Trivial problems

V.I. Arnold
Translated by Gerald Gould

Define a problem to be a decomposition of a functional space into subsets. For instance, in the problem of stability of an equilibrium for systems of ordinary differential equations one deals with the decomposition of the space of vector fields with zero at O into two classes: the class of fields for which the point O is stable, and the class of fields for which the point O is unstable.

The problem is called trivial if the decomposition of the k-jets corresponding to this problem is finite and semi-algebraic for each k. In this case the question to which class a given function belongs can be answered with the help of a finite number of arithmetical manipulations with Taylor coefficients, except for the "special case" of infinite codimension.

For instance, the problem of whether the point O is a point of strict local minimum of an infinitely differentiable function is trivial.

In this talk the following hypotheses are discussed:

1. The problem of the asymptotic (or Lyapunov) stability of equilibrium is trivial.
2. The problem of the topological classification of equilibria for the system of real differential equations is trivial. It should be noted that the similar problem considered in the complex domain is not trivial.