

# LSCM Based Non-rigid Registration for Craniofacial Surfaces

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**Abstract.** The Least Squares Conformal Maps (LSCM) is an approximation of the conformal mapping in the least-squares sense, and it can map the corresponding feature points on two 3D surfaces into the same 2D location. This paper proposes a non-rigid registration method for craniofacial surfaces based on LSCM parameterization. Firstly, craniofacial surfaces are normalized in pose and scale by using a unified coordinate system. Secondly, by pinning six landmarks, which include the outer corners of the eyes, two corners of the mouth, two side points of the nose wing, each craniofacial surface is mapped into a nearly equal 2D domain by using LSCM. Finally, an iso-parameter mesh of each craniofacial surface can be obtained by 2D to 3D mapping, which establishes a unique correspondence among different craniofacial surfaces. To evaluate the proposed method, the target surface is deformed into the reference surface using TPS algorithm with dense correspondences being control points, and then the sum of the distance between two correspondence point sets are computed, and vice versa. According to the average distance, the proposed method is compared with ICP and a TPS based method. The comparison shows that the proposed approach is more accurate and effective.

**Keywords:** non-rigid registration, LCSM, craniofacial surface.

## 1 Introduction

Non-rigid registration of 3D data set is an important and challenging issue for many applications such as atlas matching, shape retrieval, 3D object recognition etc [1,2]. The registration for craniofacial surfaces is to find an optimal transformation from one craniofacial surface to the other surface, so that each point of one craniofacial surface maps to its corresponding point in another surface which stems from the same physical point. In fact, the registration for craniofacial surfaces is not a well-defined problem. Except for some distinct points such as the tip of nose and the corners of mouth and so on, it is difficult to define exact correspondence between different craniofacial surfaces, especially for the points on smooth regions such as the cheeks and forehead.

Up to now, there are many researches on the 3D registration. The iterative closest point (ICP) algorithm [3] is a classical technique used in 3D registration. It iteratively searches the closest corresponding points in two data sets and optimized rigid

transformation to minimize the distances between these closest points. Since the original ICP algorithm has a heavy time cost and need a good pre-alignment, various improved ICP methods [4] have been proposed to enhance the registration accuracy or convergence stability of ICP. However, ICP is not suitable for alignments with large non-rigid deformation. The dominant non-rigid deformation is Thin Plate Spline (TPS)-based [5,6]. Hutton et al [7] proposed a dense 3D face model registration based on TPS. They manually pick up 9 feature points as TPS controlling points on 3D faces in the training set. However, these 9 feature points are inadequate for describing the variation of the 3D facial shapes. Schneider and Eisert [8] propose an automatic registration method by combining ICP and TPS. In this method, landmarks are first automatically defined using the ICP scheme with a re-weighted error function, and then a TPS deformation is computed based on the landmark correspondences. Hu et al. [9] propose a similar method, but they define the landmarks of TPS by first randomly selecting some points on one face, and then finding their correspondences on the other face through an iterative closest point searching strategy. However, closest point pairs may not always physically correspond to each other.

In this work, we propose a registration method for 3D craniofacial surfaces based on Least squares conformal maps parameterization (LSCM). LSCM is an approximation of the conformal mapping in the least-squares sense, and it can map the corresponding feature points on two 3D surfaces into the same 2D location, and can map a 3D surface to a 2D plane in a continuous manner with minimized local angle distortion. By pinning six corresponding landmarks on each craniofacial surface, all craniofacial surfaces are mapped into nearly equal 2D parameterizations using LSCM, that is, the corresponding points are mapped onto nearly equal 2D parameters. Compared with ICP and a TPS based method, the proposed approach is more accurate and effective.

Section 2 introduces the LSCM. Section 3 describes the proposed registration method for craniofacial surfaces. Experimental results are reported in Section 4, and followed are some conclusions in Section 5.

## 2 Least Squares Conformal Maps

In this section, we briefly introduce the notion of the LSCM (see [10] for details).

It can be proofed by Riemann's theorem that any surface homeomorphic to a disc can be parameterized to a 2D planar domain by a conformal mapping, which is one to one, onto, and angle preserving [11,12]. The mapping can be uniquely determined by any two points on the surface. However, the mapping usually is unreliable, since there is noise in the discrete data we obtained. In order to better handle the errors caused by noise and the inaccuracy of locating feature points, the LSCM introduces additional constraints in the least squares sense.

Consider a discrete 3D surface triangle mesh  $S$  and a smooth target mapping  $U : S \rightarrow (u, v)$ .  $U$  is conformal on  $S$  if and only if the following Cauchy-Riemann equation holds true on the whole of  $S$ .

$$\frac{\partial U}{\partial x} + i \frac{\partial U}{\partial y} = 0 \quad (1)$$

Since this conformal condition cannot be strictly satisfied on the whole triangulated surface  $S$ , the conformal map is constructed in the least squares sense with the constraint that the mapping  $U$  is linear on each triangle of the mesh surface  $S$ . Then the conformal map is constructed in the least-squares sense:

$$\text{Min}C(S) = \sum_{d \in S} \left| \frac{\partial U}{\partial x} + i \frac{\partial U}{\partial y} \right|^2 A(d) \quad (2)$$

where  $d$  is a triangle on the mesh  $S$ , and  $A(d)$  is the area of the triangle  $d$ . In solving this minimization problem, we can add more correspondences as additional constraints by pinning multiple points with a priori values. Thus, we can map a 3D surface to a 2D domain with multiple correspondences as constraints by using the LSCM technique.

