State Propagation in Abstracted Business Processes

Sergey Smirnov, Armin Zamani Farahani, and Mathias Weske

Hasso Plattner Institute, Potsdam, Germany
{sergey.smirnov,mathias.weske}@hpi.uni-potsdam.de,
armin.zamanifarahani@student.hpi.uni-potsdam.de

Abstract. Business process models are abstractions of concrete operational procedures that occur in the daily business of organizations. Typically one model is insufficient to describe one business process. For instance, a detailed technical model may enable automated process execution, while a more abstract model supports decision making and process monitoring by business users. Thereafter, multiple models capturing one process at various levels of abstraction often coexist. While the relations between such models are studied, little is known about the relations between process instances and abstract models.

In this paper we show how the state of an abstract activity can be calculated from the states of related, detailed process activities as they happen. The approach uses activity state propagation. With state uniqueness and state transition correctness we introduce formal properties that improve the understanding of state propagation. Algorithms to check these properties are devised. Finally, we use behavioral profiles to identify and classify behavioral inconsistencies in abstract process models that might occur, once activity state propagation is used.

1 Introduction

Recent years have seen an increasing interest in modeling business processes to better understand and improve working procedures in organizations and to provide a blue print for process implementation. With an increasing complexity of the processes and their IT implementations, technical process models become intricate. Business users can hardly grasp and analyze such exhaustive models. For instance, monitoring the state of a process instance challenges a manager, once a model enriched with technicalities is used. To support business users, less detailed models are created. As an outcome, one process is typically formalized by several models belonging to various levels of abstraction.

While methods for derivation of abstract process models from detailed ones are well understood, e.g., see [4,5,10,11,14,17], little is known about the relations between process instances and abstract process models. Meanwhile, this knowledge is essential for such tasks as monitoring of process instances by means of abstract models. Only a small share of the named approaches discusses the role of process instances [4,14]. However, even these endeavors have gaps and limitations motivating the current research. This paper assumes that each activity of an abstract process model is refined by a group of activities in a detailed model, yet each activity of the detailed model belongs to some group. Motivated by non-hierarchical activity refinement [4,12,21], we are liberal in terms of activity group definition. For instance, activities of one group can be arbitrary spread over the model. We study acyclic process models.

This paper clarifies the relations between process instances and abstract process models. To achieve this we introduce an approach to derive the state of an activity in the abstract model from the states of concrete process activities, as they happen. The approach is based on activity instance state propagation that determines the state of an abstract activity by the states of their detailed counterparts. We identify two formal properties for state propagation approaches—*state uniqueness* and *state transition correctness*. Further, we develop methods for validation of these properties. The properties should be considered during the design of any state propagation approach and can be validated by the developed algorithms. Finally, we investigate behavioral inconsistencies that might result from state propagation.

The paper is structured as follows. Section 2 motivates the work and identifies the main challenges. In Section 3 we elaborate on the state propagation, its properties and property validation methods. Further, Section 4 explains behavioral inconsistencies observable during state propagation. We position the contribution of this paper against the related work in Section 5 and conclude with Section 6.

2 Motivating Example and Research Challenges

This section provides further insights into the problem addressed by the current study. We start with a motivating example. Further, we informally outline our approach and identify the main research challenges.

Various stakeholders use models with different level of details about a given business process. In this setting several models are created for one process. Consider the example in Fig. 1. Model m describes a business process, where a forecast request is processed. Once an email with a forecast request is received, a request to collect the required data is sent. The forecast request is registered and the collected data is awaited. Then, there are two options: either to perform a full data analysis, or its quick version. The process concludes with a forecast report creation. Model m contains semantically related activities that are aggregated together into more coarse-grained ones. The groups of related activities are marked by areas with a dashed border, e.g., group g_1 includes *Receive email* and *Record request*. Model m_a is a more abstract specification of the forecast business process. Notice that further we reference the most detailed model as *initial*. Each activity group in m corresponds to a high-level activity in m_a, e.g., g_1 corresponds to *Receive forecast request*. Meanwhile, m'_a is even more abstract: its activities are refined by the activities of model m_a and are further refined by activities of m. While the forecast process can be enacted using model m, abstract models m_a and m'_a are suitable for monitoring the state of process

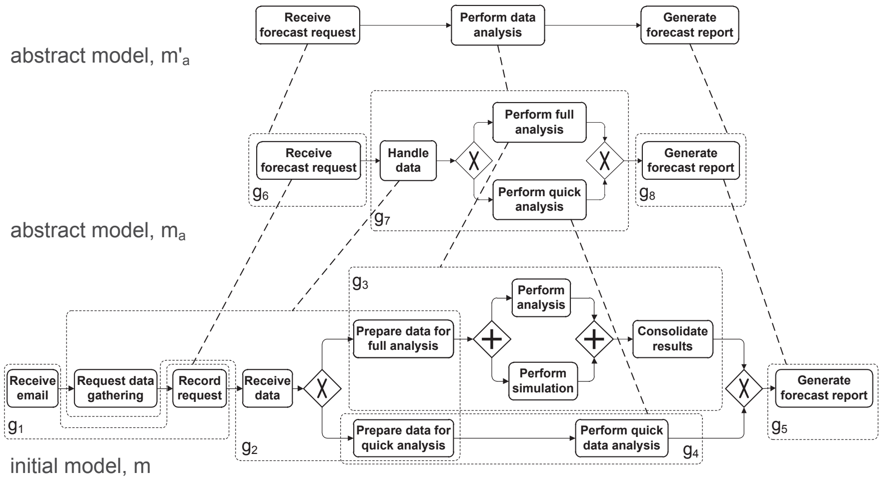


Fig. 1. Models capturing business process "Forecast request handling" at different levels of abstraction

instances. For example, a process participant might leverage model m_a, while the process owner may monitor states of instances by means of m'_a.

