\left[s_{jr}(n)s_{kd}(n)\right] = \begin{cases} 0 \;\; \forall\, j \neq k \;\; \underline{\text{or}} \;\; r \neq d \\ E\left[s_{jr}(n)^2\right] \;\; \text{for} \;\; j = k \;\; \underline{\text{and}} \;\; r = d \end{cases} \tag{7}$$

Thus, the generalized sources $s_{jr}(n)$ are *instantaneously* mutually uncorrelated, so that the matrix $\mathcal{R}_{\tilde{\mathbf{s}}}(n,\tau)$ is diagonal for $\tau = 0$. We now propose a trick to transform these generalized sources $s_{jr}(n)$ into new sources which are mutually uncorrelated for every time lag τ so as to satisfy Condition (C1) and to apply our spectral decorrelation method for LIM. This trick is based on the following theorem.

Theorem 1. *Let $u_p(n)\,(p = 1,...,\mathcal{N})$ be $\mathcal{N}$ real, zero-mean and instantaneously mutually uncorrelated random signals i.e.*

$$\forall\,(p,q) \in [1,\mathcal{N}]^2,\, p \neq q,\;\; \forall\, n,\;\; E\left[u_p(n)u_q(n)\right] = 0. \tag{8}$$

[2] In LIM, the considered source vector is defined as $\mathbf{s}(n) = [s_1(n), s_2(n), ..., s_N(n)]^T$.

Suppose $g(n)$ is a real, zero-mean, stationary, temporally uncorrelated random signal, independent from all signals $u_p(n)$. Then, the signals $u'_p(n)$ defined by $u'_p(n) = g(n)u_p(n)$ are real, zero-mean, ***temporally uncorrelated*** *and* ***mutually uncorrelated****. Moreover, each new signal $u'_p(n)$ has the same normalized variance profile as the original signal $u_p(n)$.*

Proof: See Appendix.

Multiplying each generalized observation $x_{il}(n)$ by a random signal $g(n)$ satisfying the conditions of the above theorem, we obtain new observations denoted by $x'_{il}(n) = g(n)x_{il}(n)$. These new observations are LIM of the new sources $s'_{jr}(n) = g(n)s_{jr}(n)$ with the same mixing matrix $\tilde{\mathbf{A}}$, because denoting $\tilde{\mathbf{x}}'(n) = g(n)\tilde{\mathbf{x}}(n)$ and $\tilde{\mathbf{s}}'(n) = g(n)\tilde{\mathbf{s}}(n)$ and using (3) we obtain

$$\tilde{\mathbf{x}}'(n) = g(n)\tilde{\mathbf{x}}(n) = g(n)(\tilde{\mathbf{A}}\tilde{\mathbf{s}}(n)) = \tilde{\mathbf{A}}(g(n)\tilde{\mathbf{s}}(n)) = \tilde{\mathbf{A}}\tilde{\mathbf{s}}'(n). \tag{9}$$

Moreover, thanks to the above theorem (applied to signals $u_p(n) = s_{jr}(n)$), the new sources $s'_{jr}(n) = g(n)s_{jr}(n)$ are:

1. real and non-stationary with the same normalized variance profiles as the sources $s_{jr}(n)$,
2. zero-mean and temporally uncorrelated,
3. mutually uncorrelated for each time lag, i.e. $\mathcal{R}_{\tilde{\mathbf{s}}'}(n,\tau)$ is **diagonal** $\forall\, n,\tau$.

Thus, the first condition (C1) for applying our spectral decorrelation method for LIM is now satisfied because $\mathcal{R}_{\tilde{\mathbf{s}}'}(n,\tau)$ is diagonal $\forall\, n,\tau$. Besides, if the sources $s_{jr}(n)$ have different normalized variance profiles, then the new sources $s'_{jr}(n)$ have too so that the second condition (C2) is also verified. In this case, the new frequency-domain sources $S'_{jr}(\omega)$, which are the Fourier transforms of $s'_{jr}(n)$, satisfy the following identifiability condition

$$\forall\, j \neq k \ \ \underline{\text{or}}\ \ r \neq d, \quad \frac{E\left[S'_{jr}(\omega){S'}^{*}_{jr}(\omega-\nu_q)\right]}{E\left[|S'_{jr}(\omega)|^2\right]} \neq \frac{E\left[S'_{kd}(\omega){S'}^{*}_{kd}(\omega-\nu_q)\right]}{E\left[|S'_{kd}(\omega)|^2\right]}, \tag{10}$$

