A Global Optimal Algorithm for Camera Calibration with One-Dimensional Objects

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Abstract. The emergent one-dimensional (1D) calibration is very suitable for multi-camera calibration. However its accuracy is not satisfactory. Conventional optimal algorithms, such as bundle adjustment, do not perform well for the non-convex optimization of 1D calibration. In this paper, a practical optimal algorithm for camera calibration with 1D objects using branch and bound framework is presented. To obtain the optimal solution which can provide ε-optimality, tight convex relaxations of the objective functions are constructed and minimized in a branch and bound optimization framework. Experiments prove the validity of the proposed method.

1 Introduction

Camera calibration, which is to determine camera parameters, is a necessary step to extract three-dimensional(3D) metric information from two-dimensional (2D) images. According to the dimension of the calibration objects, the camera calibration techniques can be roughly classified into four categories: 3D object based calibration[1,2], 2D plane based calibration[3,4], one-dimensional (1D) object based calibration[5,6,7,8,9] and zero-dimensional (0D) approach or self-calibration [10,11,12]. Much work has been done expect for 1D calibration. Camera calibration using 1D object was proposed by Zhang in [5]. Hammarstedt et.al. analyze the critical configuration of 1D calibration and provide simplified closed-form solutions in Zhang's setup [6]. Wu et.al. prove that the rotating 1D object used in [5] is essentially equivalent to a familiar 2D planar object, and such equivalence still holds when the 1D object undergoes a planar motion rather than the rotation around a fixed point [7]. Wang et.al. present an 1D calibration algorithm for multi-camera in which the 1D object can undergo general motions without special constraints [8]. The emergent 1D calibration has the following advantages: (1). The 1D calibration object is easy to construct. In practice, it can be constructed by marking three points on a stick. (2). The 1D calibration is very suitable for multi-camera calibration, in which the whole 1D calibration object can be simultaneously observed by all cameras, whereas 3D or 2D calibration objects hard satisfy.

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The deficiency of 1D calibration is that its accuracy is inferior to that of 3D and 2D calibration. One reason is that the 1D calibration uses the co-linearity and distance of markers on the 1D segment to compute the intrinsic matrix, which means only one dimension of 3D metric information is used. Some researchers propose some methods and resort to nonlinear optimization to refine 1D calibration results [9]. However these methods are not guaranteed to perform well since the 1D calibration optimization is a non-convex nonlinear optimization problem.

In this paper, we present a practical algorithm for 1D calibration of multi-camera system with the theoretical guarantees of global optimality. Given images captured by a camera, firstly image points of marks on 1D calibration objects are extracted with image processing techniques and initial calibration results are obtained with original 1D calibration algorithm. Normalize the initial calibration results and use them to give bounds on variables. Secondly construct tight convex relaxations of the objective functions and minimize the objective functions in a branch and bound optimization framework. Then normalized camera parameters can be obtained by the decomposition of the dual image of the absolute conic. Finally, the camera parameters can be obtained by de-normalization.

The paper is organized as follows. Some preliminaries are introduced in Section 2. In Section 3 the proposed global optimal calibration algorithm for a camera is presented. Then calibration experiments are reported in Section 4. Section 5 are some concluding remarks.

2 Preliminary

2.1 Camera Model

In this paper, a 3D point is denoted by $\mathbf{M} = [X, Y, Z]^T$, and a 2D image point by $\mathbf{m} = [u, v]^T$. The corresponding homogeneous vector is denoted respectively as $\tilde{\mathbf{M}} = [X, Y, Z, 1]^T$, $\tilde{\mathbf{m}} = [u, v, 1]^T$. With the pinhole camera model, the relationship between a 2D image point and $\tilde{\mathbf{m}}$ its corresponding 3D point$\tilde{\mathbf{M}}$ is

$$d\tilde{\mathbf{m}} = \mathbf{K}[\mathbf{R}|\mathbf{t}]\tilde{\mathbf{M}}, \quad \mathbf{K} = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where d is a scale factor (the projection depth of 3D point $\tilde{\mathbf{M}}$), $\mathbf{K}$ is called the camera intrinsic matrix, α, β denotes the scale factors along the u and v image axes respectively, γ the skew, and $[u_0, v_0]$ the principal point, $[\mathbf{R}|\mathbf{t}]$ called the extrinsic matrix, $\mathbf{R}$ is the rotation matrix and $\mathbf{t}$ translation vector which relates the world coordinate system to the camera coordinate system.