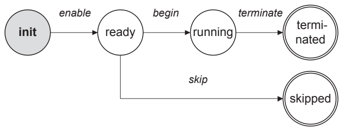


Fig. 2. Activity instance life cycle

We assume that the state of a process instance is defined by the states of its activity instances. The paper adheres to the activity instance life cycle presented in Fig. 2. When an activity is created, it is in the *init* state. We consider process models to be acyclic. Hence, once a process is instantiated, all of its activity instances are created in state *init*. The *enable* state transition brings the activity into state *ready*. If an instance is not required, *skip* transition brings it to state *skipped*. The *skipped* state has to be spread among activities that are not required. This can be realized by a well established approach of dead path elimination [13]. From the *ready* state the activity instance may evolve to *running* state by means of transition *begin*. When the instance completes its work, *terminate* transition brings it to the *terminated* state. The use of one activity instance life cycle implies that all activity instances behave according to this life cycle disregard of the abstraction level of the model an activity belongs to. Throughout this paper we frequently refer to *activity instance states*. As *activities* at the model level do not have states, we interchangeably and unambiguously use terms *activity state* and *activity instance state*.

To monitor process instance state by means of an abstract model, one needs a mechanism establishing the relation between the states of activities in the abstract model and activities of the detailed model. We reference this mechanism as activity instance state propagation. Consider a group of activities g in model m and activity x of the abstract model, such that x is refined by activities of g. State propagation maps the states of instances of activities in g to the state of x. One can design various state propagation mechanisms depending on the use case at hand. However, we identify two formal criteria to be fulfilled by any state propagation. The first criterion, *activity instance state uniqueness*, is motivated by the observation that each activity instance at every point in time is exactly in one state. Hence, this criterion requires state propagation to be a surjective mapping: each constellation of instance states in group g must result exactly one state for x. Second criterion, *activity instance state transition correctness* requires state propagation to assure that every activity instance behaves according to the declared life cycle, neither adding, nor ignoring predefined state transitions.

In the following section we design a state propagation approach that considers the activity grouping along with the states of activity instances in the groups. This state propagation is simple and can be efficiently implemented. However, this approach does not consider control flow information. Hence, one may observe *behavioral inconsistencies* taking place in the abstract model: while the model control flow prescribes one order of activity execution, state propagation results contradicting activity instance states. Section 4 elaborates on this phenomenon.

3 Activity Instance State Propagation

This section formalizes state propagation. We start by introducing the concepts of a process model and process instance. Next, we design the state propagation method. Further, Section 3.3 proposes the algorithm validating activity instance state uniqueness, while Section 3.4 elaborates on the algorithm for activity instance state transition correctness validation. The role of the algorithms is twofold. First, they validate the developed state propagation. Second, the algorithms can be reused for validation of other state propagation methods.

3.1 Preliminaries

Definition 1 (Process Model). A tuple $m = (A, G, F, s, e, t)$ is a *process model* , where A is a finite nonempty set of activities, G is a finite set of gateways, and $N = A \cup G$ is a set of nodes with $A \cap G = \emptyset$. $F \subseteq N \times N$ is a flow relation, such that (N, F) is an acyclic connected graph. The direct predecessors and successors of a node $n \in N$ are denoted, respectively, by $\bullet n = \{n' \in N | (n', n) \in F\}$ and $n \bullet = \{n' \in N | (n, n') \in F\}$. Then, $\forall\ a \in A : | \bullet a| \leq 1 \wedge |a \bullet| \leq 1$, while $s \in A$ is the only start activity, such that $\bullet s = \emptyset \wedge \forall a \in A \backslash \{s\} : | \bullet a| > 0$ and $e \in A$ is the only end activity, such that $e \bullet = \emptyset \wedge \forall a \in A \backslash \{e\} : |a \bullet| > 0$. Finally, $t : G \rightarrow \{and, xor\}$ is a mapping that associates each gateway with a type.

The execution semantics of a process model is given by a translation to a Petri net [1,8]. As the defined process model has exactly one start activity and end activity the corresponding Petri net is a WF-net. We consider *sound* process models, see [2], that can be mapped to *free-choice* WF-nets [1].

To describe the process instance level, we formalize the activity instance life cycle shown in Fig. 2 as a tuple $(\mathcal{S}, \mathcal{T}, tran, \{init\}, \mathcal{S}')$.

$\mathcal{S} = \{init, ready, running, terminated, skipped\}$ is a set of activity instance states, where *init* is the initial state and $\mathcal{S}' = \{skipped, terminated\}$ is a set of final states. $\mathcal{T} = \{enable, begin, skip, terminate\}$ is a set of state transition labels. The state transition mapping $tran : \mathcal{S} \times \mathcal{T} \rightarrow \mathcal{S}$, is defined as *tran(init, enable) = ready*, *tran(ready, begin) = running*, *tran(running, terminate) = terminated*, *tran(ready, skip) = skipped*. A *process instance* is defined as follows.

Definition 2 (Process Instance). Let $\mathcal{S}$ be the set of activity instance states. A tuple $i = (m, I, inst, stat)$ is a *process instance,* where $m = (A, G, F, s, e, t)$ is a process model, I is a set of activity instances, $inst : A \rightarrow I$ is a bijective mapping that associates an activity with an activity instance, and $stat : I \rightarrow \mathcal{S}$ is a mapping establishing the relation between an activity instance and its state.

As Definition 1 claims the process model to be acyclic, there is exactly one activity instance per process model activity, i.e., $|I| = |A|$. Finally, we formalize the activity grouping by means of function *aggregate*.

Definition 3 (Function Aggregate). Let $m = (A, G, F, s, e, t)$ be a process model and $m_a = (A_a, G_a, F_a, s_a, e_a, t_a)$—its abstract counterpart. Function $aggregate : A_a \rightarrow (\mathcal{P}(A) \backslash \emptyset)$ sets a correspondence between one activity in m_a and the set of activities in m.

Definition 4 introduces an auxiliary function st_{agg} mapping a set of activities to the set of activity instance states observed among the instances of these activities.