so that our spectral decorrelation method for LIM can be used to compute an estimate of the separating matrix $\tilde{\mathbf{A}}^{+}$, denoted $\tilde{\mathbf{A}}^{+}_{est}$. To this end, we start by computing the Fourier transform of the new observation vector $\tilde{\mathbf{x}}'(n) = \tilde{\mathbf{A}}\tilde{\mathbf{s}}'(n)$ which yields:

$$\tilde{\mathbf{X}}'(\omega) = \tilde{\mathbf{A}}\tilde{\mathbf{S}}'(\omega), \tag{11}$$

where $\tilde{\mathbf{S}}'(\omega) = \left[S'_{10}(\omega), ..., S'_{1(K+L-1)}(\omega), ..., S'_{N0}(\omega), ..., S'_{N(K+L-1)}(\omega)\right]^T$ and $\tilde{\mathbf{X}}'(\omega) = \left[X'_{10}(\omega), ..., X'_{1(L-1)}(\omega), ..., X'_{M0}(\omega), ..., X'_{M(L-1)}(\omega)\right]^T$, with $X'_{il}(\omega)$ the Fourier transform of $x'_{il}(n)$. The modified generalized sources $s'_{jr}(n)$ being zero-mean, non-stationary, temporally uncorrelated and mutually uncorrelated, their Fourier transforms $S'_{jr}(\omega)$ are wide-sense stationary, autocorrelated and mutually uncorrelated, thanks to Properties (P1) and (P2). Therefore, we can apply the *SOBI-F* algorithm to compute $\tilde{\mathbf{A}}^{+}_{est}$ as follows:

1. we compute the $N' \times M'$ *whitening matrix* $\mathbf{W}$ which yields a new observation vector $\tilde{\mathbf{Z}}'(\omega) = \mathbf{W}\tilde{\mathbf{X}}'(\omega)$ so that $E[\tilde{\mathbf{Z}}'(\omega)\tilde{\mathbf{Z}}'^H(\omega)] = \mathbf{I}_{N'}$, by diagonalizing the matrix $\mathcal{R}_{\tilde{\mathbf{X}}'}(0) = E[\tilde{\mathbf{X}}'(\omega)\tilde{\mathbf{X}}'^H(\omega)]$,
2. we compute the *rotation matrix* $\mathbf{U}$ by **J**ointly **D**iagonalizing (**JD**) several covariance matrices $\mathcal{R}_{\tilde{\mathbf{Z}}'}(\nu_q) = E[\tilde{\mathbf{Z}}'(\omega)\tilde{\mathbf{Z}}'^H(\omega - \nu_q)]$ $(q = 1, 2, ...)$,
3. an estimate of the separating matrix $\tilde{\mathbf{A}}^+$ is given by:

$$\tilde{\mathbf{A}}^+_{est} = \Re\{\mathbf{U}^H\mathbf{W}\} \simeq \mathbf{P}\mathbf{D}\tilde{\mathbf{A}}^+, \tag{12}$$

where $\mathbf{P}$ is a permutation matrix and $\mathbf{D}$ is a real diagonal matrix [5, 8].

Once $\tilde{\mathbf{A}}^+_{est}$ has been computed by this method, we can directly find an estimate of the generalized source vector $\tilde{\mathbf{s}}(n)$, denoted by $\tilde{\mathbf{s}}_{est}(n)$, using (3) as follows:

$$\tilde{\mathbf{s}}_{est}(n) = \tilde{\mathbf{A}}^+_{est}\tilde{\mathbf{x}}(n) \simeq (\mathbf{P}\mathbf{D}\tilde{\mathbf{A}}^+)(\tilde{\mathbf{A}}\tilde{\mathbf{s}}(n)) \simeq \mathbf{P}\mathbf{D}\tilde{\mathbf{s}}(n). \tag{13}$$

Thus, each generalized source $s_{jr}(n)$, and in particular each initial source $s_j(n)$ $(= s_{j0}(n))$, can be estimated up to a scale factor (and a permutation). In the following, we call our method[3] *SOBI-F-C*$_{JD}$.

