On Integral Sum Numbers of Cycles

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Abstract. In this paper we determine that integral sum number of graph C_n, namely for any integer $n \geq 5$, then $\xi(C_n) = 0$, therefore we prove that the graph $C_n (n \geq 5)$ is an integral sum graph.

Keywords: Integral sum number, Integral sum graph, Graph C_n ·

1 Introduction

The graph in this paper discussed are undirected, no multiple edges and simple graph, the unorganized state of definitions and terminology and the symbols in this paper referred to reference [1],[2].

F. Harary [3] introduce the concept of integral sum graphs. The integral sum graph $G^+(S)$ of a finite subset $S \subset Z$ is the graph (V, E), where $V = S$ and $uv \in E$ if and only if $u + v \in S$. A graph *G* is an integral sum graph if it is isomorphic to the integral sum graph number of $G^+(S)$ of some $S \subset Z$. The integral sum number of a given graph *G*, denoted by $\xi(G)$, is defined as the smallest nonnegative integer S such that $G \bigcup sk_1$ is an integral sum graph. For convincing, an integral sum graph is written as an integral sum graph in references [3, 4]. Obviously, graph *G* is an integral sum graph if $\xi(G) = 0$.

It is very difficult to determine $\xi(G)$ for a given graph *G* in general. All paths and matchings are verified to be integral sum graph in references [3], and we see from references [4] that $\xi(C_n) \leq 1$ for all $n \neq 4$. And further, an open conjecture was posed in references [4] as follows:

Conjecture [4]: Is it true that any old cycle is an integral sum graph?

Definition 1.1. If a graph is isomorphism graph $G^+(S)$, then we call graph *G* is Integral sum graph, denoted by $G \cong G^+(S)$.

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Definition 1.2. For graph *G*, if it exists nonnegative integer *S* such that $G \bigcup sk_1$ is an integral sum graph, then we call number *s* is integral sum number of *G*, denoted by $\xi(G) = s$.

2 Main Results and Certification

Theorem 2.1. For any integer $n \geq 3$, then

$$\xi(C_n) = \begin{cases} 3, & \text{when } n = 4; \\ 0, & \text{when } n \neq 4. \end{cases}$$

Proof. It is immediate from references [3, 4] that $\xi(C_3) = 0$ and $\xi(C_4) = 3$. And it is clear that $C_5 \cong G^+\{2, 1, -2, 3, -1\}$ and $C_7 \cong G^+\{1, 2, -5, 7, -3, 4, 3\}$.

Next we consider two cases: For all $C_{2j}\,(j \geq 3)$ and $C_{2j+1}\,(j \geq 4)$, we will show that the two classes of cycle are integral sum graph.

Let the vertices of $C_{2j}\,(j \geq 3)$ and $C_{2j+1}\,(j \geq 4)$ be marked as the methods in Fig.1 and Fig.2.

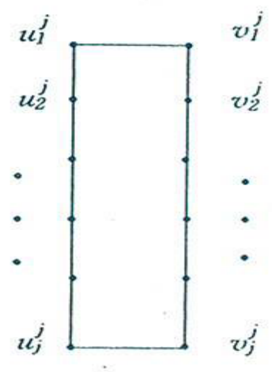

Fig. 1. Marking of C_{2j}

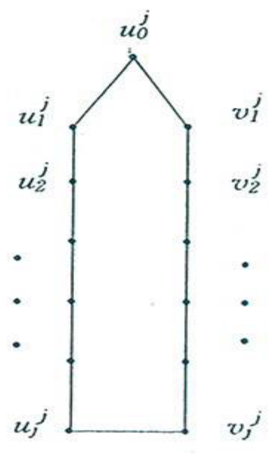

Fig. 2. Marking of C_{2j+1}

Case 1. When $n = 2j\,(j \geq 3)$

We first give the labels of C_6 and C_8 as follows:

Let $u_1^3 = 4,\ u_2^3 = -1,\ u_3^3 = 5;\ \ v_1^3 = 1,\ v_2^3 = 3,\ v_3^3 = -2$.

and

$$u_1^4 = 7,\ u_2^4 = -2,\ u_3^4 = 9,\ u_4^4 = -11;\ v_1^4 = 2,\ v_2^4 = 5,\ v_3^4 = -3,\ v_4^4 = 8.$$

Then $C_6 \cong G^+\{4,-1,5,-2,3,1\}$ and $C_8 \cong G^+\{7,-2,9,-11,8,-3,5,2\}$, therefore $\xi(C_6)=0$ and $\xi(C_8)=0$.