Definition 4 (Function State Aggregate). Let $m = (A, G, F, s, e, t)$ be a process model and $m_a = (A_a, G_a, F_a, s_a, e_a, t_a)$—the abstract model of the same process. Function $st_{agg} : A_a \rightarrow (\mathcal{P}(\mathcal{S}) \backslash \emptyset)$ is defined as $st_{agg}(x) = \bigcup_{\forall a \in aggregate(x)} \{stat(inst(a))\}$, where $x \in A_a$.

Fig. 1 illustrates function *aggregate* as follows *aggregate(Receive forecast request)* = $\{$*Receive email, Record request*$\}$. In the subsequent examples we denote the coarse-grained activities as x and y, where $x, y \in A_a$, while a and b are such activities of the model m, i.e., $a, b \in A$ that $a \in aggregate(x), b \in aggregate(y)$.

3.2 State Propagation

State propagation implies that the state of an activity x in the abstract model m_a is defined by the states of activities $aggregate(x)$ in model m. Consider the example in Fig. 3, where the instances of *Receive email* and *Record request* define the state of *Receive forecast request* instance. We develop *one possible* approach establishing the relation between activity instance states. To formalize state propagation we introduce five predicates, each corresponding to one activity instance state and "responsible" for propagation of this state to an abstract activity. An argument of a predicate is a nonempty set of states $S \subseteq \mathcal{S}$. Set S is populated by the states of activity instances observed in the activity group *aggregate*(x), i.e., $S = st_{agg}(x)$. If a predicate evaluates to true, it propagates the respective state to the instance of x. For example, predicate p_{ru} corresponds to the state *running*. Given an instance of *Receive forecast request* and instances of *Receive email* and *Record request*, we evaluate predicate p_{ru} against the set $\{init, terminated\}$. If p_{ru} evaluates to true, we claim the instance of *Receive forecast request* to be *running*, see Fig. 3. The predicates are defined as follows.

- $p_{in}(S) := \forall s \in S : s = init$
- $p_{re}(S) := (\exists s' \in S : s' = ready \wedge \forall s \in S : s \in \{init, ready, skipped\}) \vee (\exists s', s'' \in S : s' = init \wedge s'' = skipped \wedge \forall s \in S : s \in \{init, skipped\})$
- $p_{ru}(S) := \exists s \in S : s = running \vee (\exists s', s'' \in S : s' = terminated \wedge s'' \in \{init, ready\})$
- $p_{te}(S) := \exists s \in S : s = terminated \wedge \forall s' \in S : s' \in \{skipped, terminated\}$
- $p_{sk}(S) := \forall s \in S : s = skipped$

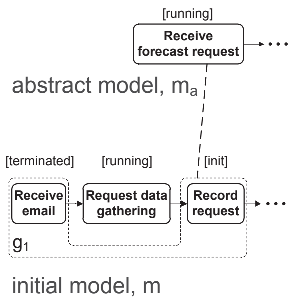


Fig. 3. State propagation example

We name this set of predicates *ps*. The predicate design implies that activity instance state uniqueness holds. The predicates p_{in} and p_{sk} propagate, respectively, states *init* and *skipped*, if only initialized or skipped activities are observed. The predicate p_{re} propagates state *ready* containing two conditions. The first part of its disjunction requires at least one *ready* activity, while others can be *skipped* or initialized. The second part of the disjunction propagates state *ready*, if in S exists a *skipped* activity, i.e., this activity *was* in state *ready*, and there exists an initialized activity, i.e., that activity *will be* in state *ready*. Predicate p_{ru} propagates state *running*, if 1) a *running* activity is observed or 2) in S exists a *terminated* activity, i.e., this activity *was* in state *running* and there is an initialized or *ready* activity, i.e., that activity *can be* in state *running*. The additional conditions of p_{re} and p_{ru} assure that activity instance state transition correctness holds. Finally, p_{te} propagates state *terminated*, once each activity of the group is either *skipped* or *terminated*. The five predicates construct *activity instance state propagation function* defining the state of activity x instance.

Definition 5 (Activity Instance State Propagation Function). *Activity instance state propagation function stp* : $\mathcal{P}(\mathcal{S}) \rightarrow \mathcal{S}$ maps a set of activity instance states to one activity instance state:

$$stp(S) = \begin{cases} init, & \textit{if } p_{in}(S) \\ ready, & \textit{if } p_{re}(S) \\ running, & \textit{if } p_{ru}(S) \\ terminated, & \textit{if } p_{te}(S) \\ skipped, & \textit{if } p_{sk}(S). \end{cases}$$

Let $m = (A, G, F, s, e, t)$ be a process model with its process instance $i = (m, I, inst, stat)$ and $m_a = (A_a, G_a, F_a, s_a, e_a, t_a)$—the abstract model with

Algorithm 1. Verification of activity instance state uniqueness

```
1: checkStateUniqueness(Predicate[] $ps$, LifeCycle $lifeCycle$)
2: for all $S \subseteq lifeCycle.\mathcal{S}$ do
3:    if $S \neq \emptyset$ then
4:       propagated = false;
5:       for all $p$ in $ps$ do
6:          if !propagated then
7:             if $p(S)$ then
8:                propagated = true;
9:          else
10:            if $p(S)$ then
11:               return false
12:      if !propagated then
13:         return false
14: return true
```

process instance $i_a = (m_a, I_a, inst_a, stat_a)$. Then, function $stat_a : I_a \rightarrow \mathcal{S}$ is defined as $stat_a(inst_a(x)) = stp(st_{agg}(x))$.

3.3 Activity Instance State Uniqueness

State propagation mechanism maps the states of activity instances of $aggregate(x)$ to the state of $inst(x)$. Definition 6 formalizes activity instance state uniqueness property.