3 Simulation Results

In this section, we present our simulation results using $M = 3$ artificial FIR convolutive mixtures of $N = 2$ artificial sources containing $N_s = 65536$ samples. The sources are generated using $s_j(n) = r_j(n)\mu_j(n)$, where $r_j(n)$ are mutually uncorrelated, zero-mean i.i.d. (independent and identically distributed) Gaussian signals, $\mu_1(n) = \cos(\omega_0 n)$ and $\mu_2(n) = \sin(\omega_0 n)$ with $\omega_0 = \pi/7$. This choice allows us to generate two non-stationary and temporally uncorrelated initial sources $s_1(n)$ and $s_2(n)$ with different normalized variance profiles. The mixtures are generated using FIR filters of order $K \in \{1, 3, 5\}$. The coefficients $a_{ij}(k)$ of each transfer function $A_{ij}(z) = \sum_{k=0}^{K} a_{ij}(k)z^{-k}$ are generated randomly. For each value of K we choose in the model (2) the integer L equal to $2K$. This choice provides $M' = 6K$ generalized observations $x_{il}(n)$ and $N' = 6K (\in \{6, 18, 30\})$ generalized sources $s_{jr}(n)$ so that the matrix $\tilde{\mathbf{A}}$ is square[4].

To apply our *SOBI-F-C*$_{JD}$ method, we first multiply all generalized observations $x_{il}(n)$ by an i.i.d., real, zero-mean and uniformly distributed signal $g(n)$, independent from the generalized sources. After whitening data as explained in the previous section, we jointly diagonalize 4 covariance matrices corresponding to 4 different frequency shifts, yielding an estimate of each of the generalized sources $s_{jr}(n)$ up to a scale factor.

We compare our results with those obtained using the time-domain method *BGML-C* [2] which exploits the non-stationarity of signals without requiring

[3] 'C' for Convolutive and 'JD' for Joint Diagonalization.

[4] Having originally 3 FIR mixtures of 2 sources, i.e. $M = 3$ et $N = 2$, we obtain $M' = ML = 6K$ and $N' = N(K + L) = 6K$ after reformulation as in (2) .

them to be temporally autocorrelated. To apply *BGML-C*, we consider 128 covariance matrices computed over 128 adjacent frames of 512 samples. To block-diagonalize these matrices, we use the orthogonal algorithm proposed by Févotte et al. in [3]. After the JBD stage, we obtain $K+L$ filtered versions of each initial source $s_j(n)$. Then, we use a blind deconvolution method proposed in [3] which allows us to estimate each of the generalized sources $s_{jr}(n)$ up to a scale factor. Performance is measured using the Signal to Interference Ratio (SIR) defined as $SIR = \frac{1}{2}(SIR_1 + SIR_2)$ where:

$$SIR_j = \max_r \left\{ 10 \log_{10} \left[\frac{E\{s_{jr}(n)^2\}}{E\{(\hat{s}_{jr}(n) - s_{jr}(n))^2\}} \right] \right\}, (j,r) \in [1,2] \times [0, K+L-1],$$

after normalizing the estimated generalized sources $\hat{s}_{jr}(n)$ so that they have the same variances and signs as the original generalized sources $s_{jr}(n)$. The SIR as well as the computation time[5] are given in Table 1 for our method and the *BGML-C* method. We also repeat our simulations by varying the number of samples N_s. The results for $N_s \in \{2^{17}, 2^{18}, 2^{19}\}$ and $K = 1$ are shown in Table 2.

Table 1. SIR (in dB) and computation time T_j (in minutes) versus filter order K for $N_s = 2^{16} = 65536$

	$K=1\,(N'=6)$			$K=3\,(N'=18)$			$K=5\,(N'=30)$		
Method	SIR	$T_j(mn)$	T_2/T_1	SIR	$T_j(mn)$	T_2/T_1	SIR	$T_j(mn)$	T_2/T_1
SOBI-F-C$_{JD}$	**40.43**	$T_1 = 0.04$		**32.84**	$T_1 = 0.20$		**26.28**	$T_1 = 0.50$	
BGML-C	**28.07**	$T_2 = 0.06$	1.50	**10.18**	$T_2 = 1.54$	7.70	**7.14**	$T_2 = 13.31$	26.62

Table 2. SIR (in dB) and computation time T_j (in minutes) versus number of samples N_s for $K = 1$

	$N_s = 131072$			$N_s = 262144$			$N_s = 524288$		
Method	SIR	$T_j(mn)$	T_2/T_1	SIR	$T_j(mn)$	T_2/T_1	SIR	$T_j(mn)$	T_2/T_1
SOBI-F-C$_{JD}$	**42.90**	$T_1 = 0.08$		**46.22**	$T_1 = 0.15$		**53.33**	$T_1 = 0.31$	
BGML-C	**32.94**	$T_2 = 0.09$	1.13	**33.09**	$T_2 = 0.18$	1.20	**37.86**	$T_2 = 0.36$	1.16