2.2 1D Calibration Object

In this paper, the minimal configuration of 1D calibration object which only consists of three collinear points is considered. Assume three points of 1D calibration object are $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, where $||\mathbf{A} - \mathbf{C}|| = L_1$, $||\mathbf{B} - \mathbf{C}|| = L_2$ (see

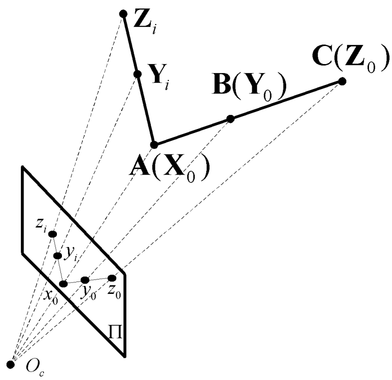


Fig. 1. Multi-camera system calibration with a 1D object

Fig.1). For the convenience of statement, the 1D calibration object is also called the line-segment (**ACB**) and denoted as $\mathbf{L_{ACB}}$.

2.3 Branch and Bound Theory

Branch and bound algorithms are methods for global optimization of non-convex problems [14]. They maintain a provable upper or(and) lower bound on the objective function and terminate with a ϵ-suboptimal for arbitrarily small ϵ.

For a multivariate, non-convex, scalar-valued objective function $f : \mathbf{R}^m \rightarrow \mathbf{R}$, to seek a global minimum over an m-dimensional rectangle Q_{init}, firstly introduce an auxiliary function $\Phi_{1b}(Q)$ which satisfies the following conditions for every region $Q \subset Q_{init}$.

(**R**1). $\Phi_{1b}(Q) \leq f_{min}(Q)$. Thus, the function $\Phi_{1b}(Q)$ compute a lower bound on $f_{min}(Q)$ of $f(x)$ for all $x \in Q$.

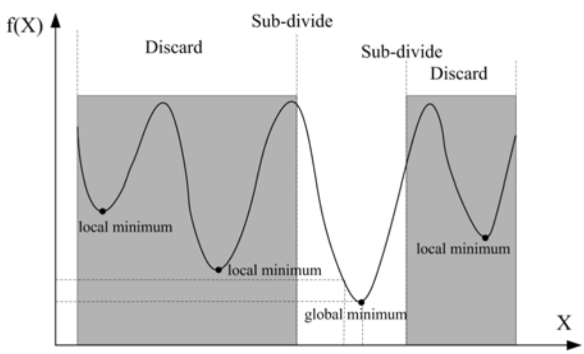


Fig. 2. The mechanism of global minimization with branch and bound algorithm for a univariate function

(**R**2). $\forall\epsilon > 0, \exists\delta > 0$ such that $\forall Q \subset Q_{init}, \mid Q \mid \leq \delta \Rightarrow f(x) - \Phi_{1b}(x) \leq \epsilon$, i.e., as $\mid Q \mid$, the length of the longest edge of a rectangle Q, goes to zero, the relaxation gap $f(x) - \Phi_{1b}(x)$ uniformly converges to zero.

Bounding refers to computing the value of $\Phi_{1b}(Q)$, while branching refers to choosing and subdividing a rectangle Q. Fig.2 illustrates the basic mechanism of a branch and bound routine for the case of a univariate function. Take the rectangle with the smallest minimum of Φ_{1b} as the most promising to contain the global minimum and pick it for refinement. Then subdivide the picked rectangle into k rectangles along the largest dimension. Repeat this process iteratively until the optimal solution is reached. The key problem of branch and bound methods is designing a proper bounding function $\Phi_{1b}(Q)$, which should be easy to estimate.