Definition 6 (Activity Instance State Uniqueness). Let $(\mathcal{S}, \mathcal{T}, tran, \{init\}, \mathcal{S}')$ be an activity instance life cycle. Activity instance state propagation defined by function *stp* based on predicates *ps* fulfills *activity instance state uniqueness* iff $\forall S \subseteq \mathcal{S}$ exactly one predicate of *ps* evaluates to *true*.

The set of states S is observed within $aggregate(x)$. However, an activity group is defined by the user and is not known in advance. Hence, it is not efficient to reason about state uniqueness property explicitly enumerating all activity instance states that occur within activity instance groups. Instead of dealing with concrete activity instance groups, we introduce activity instance group equivalence classes. For a process instance $i = (m, I, inst, st)$ two activity instance groups $I_1, I_2 \subseteq I$ belong to one equivalence class, if in both groups the same set of activity instance states is observed, i.e., $\forall i_1 \in I_1 \exists i_2 \in I_2 : stat(i_1) = stat(i_2) \wedge \forall i_2 \in I_2 \exists i_1 \in I_1 : stat(i_2) = stat(i_1)$. For instance, a pair of activity instances with states (*init*, *terminated*) and an instance triple with states (*init*, *init*, *terminated*) belong to one class with observed states $S = \{init, terminated\}$. As this classification covers all possible state combinations, the algorithm checks all cases. We can consider such classes of activity instance groups, since the predicates make use of existential and universal quantifiers.

Algorithm 1 validates activity instance state uniqueness. The algorithm takes a set of predicates and an activity instance life cycle as inputs; it returns *true*, once the property holds. As the number of equivalence classes is exponential to the number of states in the activity instance life cycle, the computational complexity of Algorithm 1 is also exponential.

Algorithm 2. Validation of activity instance state transition correctness

```
1: checkStateTransitionCorrectness(Predicate[] $ps$, LifeCycle $lifeCycle$)
2: for all $p$ in $ps$ do
3:     for all $S \subseteq \mathcal{S} : p(S) = true$ do
4:         for all $s \in (S \backslash lifeCycle.\mathcal{S}')$ do
5:             for all $t \in lifeCycle.\mathcal{T}$, where $tran(s, t)$ is defined do
6:                 $S' = S \cup \{tran(s, t)\}$
7:                 if $stp(S') \neq stp(S)$ and $tran(stp(S), t) \neq stp(S')$ then
8:                     return false
9:                 $S' = S' \backslash \{s\}$
10:                if $stp(S') \neq stp(S)$ and $tran(stp(S), t) \neq stp(S')$ then
11:                    return false
12: return true
```

3.4 Activity Instance State Transition Correctness

Activity instance state propagation must assure that instances of activities in abstract models behave according to the predefined life cycle, see Definition 7.

Definition 7 (Activity Instance State Transition Correctness). Let $(\mathcal{S}, \mathcal{T}, tran, \{init\}, \mathcal{S}')$ be an activity instance life cycle. Activity instance state propagation defined by function *stp* fulfills *activity instance state transition correctness* iff $\forall S \subseteq \mathcal{S}$ each state transition allowed by $tran$ from $\forall s \in S$ through $t \in \mathcal{T}$ produces a set $S' = S \cup \{tran(s,t)\}$ such that either $stp(S) = stp(S') \vee stp(S) = stp(S' \backslash \{s\})$ or $tran(stp(S), t) = stp(S') \vee tran(stp(S), t) = stp(S' \backslash \{s\})$.

Algorithm 2 validates activity instance state transition correctness. Its inputs are a set of predicates and an activity instance life cycle. It returns $true$, if state transitions are correct. The key idea of the algorithm is the observation that an instance of an activity x in the abstract model changes its state, when *one* of the activity instances that refines x changes its state. Hence, the validation considers all possible state transitions. For each predicate p of ps the algorithm constructs state sets $S \subseteq \mathcal{S}$, where the predicate evaluates to $true$ (lines 2–3). For instance, predicate p_{in} has one such set $\{init\}$. Then, the validation constructs state set S' reachable from S by one state transition of the activity instance life cycle (lines 4–6 and line 9). In the example $S = \{init\}$ evolves to $\{ready\}$ or $\{init, ready\}$. For each of those reachable state sets S' function *stp* is evaluated. If for each S' the state $stp(S')$ equals $stp(S)$ or can be reached from $stp(S)$ using the same state transition as required to reach S' from S, the state propagation rules are valid. Algorithm 2 realizes the checks in lines 7 and 10 and reports correctness in line 12. The algorithm has the running time of $O(2^{|\mathcal{S}|})$.

4 Behavioral Inconsistencies

This section elaborates on the problem of behavioral inconsistencies. We start with the motivation, then introduce auxiliary formal concepts and define the notion of behavioral inconsistency. Finally, we classify behavioral inconsistencies.

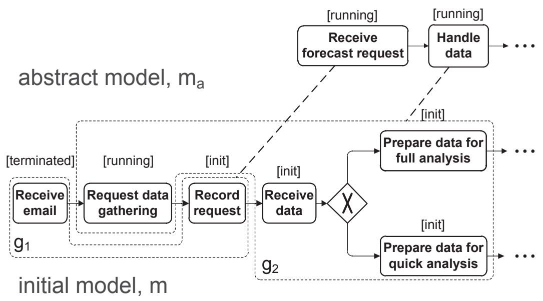


Fig. 4. Behavioral inconsistency in a process instance for the business process in Fig. 1

4.1 Example

An abstract process model dictates activity execution order. Meanwhile, the designed state propagation disregards the control flow, but influences the states of activities in m_a. In this setting one can observe *behavioral inconsistencies.* Fig. 4 exemplifies a behavioral inconsistency. Activities *Receive forecast request* and *Handle data* are refined with groups g_1 and g_2, respectively. According to the state propagation mechanism, once *Receive email* terminates, *Receive forecast request* runs until *Record request* terminates. While *Request data gathering* runs, *Handle data* is in the state *running.* According to the state propagation we observe activities *Receive forecast request* and *Handle data* in state *running* at the same time. However, model m_a prescribes sequential execution of the activities: *Handle data* can be executed, once *Receive forecast request* terminates. Hence, these states are inconsistent with the control flow of m_a.