Compared to conformal maps, LSCM has the following advantages [12]: LSCM can map a 3D shape to a 2D domain in a continuous manner with minimized local angle distortion; LSCM can handle missing boundaries and occlusion; LSCM is independent of mesh resolution, LSCM can be linearly solved; LSCM can better handle noise from the feature point detection due to multiple feature constraints.

### 3 The Registration Algorithm for Craniofacial Surfaces

#### 3.1 Normalization of Craniofacial Surfaces

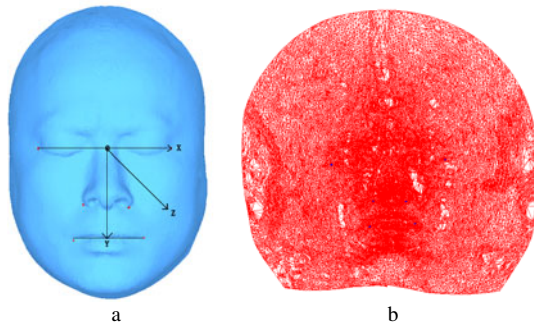
Since different craniofacial data can be obtained by different 3D data acquisition, and the poses and the scales of the craniofacial surfaces are various, we need to perform a pose and scale normalization before registration. According to human physiological characteristics, the four landmarks, which include the two outer corners of eyes and two corners of the mouth, are coplanar. We build a unified coordinate system as follows: The midpoint of the two landmarks of two eyes is the origin of the coordinate system; The direction from the coordinate origin to the right landmark of the eyes is the x-direction; The normal of the plane is the z-direction; Then, y-direction is determined according to the right-hand rule; Finally, the metric unit of the coordinate system is defined according to the distance from the coordinate origin to the right landmark of the eyes. Figure.1a shows the unified coordinate system. When transformed to this unified coordinate system, each craniofacial surface can be adjusted to a same pose and a same scale.

#### 3.2 Registration Based on LCSM

A craniofacial surface is approximately homeomorphic to a disc, so we can use LSCM to map a craniofacial surface to a 2D domain. When adding additional feature constraints that the corresponding anatomy features on any craniofacial surface should be mapped onto the same 2D locations, we expect that corresponding points on different craniofacial surfaces have nearly the same 2D values. Then from 2D correspondences, we can obtain the 3D point matching.

To generate feature constraints in LSCM, we have to pin some feature points. As we know, except for some distinct points such as the tip of nose and the corners of mouth and so on, it is difficult to find the exact anatomy correspondence between different craniofacial surfaces. In addition, as Figure.1 shows, the six landmarks in a craniofacial surface including the outer corners of the two eyes, two corners of the mouth, two side points of the nose wing, control the main feature region of a craniofacial surface. So we choose the six landmarks to generate the feature constraints in LSCM. However, improper constraints will generate bad mapping, and how to assign proper prior 2D values to these six landmarks is not easy. We firstly map one reference surface into 2D domain using LSCM by pinning the two landmarks of eyes, where the two landmarks are assigned  $(-1, 0)$  and  $(1, 0)$  respectively. The 2D domain of the reference surface is shown in Figure.1b, where the 2D locations of the six landmarks are also labeled in Figure.1b. Table.1 shows the 2D values of the six landmarks. Then any craniofacial surface can be mapped to a 2D domain by LSCM with pinning the six landmarks according to the obtained 2D values, Fig.2 shows another craniofacial surface and the 2D domain mapped by LSCM with pinning the six landmarks.

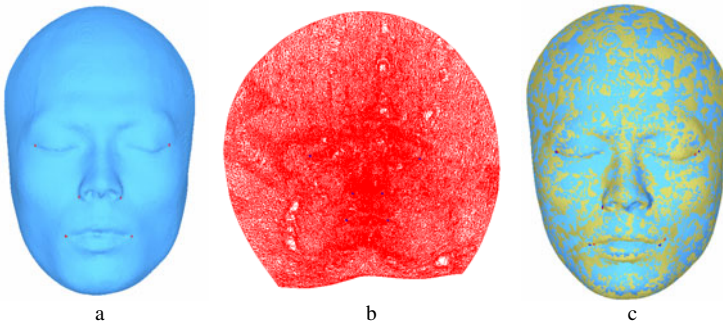
Since the LSCM subjects to the constraint that the mapping is linear on each triangle of the mesh surface, the iso-parametric mesh of a craniofacial surface can be obtained by linear interpolation, and a 3D point correspondence is formed according to the iso-parametric mesh. Fig.2c shows the registration result between two craniofacial surfaces in Fig.1 and Fig.2.