3 Global Optimal 1D Calibration Algorithm

The goal of camera 1D calibration is to compute the camera intrinsic matrix metric $\mathbf{K}$, with image points $\{\mathbf{a}_{ij}, \mathbf{b}_{ij}, \mathbf{c}_{ij} | i = 0, 1, 2, ..., m, j = 1, 2, ..., n\}$ of $\mathbf{L_{ACB}}$ under the jth rotation around the fixed point $\mathbf{A}$ (see Fig. 1). A camera system can be linearly calibrated with 1D calibration objects undertaken rotations [5]. Firstly express 3D coordinates with image points of markers $\mathbf{A}_i$, $\mathbf{B}_i$ and $\mathbf{C}_i$ on 1D objects. Secondly construct equations with estimated 3D coordinates corresponding to image points and geometry constraints of 1D calibration objects. However the linear 1D calibration algorithm is susceptible to fail due to image noise. On the other hand, the conventional bundle adjustment optimization algorithm possibly converges to a local rather than a global minimum. So a 1D global optimal calibration algorithm using branch and bound framework is proposed to reduce the effect of image noises and improve the calibration accuracy.

3.1 Traditional Solution

As shown in Fig. 1, let the line segment **AC** rotates N times around the fixed point **A**. Denote the coordinate of **A** by $\mathbf{X}_0$ under the camera coordinate system, and coordinates of **B** and **C** under the i^{th} rotation by $\mathbf{Y}_i$ and $\mathbf{Z}_i$, respectively, $(i = 0, 1, 2, \ldots, N)$, where $\mathbf{Y}_0$, $\mathbf{Z}_0$ is the coordinates of **B** and **C** at the initial position. Then, we have

$$\mathbf{Y}_i = \lambda_1\mathbf{X}_0 + \lambda_2\mathbf{Z}_i, \tag{2}$$

where $\lambda_1 = (L_1 - L_2)/L_1$ and $\lambda_2 = L_2/L_1$. Denote the image point of $\mathbf{X}_0$, $\mathbf{Y}_i$, $\mathbf{Z}_i$ by $\mathbf{x}_0$, $\mathbf{y}_i$, $\mathbf{z}_i$. With equation (1) under the camera coordinate system, we have

$$d_0^x\mathbf{x}_0 = \mathbf{K}\mathbf{X}_0, \quad d_i^y\mathbf{y}_i = \mathbf{K}\mathbf{Y}_i, \quad d_i^z\mathbf{z}_i = \mathbf{K}\mathbf{Z}_i. \tag{3}$$

Substitute equation (2) into $d_i^y\mathbf{y}_i = \mathbf{K}\mathbf{Y}_i$, then we have

$$d_i^y\mathbf{y}_i = d_0^x\lambda_1\mathbf{x}_0 + d_i^z\lambda_2\mathbf{z}_i, \quad i = 0, 1, 2, \cdots, N. \tag{4}$$

By some simple vector manipulation, we obtain the following constraints

$$d_i^y = d_0^x d'^y_i, \quad d_i^z = d_0^x d'^z_i, \tag{5}$$

where d'^y_i, d'^z_i is the relative depth of $\mathbf{Y}_i$, $\mathbf{Z}_i$ to $\mathbf{X}_0$:

$$d'^y_i = \frac{\lambda_1 (\mathbf{y}_i \times \mathbf{z}_i)^T (\mathbf{x}_0 \times \mathbf{z}_i)}{\lambda_2 (\mathbf{y}_i \times \mathbf{z}_i)^T (\mathbf{y}_i \times \mathbf{z}_i)}, \tag{6}$$

$$d'^z_i = \frac{\lambda_1 (\mathbf{x}_0 \times \mathbf{y}_i)^T (\mathbf{z}_i \times \mathbf{y}_i)}{\lambda_2 (\mathbf{z}_i \times \mathbf{y}_i)^T (\mathbf{z}_i \times \mathbf{y}_i)}. \tag{7}$$

With the geometry information of the 1D object, for each i ($i = 1, 2, \ldots, N$) we have the following constraint

$$L = \|\mathbf{Z}_i - \mathbf{X}_0\| = \|\mathbf{K}^{-1}(d_i^z \mathbf{z}_i - d_0^x \mathbf{x}_0)\|. \tag{8}$$