Behavioral inconsistencies have two reasons. The first reason is activity grouping. Consider the example in Fig. 4, where activities in groups g_1 and g_2 interleave: *Receive email* precedes *Request data gathering* and *Request data gathering* precedes *Record request.* The second reason is the loss of activity *optionality* or *causality* in the abstract model. We say that an activity is optional, if there is such a process trace, where this activity is not observed. Considering the example in Fig. 4 *Prepare data for full analysis* is optional. Activity causality implies that 1) an order of execution for two activities is given and 2) two activities appear together in all process executions. One can observe causality relation for *Receive email* and *Receive data*, but not for *Receive email* and *Prepare data for full analysis.* The next section formalizes the notion of behavioral inconsistencies.

4.2 Formalization of Behavioral Inconsistencies

To formalize the discussion of behavioral inconsistencies we exploit the notion of behavioral profiles [23]. While this discussion can be based on alternative formalisms, we stick to behavioral profiles, as 1) they can be efficiently computed and 2) there are techniques enabling navigation between process models of different abstraction levels based on behavioral profiles [17]. To introduce behavioral profiles we inspect the set of all traces from s to e for a process model

$m = (A, G, F, s, e, t)$. The set of *complete process traces* $\mathcal{W}_m$ for m contains lists of the form $s \cdot A^* \cdot e$, where a list captures the activity execution order. To denote that an activity a is a part of a complete process trace we write $a \in w$ with $w \in \mathcal{W}_m$. Within this set of traces the *weak order* relation for activities is defined.

Definition 8 (Weak Order Relation). Let $m = (A, G, F, s, e, t)$ be a process model, and $\mathcal{W}_m$—its set of traces. The *weak order relation* $\succ_m \subseteq (A \times A)$ contains all pairs (x, y), where there is a trace $w = n_1, \ldots, n_l$ in $\mathcal{W}_m$ with $j \in \{1, \ldots, l-1\}$ and $j < k \leq l$ for which holds $n_j = x$ and $n_k = y$.

Two activities of a process model are in weak order, if there exists a trace in which one activity occurs after the other. Depending on how weak order relates two process model activities, we define relations forming the behavioral profile. While behavioral profiles enable judgment on activity ordering, they do not capture causality. Following on [24] we make use of auxiliary *co-occurrence relation* and *causal behavioral profile.*

Definition 9 (Behavioral Profile and Causal Behavioral Profile). Let $m = (A, G, F, s, e, t)$ be a process model and $\mathcal{T}_m$ be its set of traces. A pair $(a, b) \in (A \times A)$ is in one of the following relations: *1)* strict order relation $\rightsquigarrow_m$, if $a \succ_m b$ and $b \not\succ_m a$; *2)* exclusiveness relation $+_m$, if $a \not\succ_m b$ and $b \not\succ_m a$; *3)* interleaving order relation $||_m$, if $a \succ_m b$ and $b \succ_m a$. The set of all three relations is the *behavioral profile* of m. A pair $(a, b) \in (A \times A)$ is in the *co-occurrence relation* $\gg_m$ iff for all traces $\sigma = n_1, \ldots, n_l$ in $\mathcal{W}_m$ it holds $n_i = a, i \in \{1, \ldots, l\}$ implies that $\exists j \in \{1, \ldots, l\}$ such that $n_j = b$. Then $\{\rightsquigarrow_m, ||_m, +_m, \gg_m\}$ is the *causal behavioral profile* of m.

The behavioral profile relations along with the inverse strict order $\rightsquigarrow^{-1} = \{(x, y) \in (A \times A) \mid (y, x) \in \rightsquigarrow\}$, partition the Cartesian product of activities in one process model. The causality relation holds for $a, b \in A$ if $a \rightsquigarrow_m b \wedge a \gg_m b$.

The example in Fig. 4 witnesses that state propagation allows concurrent activity execution. However, the behavioral profile relations are defined on the trace level and do not capture concurrency. To formalize the *observed* behavior of activities, we introduce relations defined on the process instance level. These relations build on top of causal behavioral profile relations. However, they consider not traces, but process instances.

We say $(x, y) \in \rightsquigarrow_{obs}$ if there is a process instance where x is executed before y, but no instance, where y is executed before x. Similarly, relation $x \rightsquigarrow_{obs}^{-1} y$ means that there is a process instance where y is executed before x, but no instance, where x is executed before y. Relation $x +_{obs} y$ holds if there is no instance where x and y both take place. Relation $||_{obs}$ corresponds to the existence of 1) an instance where x is executed before y, 2) an instance where y is executed before x and 3) an instance where x and y are executed concurrently. Finally, $x \gg_{obs} y$ holds if for every instance, where x is executed, y is executed as well. Then, the behavioral inconsistency can be defined as follows.

Definition 10 (Behavioral Inconsistency). Let $m = (A, G, F, s, e, t)$ be a process model and $i = (m, I, inst, stat)$—its instance. $m_a = (A_a, G_a, F_a, s_a, e_a, t_a)$ is the abstract model obtained from m and having the instance $i_a = (m_a, I_a, inst_a, stat_a)$, where function $stat_a$ is defined as $stat_a(inst_a(x)) = stp(st_{agg}(x))$. We say that there is a *behavioral inconsistency,* if $\exists(x, y) \in (A_a \times A_a)$ for which the causal behavioral profile relations do not coincide with the observed behavioral relations: *1)* $(x, y) \in \rightsquigarrow_{m_a} \wedge (x, y) \notin \rightsquigarrow_{obs}$; or *2)* $(x, y) \in \rightsquigarrow_{m_a}^{-1} \wedge (x, y) \notin \rightsquigarrow_{obs}^{-1}$; or *3)* $(x, y) \in +_{m_a} \wedge (x, y) \notin +_{obs}$; or *4)* $(x, y) \in ||_{m_a} \wedge (x, y) \notin ||_{obs}$; or *5)* $(x, y) \in \gg_{m_a} \wedge (x, y) \notin \gg_{obs}$.