When $j \ge 5$, we give the labels of C_{2j} as follows:

Let $u_1^j = u_1^{j-1} + u_1^{j-2} \quad (j \ge 5)$; $u_2^j = u_2^{j-1} + u_2^{j-2} \quad (j \ge 5)$;

$v_1^j = v_1^{j-1} + v_1^{j-2} \quad (j \ge 5)$; $v_2^j = v_2^{j-1} + v_2^{j-2} \quad (j \ge 5)$.

and

$$u_k^j = u_{k-2}^j - u_{k-1}^j \quad (k = 3,4,\cdots,j);$$

$$v_k^j = v_{k-2}^j - v_{k-1}^j \quad (k = 3,4,\cdots,j).$$

By labeling of above, we know $\xi(C_{2j})=0\,(j \ge 5)$.

The labeling of above is illustrated in Fig.3.

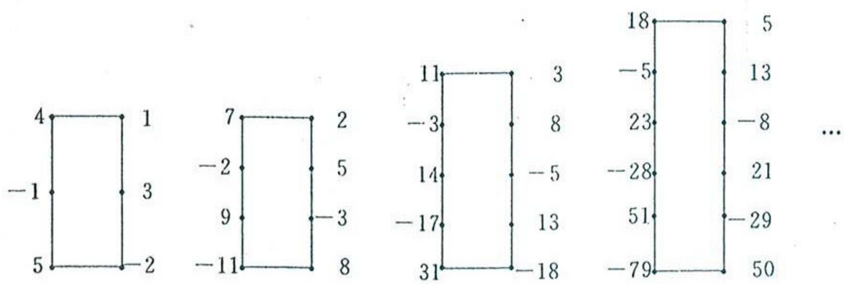

Fig. 3. Labeling of $C_{2j}\,(j \ge 3)$

Case 2. When $n = 2j+1\,(j \ge 4)$

We first give the labels of C_9 and C_{11} as follows:

Let $u_0^4 = 2,\ u_1^4 = 5,\ u_k^4 = u_{k-2}^4 - u_{k-1}^4\ (k = 2,3,4)$;

$v_1^4 = -5,\ v_2^4 = 7,\ v_k^4 = v_{k-2}^4 - v_{k-1}^4\ (k = 3,4)$.

and

$$u_0^5 = 3,\ u_1^5 = 8,\ u_k^5 = u_{k-2}^5 - u_{k-1}^5\ (k = 2,3,4,5);$$

$$v_1^5 = -8,\ v_2^5 = 11,\ v_k^5 = v_{k-2}^5 - v_{k-1}^5\ (k = 3,4,5).$$

Then $C_9 \cong G^+\{2,5,-3,8,-11,19,-12,7,-5\}$ and $C_{11} \cong G^+\{3,8,-5,13,-18,31,-49,30,-19,11,-8\}$, therefore $\xi(C_9)=0$ and $\xi(C_{11})=0$.

When $j \ge 6$, we give the labels of C_{2j+1} as follows:

Let $u_0^j = u_0^{j-1} + u_0^{j-2} \quad (j \ge 6)$; $u_1^j = u_1^{j-1} + u_1^{j-2} \quad (j \ge 6)$;

$v_1^j = v_1^{j-1} + v_1^{j-2} \quad (j \ge 6)$;

and

$$u_k^j = u_{k-2}^j - u_{k-1}^j \quad (k = 2, 3, \cdots, j);$$
$$v_k^j = v_{k-2}^j - v_{k-1}^j \quad (k = 2, 3, \cdots, j).$$

Where $v_0^j = u_0^j$ for $j \geq 6$.

By labeling of above, we know $\xi(C_{2j+1}) = 0\,(j \geq 6)$.

The labeling of above is illustrated in Fig.4.

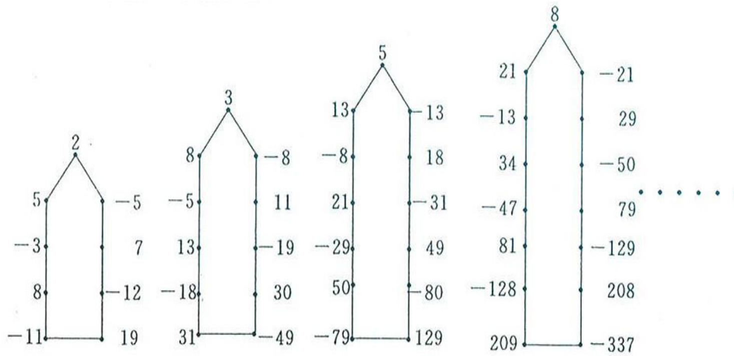

Fig. 4. Labeling of $C_{2j+1}(j \geq 6)$

Theorem 2.2. For any integer $n \geq 5$, the graph C_n is integral sum graph.

Proof. From theorem 2.1, we know $\xi(C_n) = 0\,(n \geq 5)$, therefore for any integer $n \geq 5$, the graph C_n is integral sum graph.

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