Chapter 9
Final Remarks and Outlook

In this thesis, trapped Bose-Einstein condensates were investigated based on the interacting many-body Schrödinger equation. Studies of this kind are (still) rare due to their computational complexity. Here, numerically *exact* results were obtained for the dynamics of up to one hundred identical bosons. This system size is unprecedented in literature. For the solution of the time-dependent many-body Schrödinger equation the recently developed MCTDHB method was used. Whenever appropriate, the results of the many-body Schrödinger equation were compared to the standard methods of the field, Gross–Pitaevskii theory and the Bose-Hubbard model. Thereby it was shown that the range of validity of these approximations is far more limited than what is commonly believed, and that no reliable validity criteria for these approximations exist to date. The true many-body physics of ultracold bosons turns out to be much richer than what can be anticipated based on Gross–Pitaevskii theory and the Bose-Hubbard model. This is even more so, as we have treated one of the conceptually simplest cases only, namely identical bosons trapped in a one-dimensional double-well potential. Therefore, the phenomena discovered here can only give a glimpse of what is to be expected in more complicated systems, e.g. optical lattices, higher dimensions, mixtures of bosons, boson-fermion mixtures and so on. Apart from the newly discovered physical phenomena, we have presented a conceptual innovation in this work: time-dependent Wannier functions. We have shown that lattice models can be greatly improved by letting the Wannier functions of a lattice model become time-dependent. The Bose-Hubbard model was used as an example to demonstrate this. However, the concept is general and can be applied to any lattice model that employs Wannier functions. It will be interesting to investigate how much other lattice models can be improved by using time-dependent Wannier functions. We have not included any treatment of higher dimensional systems in this thesis, but work is in progress. Another very interesting direction that we are currently pursuing is to map out explicitly the ranges of validity of Gross–Pitaevskii theory and the Bose-Hubbard model. Also here work is still in progress and results will soon be available.

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