**Fig. 1.** a)The unified coordinate system and the six landmarks, b)The 2D domain by LSCM

**Table 1.** The 2D values of the six landmarks

2D values of landmarks	1	2	3	4	5	6
u	-1.0	1.0	-0.29	0.28	-0.37	0.43
v	0.0	0.0	-0.68	-0.71	-1.13	-1.11



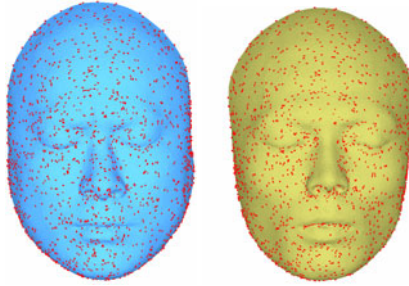
**Fig. 2.** a) A craniofacial surface, b) The 2D domain by LSCM with pinning six feature points, c) The registration result

## 4 Result

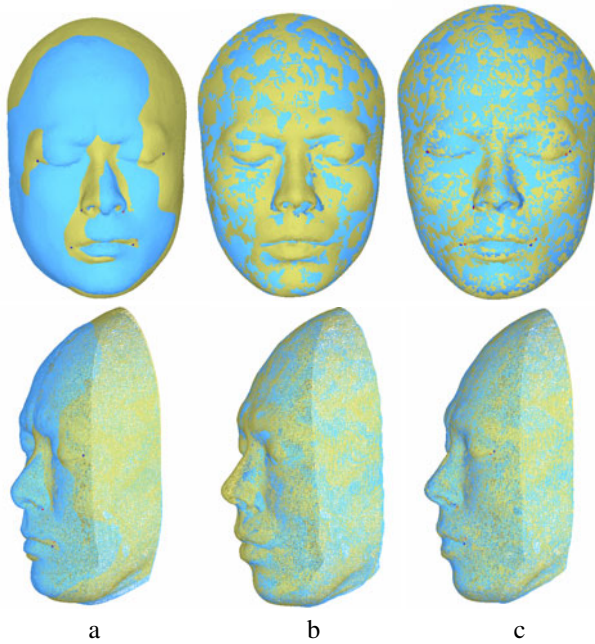
In this section, we do some experiments to evaluate our algorithm. We also use the two craniofacial surfaces in Section 3, which are acquired by reconstructing from real CT images. The craniofacial surface in Fig. 1 has 28610 vertices and 55917 triangles, and the one in Fig. 2 has 28421 points and 55296 triangles. It obviously has much differences between them in shape and geometry feature. In experiments, since the back of a head has less recognition features, a vertical cut is performed manually to retain the face part before the ears. The platform of the experiments is Dell precision 390 workstation with Intel(R) Core(TM) 2.39GHz and 3GB memory. The registration result in Fig. 2c shows that the faces can match well with each other, which indicates the proposed method is effective. In following, we compare our method with ICP and TPS.

Since the point number of the models is large, it will take a long time for registration using all points. So, we do random sampling to select points for the ICP registration, which distribute uniformly on the surfaces, and the distances among these points are forced being greater than a given threshold. In the experiment, 2500 points are select on each face. Fig. 3 shows the selected points, including most of characters on the reference face. The 2500 points are used to compute the transform between two surfaces in ICP. The result using ICP is shown in Fig. 4a. Using these 2500 point correspondences obtained by ICP as controlling points, the reference craniofacial is deformed to target craniofacial by TPS, and the result using TPS is shown in Fig. 4b. The result using the proposed method is also shown in Fig. 4c. To make the result clearer, frontal faces are displayed with solid, and side faces use triangles. We can see that the quality of ICP registration is the worst, and most of the points match with each other perfectly by the proposed method, which is better than the TPS method.

Furthermore, in order to evaluate the three methods quantitatively, we compute the mean distance between dense correspondences for ICP and TPS respectively, and for the proposed method, the target surface is deformed into the reference surface using TPS algorithm with dense correspondences being control points, and then the distance sum between two correspondence point sets are computed, and vice versa. The mean distances for the three methods are shown in Table 2. From the table, we can see the non-rigid method based on LSCM performs much better than ICP and TPS.



**Fig. 3.** The random sampled points on the reference face and the target face for TPS



**Fig. 4.** a)The alignment of the two faces after the ICP transformation, b) The alignment of the two faces after the TPS transformation, c)The alignment of the two faces after our method

**Table 2.** The result of the criteria of ICP and Our method

Methods	ICP	TPS	Our method
Mean distance(mm)	3.01	0.71	0.49

## 5 Conclusions

Non-rigid registration of 3D data set is an important and challenging issue for many applications such as atlas matching; shape retrieval, 3D object recognition etc. This

paper proposes a non-rigid registration method for craniofacial surfaces based on LSCM parameterization. Since LSCM can map the corresponding feature points on two 3D surfaces into the same 2D location, each craniofacial surface can be mapped into a nearly equal 2D domain by using LSCM. Then from 2D correspondences, we can obtain the 3D point matching. The proposed method is compared with ICP and a TPS based method. The comparison shows that the proposed approach is more accurate and effective. The future work we will do is to refine the registration result in 2D domain.

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