4.3 Classification of Behavioral Inconsistencies

Table 1 classifies behavioral inconsistencies comparing the declared and observed behavioral constraints for abstract process model activities x and y. A table row corresponds to behavioral profile relations declared by an abstract model. Columns capture the observed behavioral relations. A cell of Table 1 describes an inconsistency between the observed and declared behavioral relations. The table presents a complete analysis of inconsistencies, due to extensive exploration of all possible relation combinations.

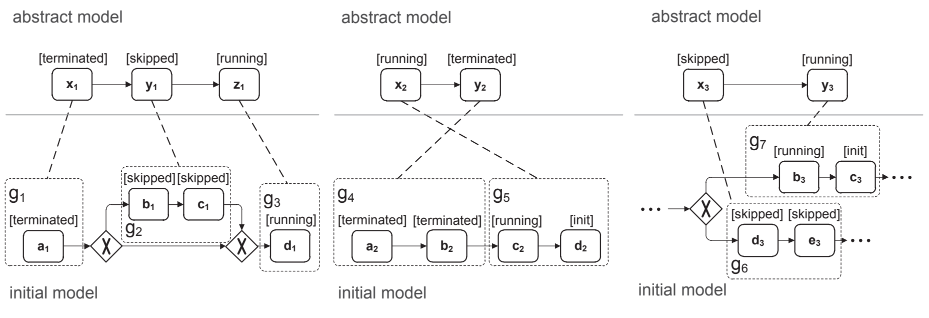
The "+" sign witnesses no inconsistency since the declared and observed constraints coincide. We identify one class of activity groups causing no inconsistency. Consider a pair of activities $x, y \in A_a$. If $\forall(a, b) \in aggregate(x) \times aggregate(y)$ the same causal behavioral profile relation holds, no behavioral inconsistency is observed. A prominent example of activity groups that fulfill the defined requirement are groups resulting from the canonical decomposition of a process model into single entry single exit fragments, see [19,20].

Every cell marked with "±" symbol corresponds to an inconsistency, where no contradiction takes place: an observed relation restricts a declared behavioral relation. Consider, for instance, the behavioral inconsistency, where $x||_{m_a}y$, while $x \rightsquigarrow_{obs}^{-1} y$ and $x \gg_{obs} y$. This inconsistency has no contradiction, as the observed behavior only restricts the declared one. We identify five classes of behavioral inconsistencies marked in Table 1 and illustrate them by the examples in Fig. 5.

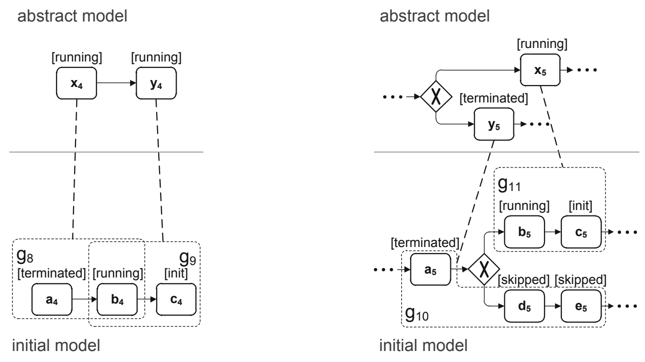
`A: Co-occurrence loss` Behavioral inconsistencies of this type take place if the model declares co-occurrence for an activity pair, while both activities are observed not in every process instance. The cause of inconsistency is the loss of

Table 1. Classification of behavioral inconsistencies

		$x \rightsquigarrow_{obs} y$		$x \rightsquigarrow_{obs}^{-1} y$		$x +_{obs} y$	$x\|\|_{obs}y$
		$x \gg_{obs} y$	$x \not\gg_{obs} y$	$x \gg_{obs} y$	$x \not\gg_{obs} y$		
$x \rightsquigarrow_{m_a} y$	$x \gg_{m_a} y$	+	A	B	B	C	D
	$x \not\gg_{m_a} y$	±	+	B	B	C	D
$x \rightsquigarrow_{m_a}^{-1} y$	$x \gg_{m_a} y$	B	B	+	A	C	D
	$x \not\gg_{m_a} y$	B	B	±	+	C	D
$x +_{m_a} y$		E	E	E	E	+	E
$x\|\|_{m_a}y$		±	±	±	±	C	+


abstract model
[terminated] [skipped] [running]
x1 y1 z1
g1 [terminated] a1
[skipped] [skipped] b1 c1 g2
g3 [running] d1
initial model
[running] [terminated] x2 y2
g4 [terminated] [terminated] a2 b2
g5 [running] [init] c2 d2
[skipped] [running] x3 y3
g7 [running] [init] b3 c3
[skipped] [skipped] d3 e3 g6

(a) Co-occurrence loss (b) Inverse order (c) Activity loss


abstract model
[running] [running] x4 y4
g8 [terminated] [running] [init] g9 a4 b4 c4
initial model
[running] x5
[terminated] y5
g11 [running] [init] b5 c5
[terminated] a5
[skipped] [skipped] d5 e5 g10

(d) Order loss (e) Exclusiveness loss

Fig. 5. Examples of behavioral inconsistencies: one example per class

information about the causal coupling of an activity pair. The example in Fig. 5(a) illustrates this inconsistency type. Since activities of group g_2 are skipped, activity y_1 is in state *skipped* as well. However, it cannot be skipped according to the abstract model control flow.