As can be seen:

- our method outperforms *BGML-C* in all of the tested configurations, especially for higher-order filters. For $K = 5$, it is about 26 times faster and leads to an SIR about 20 dB higher than *BGML-C*. This can be justified considering that *BGML-C* supposes the non-stationary signals to be piecewise stationary while this condition is not satisfied by our test signals, and

[5] The algorithms were implemented on a 2.10 GHz Dual-Core Pentium processor with 3GB memory.

it uses a JBD algorithm which is more time consuming than a JD algorithm,
- not surprisingly, for both methods the SIR increases with N_s and decreases with K, while the computation time increases with N_s and K.

4 Conclusion and Perspectives

In this paper, we proposed an extension of our spectral decorrelation method, initially developed for LIM, to convolutive mixtures. The proposed method, called *SOBI-F-*C_{JD}, may be used for separating convolutive mixtures of non-stationary and temporally uncorrelated sources. Just by using a joint diagonalization algorithm, it provides an estimate of each generalized source up to a scale factor, contrary to the existing approaches [1–4] which need a block-joint diagonalization algorithm followed by a blind deconvolution to achieve the same result. The first simulations confirmed the better performance of our method in terms of both separation quality and rapidity compared to the *BGML-C* method. Nevertheless, it would be interesting to confirm these results using more statistical tests. For example, increasing the number of covariance matrices used in JD algorithm would improve the performance.

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Appendix: Proof of Theorem 1

Denote $\mathbf{u}(n) = [u_1(n), u_2(n), ..., u_{\mathcal{N}}(n)]^T$ and $\mathbf{u}'(n) = g(n)\mathbf{u}(n)$.

1. Since $g(n)$ is independent from all the zero-mean signals $u_p(n)$, we can write

$$\forall\, p \in [1, \mathcal{N}],\;\; E\left[u'_p(n)\right] = E\left[g(n)u_p(n)\right] = E\left[g(n)\right]E\left[u_p(n)\right] = 0. \qquad (14)$$

   Hence, the new signals $u'_p(n)\,(p = 1, ..., \mathcal{N})$ are also zero-mean.
2. Whatever the times n_1 and n_2, we have

$$E\left[\mathbf{u}'(n_1)\mathbf{u}'(n_2)^T\right] = E\left[g(n_1)g(n_2)\mathbf{u}(n_1)\mathbf{u}(n_2)^T\right]. \qquad (15)$$

   The independence of $g(n)$ from all the signals $u_p(n)$ yields

$$E\left[\mathbf{u}'(n_1)\mathbf{u}'(n_2)^T\right] = E\left[g(n_1)g(n_2)\right]E\left[\mathbf{u}(n_1)\mathbf{u}(n_2)^T\right], \qquad (16)$$

   and since $g(n)$ is zero-mean, stationary and temporally uncorrelated

$$E\left[\mathbf{u}'(n_1)\mathbf{u}'(n_2)^T\right] = \sigma_g^2\delta(n_1 - n_2)E\left[\mathbf{u}(n_1)\mathbf{u}(n_1)^T\right] \qquad (17)$$

   where σ_g^2 is the variance of $g(n)$. The signals $u_p(n)$ being zero-mean and instantaneously mutually uncorrelated, the matrices $E\left[\mathbf{u}(n_1)\mathbf{u}(n_1)^T\right]$ and so $E\left[\mathbf{u}'(n_1)\mathbf{u}'(n_2)^T\right]$ are diagonal. As a result, the new zero-mean signals $u'_p(n)$ are mutually uncorrelated. Moreover, according to Eq. (17), the diagonal entries of the matrix $E\left[\mathbf{u}'(n_1)\mathbf{u}'(n_2)^T\right]$ can be written as

$$E\left[u'_p(n_1)u'_p(n_2)\right] = \sigma_g^2\delta(n_1-n_2)E\left[u_p(n_1)u_p(n_1)\right] = \sigma_g^2\delta(n_1-n_2)E\left[u_p^2(n_1)\right]. \qquad (18)$$

   Hence, the new signals $u'_p(n)$ are temporally uncorrelated. Furthermore, by choosing $n_1 = n_2 = n$, Eq. (18) becomes $E\left[u'^2_p(n)\right] = \sigma_g^2 E\left[u_p^2(n)\right]$ which means that the new signals $u'_p(n)$ have the same normalized variance profiles as the original signals $u_p(n)$.