

# Utilizing Model to Optimize Crop Plant Density: A Case Study on Tomato

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**Abstract.** Based on the functional-structural plant model GreenLab, a common plant category tomato was selected as a test case in order to analyze the impact of plant density on the yield. A yield optimization model was set up based on minimum ripe fruit weight constraints, fruit weight serve as the objective function for the bound-constrained optimization problem. Particle Swarm Optimization (PSO) algorithm was applied for maximizing the crop production, which will provide a theoretical reference for agronomy measures based on plant spacing.

**Keywords:** Tomato, Plant density, GreenLab model, Yield optimization, PSO.

## 1 Introduction

Tomato's economic benefit is related to fruit yield and quality, and these two factors are considerably affected by light condition [1][2]. Manipulation of plant spacing is a possible means to change light interception for peasant. Traditionally, plant density is based on empirical knowledge from filed experiments, and this is quite time consuming and expensive. While plant growth models provide a very important tool for realizing optimal yields of crops with an efficient means [3]. Functional-structure model GreenLab can simulate plant growth based on organ size, so it provides possibility for plant density optimization [4]. Combining plant mathematical model with optimization method can provide guiding for the optimization of planting density and horticultural practice such as pruning and environmental control in special constrained environment.

## 2 The Description of Optimization Problem

### 2.1 Greenlab Model

In GreenLab model, time scale, called a growth cycle, is often the phyllochron between appearances of two successive leaves in main stem [5]. Plant total product biomass at growth cycle  $n$  is  $Q(n)$ .

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$$Q(n) = AB \left( 1 - \ell^{-\frac{S(n)}{A}} \right) \tag{1}$$

A is the projection surface of single plant (m<sup>2</sup>/m<sup>2</sup>), which is in response to plant density. B is plant growth potential multiplied by an empirical resistance, which can be computed by environment data. S(n) is total leaves surface at n<sup>th</sup> growth cycle.

D(n) is all organ demands at the n<sup>th</sup> growth cycle

$$D(n) = \sum_o P_o \sum_{t=1}^n f_o(t) N_o(n-t+1) \tag{2}$$

Where P<sub>o</sub>(unitless) is the sink of organ o (o=internode (i), blade (b), petiole (p), fruit (f)), which is defined as the ability of organs to compete for biomass. N<sub>o</sub>(n-t+1) is the number of organ o in t growth cycle. f<sub>o</sub>(t) is sink variation of organ o at t<sup>th</sup> growth cycle, which is described by an empirical Beta function. α<sub>o</sub> and β<sub>o</sub> are the coefficients of the Beta function, and α<sub>o</sub> is set constant.

A, P<sub>o</sub>(unitless) and β<sub>o</sub> (unitless) are hidden parameter. These parameters were identified by fitting the periodical destructively-measured data and environment data with the corresponding model output.

### 2.2 Fruit Weight Calculation

At the end of n<sup>th</sup> growth cycle, biomass for one chronological aged j (growth cycle) fruit can be calculated as below:

$$q_f(n, j) = \sum_{k=1}^j p_f(k) \frac{Q(n-j+k)}{D(n-j+k)} \tag{3}$$

k is fruit growth cycle, all organs including fruit compete for biomass in a common pool. The formula for calculating the total fruit weight of an aged n (growth cycle) plant as follows:

$$Y = \sum_{k=1}^{t_{x-f}} p_f(k) \frac{Q_{n-(t_{x-f}-k)}}{D_{n-(t_{x-f}-k)}} \tag{4}$$

t<sub>x-f</sub> is fruit extension time, which is an observation value.

### 2.3 Model Parameter Estimation of Greenlab for Tomato

For establishing the relationships between GreenLab model parameters and plant density, four gradient density tomatoes named by D1,D2,D3,D4 from low density to high density were planted in solar greenhouse at the Chinese Academy of Agricultural Science in Beijing (39.55°N, 116.25°E)[6]. The generalized least square method was used for fitting [6]. Four plant density model parameters are shown in table 1:

**Table 1.** Estimated parameter values of model

Parameters	Values			
	D1	D2	D3	D4
$P_i$	0.47	0.46	0.58	0.56
$P_p$	0.65	0.57	0.83	0.72
$P_f$	23.4	16	15	8
$\beta_i$	2.41	2.24	2.76	1.67
$\beta_p$	1.91	1.86	2	1.35
$\beta_b$	1.48	1.28	1.83	1.46
$\beta_f$	1.58	1.03	1.48	1.44
A	2228	2186	1783	1592

An equation is used to describe model parameter variation trend with plant density. The supreme fitting degree function is shown as below:

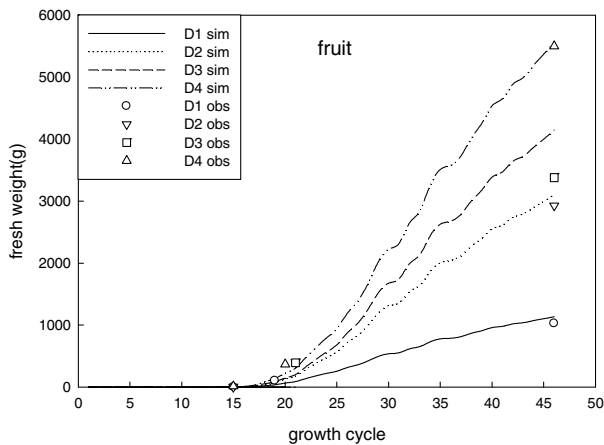
$$\Omega = a + b \exp(-cx) \quad (5)$$

x is plant density, a, b, c are constants which can be got by fitting,  $\Omega$  is GreenLab parameters as table1 listed.

### 3 Tomato Optimal Plant Density Solution

#### 3.1 Tomato Growth Simulation at Different Plant Density

Different density plants growth and development can be simulated using GreenLab model with parameters listed in table 1.



**Fig. 1.** Fitting results on fruit fresh weight evolution for a ground area plant of four densities symbols- measurement, lines- model

Model outputs show that increasing plant density results in a reduction of single plant fruit fresh weight (Figure is not shown) but in an increase of fruit fresh weight on a ground area (Fig.1), which is consistent with measured data.

### 3.2 Mathematical Solution for the Tomato Plant Density Optimization Problem

Considering market economic benefit, we define a minimum harvestable fruit weight as constraint. A single object non-linear optimization function is built as follow,

$$\begin{cases} \max_{d \in R} Y = S(d, \Omega) \\ G \geq g_{\min} \end{cases}$$

Optimal object is maximum yield Y. Plant density d is an optimizing object variable and it varies in real field. All the GreenLab parameters  $\Omega$  are control variables.  $g_{\min}$  is a threshold value of tomato fruit referred to practice(100g). G is single fruit weight. The PSO(Particle Swarm Optimization) algorithm was implied to optimize.

The fruit weight output with respect to different plant density is shown as Fig 2. Fruit yield variation with plant density can be divided into three phases. During the first phase (1 to 8.2 plants per square meter) fruit yield grow bigger with density increasing. During the second phase (8.2 to 10.5 plants per square meter) fruit yield grow less with density increasing. In the following, fruit yield drop to zero. Because fruits get smaller and smaller with later density increasing, even it can't up to harvestable standard. Optimal plant density is 8.2 plants per square meter, and the fruit weight is close to 5kg per square meter.

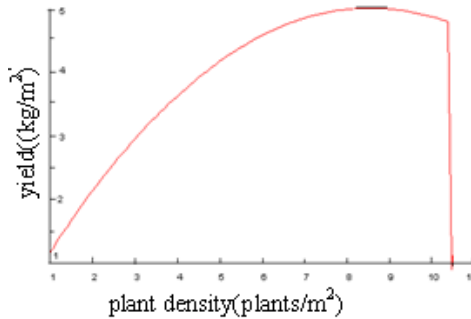


Fig. 2. Tomato yield evolution with planting density

## 4 Conclusion

Model output is well conformity with measurement. Results illustrate that model has the ability of reflecting plant density effect on yield in aspect of biomass production, distribution and organ number changing. We can infer model parameter, plant growth and development process in any environment from model calibration and parameter interpolation. Combining with optimization algorithm, we can get optimum plant

conditions (in this paper plant density as an example). It provides a reference for decision-makers using limited experiment, and it can be used in plant growth prediction, manufacture management and so on.

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