`B: Inverse order.` We say that an *inverse order* inconsistency takes place if the model prescribes $x \rightsquigarrow_{m_a} y$ ($x \rightsquigarrow_{m_a}^{-1} y$), whilst the user observes $x \rightsquigarrow_{obs}^{-1} y$ ($x \rightsquigarrow_{obs} y$). Fig. 5(b) gives an example of such an inconsistency baring its cause: for each $(a, b) \in aggregate(x) \times aggregate(y)$ activities a and b have the order opposite to the order of x and y.

`C: Activity loss.` Once the process model specifies two activities to appear within one instance, whereas only one activity is observed within an instance, *activity loss* inconsistency takes place. Fig. 5(c) exhibits one example of such an inconsistency. While activity groups g_6 and g_7 are exclusive, the corresponding abstract activities x_3 and y_3 are in strict order. Accordingly, either x_3 or y_3 is observed within each instance.

`D: Order loss.` For a pair of activities in (inverse) strict order, the user observes interleaving execution. A behavioral inconsistency of this type is exemplified in Fig. 5(d). Such inconsistencies have the following roots: 1) $aggregate(x) \cap aggregate(y) \neq \emptyset$ or 2) exist $a_1, a_2 \in aggregate(x)$ and $b_1, b_2 \in aggregate(y)$ such that it holds $a_1 \succ_m b_1$ and $b_2 \succ_m a_2$. In Fig. 5(d) activity b_2 belongs to groups g_1 and g_2. As a consequence, once b_2 runs both sequential activities x_2 and y_2 are running concurrently.

`E: Exclusiveness loss.` While the model prescribes exclusiveness relation for x and y, both activities are observed within one instance. These inconsistencies take place, once in the initial model there exist such a and b that $a \succ_m b$ or $b \succ_m a$. Fig. 5(e) exemplifies this inconsistency. According to the abstract model $x_3 +_{m_a} y_3$. However, in the presented process instance both x_3 and y_3 take place.

5 Related Work

We identify two directions of the related work. The first one is the research on business process model abstraction. The second one is the body of knowledge discussing similarity of process models.

Business process model abstraction has been approached by several authors. The majority of the solutions consider various aspects of model transformation. For instance, [5,10,11,15,17] focus on the structural aspects of transformation. Among these papers [17] enables the most flexible activity grouping. Several papers study how the groups of semantically related activities can be discovered [6,16]. A few works elaborate on the relation between process instances and abstract process models, e.g. [4,14]. In [4] Bobrik, Reichert, and Bauer discuss state propagation and associated behavioral inconsistencies, but do not use the concept of activity instance life cycle. [14] suggests state propagation approach that builds on the activity instance life cycle shown in Fig. 6. In [14] Liu and Shen order three states according to how "active" they are: *not_started* < *suspended* < *running*. The state propagation rules make use of this order, e.g., if a coarse-grained activity is refined by activities in one of the *open* states, the high-level activity is in the most "active" state. Against this background, consider an example activity pair evolving as follows: (*not_started*, *not_started*) to (*not_started*, *running*) to (*not_started*, *completed*). According to the rules defined in [14] the high level activity evolves as *not_started* to *running* to *not_started*, which contradicts the activity instance life cycle. As we mentioned above, the majority of works on business process model abstraction consider only the *model* level. Meanwhile, the papers that take into account process instances have gaps and limitations. For instance, [4,14] motivated us not only to introduce the state propagation approach, but also to identify formal properties for such approaches and develop validation algorithms.

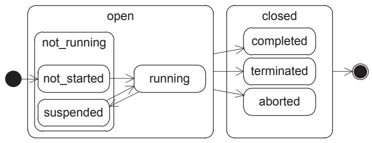


Fig. 6. Activity instance life cycle as presented in [14]

The works on similarity of process models can be refined into two substreams. A series of papers approaches process model similarity analyzing model structure and labeling information, see [7,21]. These works provide methods to discover matching model elements. Several research endeavors analyze behavioral similarity of process models. In particular, [3] introduces several notions of inheritance and operations on process models preserving the inheritance property. Recently, Weidlich, Dijkman, and Weske investigated behavioral compatibility of models capturing one business process [22]. [9] elaborates on process model similarity considering both model element labeling and model behavior. Considering that processes are inherently concurrent systems, various notions of behavioral equivalence for concurrent systems can be leveraged to compare the behavior of initial and abstract process models [18]. The enumerated papers help to compare the behavior of initial and abstract process models. As such, the notions of behavioral equivalence and behavioral compatibility might give additional insights into the causes of behavioral inconsistencies, see Section 4, and classify them further.

6 Conclusion and Future Work

Although the relations between models capturing one business process on different levels of abstraction have been thoroughly studied earlier, the relations between process instances and abstract process models have been barely explored. The current paper bridged this gap. First, we developed activity instance state propagation mechanism that allows to describe the process instance state by means of an abstract process model. Second, we have identified two formal properties for state propagation and proposed methods for their validation. Finally, we elaborated on behavioral inconsistencies that can be observed, once the assumed abstraction and state propagation mechanisms are used.

We foresee several directions of the future work. The direct next step is the extension of the considered model class. As we leverage dead path elimination to spread activity instance state *skipped* over not executed activities, the state propagation approach is limited to acyclic models. Substitution of dead path elimination with an alternative approach would facilitate handling of cyclic models. Another direction is the further study of the behavioral inconsistencies and methods for their resolution. With that respect, it is valuable to integrate control flow information into state propagation mechanism. Finally, the applications of the introduced technique call for deep investigation. One direct application of our approach is business process monitoring [25], where abstract models help users to follow the progress of running business processes.

References

1. van der Aalst, W.M.P.: The Application of Petri Nets to Workflow Management. Journal of Circuits, Systems, and Computers 8(1), 21–66 (1998)
2. van der Aalst, W.M.P.: Workflow Verification: Finding Control-Flow Errors Using Petri-Net-Based Techniques. In: van der Aalst, W.M.P., Desel, J., Oberweis, A. (eds.) BPM. LNCS, vol. 1806, pp. 161–183. Springer, Heidelberg (2000)
3. van der Aalst, W.M.P., Basten, T.: Life-Cycle Inheritance: A Petri-Net-Based Approach. In: Azéma, P., Balbo, G. (eds.) ICATPN 1997. LNCS, vol. 1248, pp. 62–81. Springer, Heidelberg (1997)

4. Bobrik, R., Reichert, M., Bauer, T.: Parameterizable Views for Process Visualization. Technical Report TR-CTIT-07-37, Centre for Telematics and Information Technology, University of Twente, Enschede (April 2007)
5. Bobrik, R., Reichert, M., Bauer, T.: View-Based Process Visualization. In: Alonso, G., Dadam, P., Rosemann, M. (eds.) BPM 2007. LNCS, vol. 4714, pp. 88–95. Springer, Heidelberg (2007)
6. Di Francescomarino, C., Marchetto, A., Tonella, P.: Cluster-based Modularization of Processes Recovered from Web Applications. Journal of Software Maintenance and Evolution: Research and Practice (2010)
7. Dijkman, R.M., Dumas, M., García-Bañuelos, L.: Graph Matching Algorithms for Business Process Model Similarity Search. In: Dayal, U., Eder, J., Koehler, J., Reijers, H.A. (eds.) BPM 2009. LNCS, vol. 5701, pp. 48–63. Springer, Heidelberg (2009)
8. Dijkman, R.M., Dumas, M., Ouyang, C.: Semantics and Analysis of Business Process Models in BPMN. Information and Software Technology 50(12), 1281–1294 (2008)
9. van Dongen, B., Dijkman, R., Mendling, J.: Measuring Similarity between Business Process Models. In: Bellahsène, Z., Léonard, M. (eds.) CAiSE 2008. LNCS, vol. 5074, pp. 450–464. Springer, Heidelberg (2008)
10. Eshuis, R., Grefen, P.: Constructing Customized Process Views. Data & Knowledge Engineering 64(2), 419–438 (2008)
11. Günther, C.W., van der Aalst, W.M.P.: Fuzzy Mining–Adaptive Process Simplification Based on Multi-perspective Metrics. In: Alonso, G., Dadam, P., Rosemann, M. (eds.) BPM 2007. LNCS, vol. 4714, pp. 328–343. Springer, Heidelberg (2007)
12. Knoepfel, A., Groene, B., Tabeling, P.: Fundamental Modeling Concepts: Effective Communication of IT Systems. John Wiley & Sons, Ltd. (2005)
13. Leymann, F., Roller, D.: Production Workflow: Concepts and Techniques. Prentice Hall PTR, Upper Saddle River (2000)
14. Liu, D.-R., Shen, M.: Business-to-business Workflow Interoperation based on Process-Views. Decision Support Systems 38, 399–419 (2004)
15. Polyvyanyy, A., Smirnov, S., Weske, M.: The Triconnected Abstraction of Process Models. In: Dayal, U., Eder, J., Koehler, J., Reijers, H.A. (eds.) BPM 2009. LNCS, vol. 5701, pp. 229–244. Springer, Heidelberg (2009)
16. Smirnov, S., Dijkman, R.M., Mendling, J., Weske, M.: Meronymy-Based Aggregation of Activities in Business Process Models. In: Parsons, J., Saeki, M., Shoval, P., Woo, C., Wand, Y. (eds.) ER 2010. LNCS, vol. 6412, pp. 1–14. Springer, Heidelberg (2010)
17. Smirnov, S., Weidlich, M., Mendling, J.: Business Process Model Abstraction Based on Behavioral Profiles. In: Maglio, P.P., Weske, M., Yang, J., Fantinato, M. (eds.) ICSOC 2010. LNCS, vol. 6470, pp. 1–16. Springer, Heidelberg (2010)
18. van Glabbeek, R.J.: The Linear Time-Branching Time Spectrum (Extended Abstract). In: Baeten, J.C.M., Klop, J.W. (eds.) CONCUR 1990. LNCS, vol. 458, pp. 278–297. Springer, Heidelberg (1990)
19. Vanhatalo, J., Völzer, H., Koehler, J.: The Refined Process Structure Tree. In: Dumas, M., Reichert, M., Shan, M.-C. (eds.) BPM 2008. LNCS, vol. 5240, pp. 100–115. Springer, Heidelberg (2008)
20. Vanhatalo, J., Völzer, H., Leymann, F.: Faster and More Focused Control-Flow Analysis for Business Process Models Through SESE Decomposition. In: Krämer, B.J., Lin, K.-J., Narasimhan, P. (eds.) ICSOC 2007. LNCS, vol. 4749, pp. 43–55. Springer, Heidelberg (2007)

21. Weidlich, M., Dijkman, R., Mendling, J.: The ICoP Framework: Identification of Correspondences between Process Models. In: Pernici, B. (ed.) CAiSE 2010. LNCS, vol. 6051, pp. 483–498. Springer, Heidelberg (2010)
22. Weidlich, M., Dijkman, R., Weske, M.: Deciding Behaviour Compatibility of Complex Correspondences between Process Models. In: Hull, R., Mendling, J., Tai, S. (eds.) BPM 2010. LNCS, vol. 6336, pp. 78–94. Springer, Heidelberg (2010)
23. Weidlich, M., Mendling, J., Weske, M.: Efficient Consistency Measurement based on Behavioural Profiles of Process Models. In: IEEE TSE (2010) (to appear)
24. Weidlich, M., Polyvyanyy, A., Mendling, J., Weske, M.: Efficient Computation of Causal Behavioural Profiles Using Structural Decomposition. In: Lilius, J., Penczek, W. (eds.) PETRI NETS 2010. LNCS, vol. 6128, pp. 63–83. Springer, Heidelberg (2010)
25. zur Muehlen, M.: Workflow-based Process Controlling - Foundation, Design and Application of Workflow-Driven Process Information Systems. PhD thesis, University of Münster (2002)