It can also be expressed as the following equation system,

$$(d'^z_i \mathbf{z}_i - \mathbf{x}_0)^T \varpi (d'^z_i \mathbf{z}_i - \mathbf{x}_0) = L^2 \quad i = 0, \cdots, N. \tag{9}$$

where $\varpi = (d_0^x)^2 \mathbf{K}^{-T}\mathbf{K}^{-1}$. Considering that

$$\begin{aligned} &\mathbf{m}_i^T \varpi \mathbf{m}_i = \mathbf{M}_i \Theta \\ &= [\mathbf{m}_{i1}^2 \quad 2\mathbf{m}_{i1}\mathbf{m}_{i2} \quad 2\mathbf{m}_{i1}\mathbf{m}_{i3} \quad \mathbf{m}_{i2}^2 \quad 2\mathbf{m}_{i2}\mathbf{m}_{i3} \quad \mathbf{m}_{i3}^2]\Theta \end{aligned} \tag{10}$$

with

$$\Theta = [\varpi_{11} \quad \varpi_{12} \quad \varpi_{13} \quad \varpi_{22} \quad \varpi_{23} \quad \varpi_{33}]^T \tag{11}$$

and $\mathbf{m}_i = (d'^z_i \mathbf{z}_i - \mathbf{x}_0)$, (9) can be rewritten as

$$\mathbf{M}_i \Theta = L^2 \quad i = 0, \cdots, N. \tag{12}$$

With $\mathbf{M} = [\mathbf{M}_0^T, \mathbf{M}_1^T, \cdots, \mathbf{M}_N^T]^T$, $\mathbf{L} = [L^2, L^2, \cdots, L^2]^T$, we can obtain Θ with the total (orthogonal) least squares method by sloving

$$\mathbf{M}\Theta = \mathbf{L}. \tag{13}$$

Once Θ is obtained with equation (11) we can compute ϖ, then obtain $d_0^x \mathbf{K}^{-1}$ by the Cholesky decomposition. Finally the inner matrix $\mathbf{K}$ is obtained.

3.2 Global Optimization with Branch and Bound

The above solution is obtained by minimizing algebraic distances which is susceptible to image noise. The conventional way to overcome this deficiency is resort to bundle adjustment, however it often traps in local optimal solution. We instead consider branch and bound algorithm outlined in this section to refine 1D calibration results.

In the linear 1D calibration, the main problem is to estimate the image of absolute quadratic (IAC)

$$\varpi_0 = \mathbf{K}^{-T}\mathbf{K}^{-1} = \frac{1}{\alpha^2\beta^2}\begin{bmatrix} \beta^2 & -\gamma\beta & -u_0\beta^2+\gamma v_0\beta \\ -\gamma\beta & \alpha^2+\gamma^2 & \gamma u_0\alpha - v_0\alpha^2-\gamma^2 v_0 \\ -u_0\beta^2+\gamma v_0\beta & \gamma u_0\alpha - v_0\alpha^2-\gamma^2 v_0 & \alpha^2\beta^2+\alpha^2 {v_0}^2+(\beta u_0-\gamma v_0)^2 \end{bmatrix} \tag{14}$$

It can be posed as the following least squares problem:

$$\min_{\varpi_0}\sum_i (L_1^2 - (\mathbf{Z}_i-\mathbf{X}_0)^T\varpi_0(\mathbf{Z}_i-\mathbf{X}_0))^2 \tag{15}$$

Considering equation (8), equation (15) can be rewritten as

$$\min_{\hat{\varpi}_0}\sum_i (L_1^2 - k({d'}_i^z\mathbf{z}_i-\mathbf{x}_0)^T\hat{\varpi}_0({d'}_i^z\mathbf{z}_i-\mathbf{x}_0))^2 \tag{16}$$

where $k = (d_0^x)^2/\alpha^2/\beta^2$,

$$\hat{\varpi}_0 = \begin{bmatrix} \beta^2 & -\gamma\beta & -u_0\beta^2+\gamma v_0\beta \\ -\gamma\beta & \alpha^2+\gamma^2 & \gamma u_0\alpha - v_0\alpha^2-\gamma^2 v_0 \\ -u_0\beta^2+\gamma v_0\beta & \gamma u_0\alpha - v_0\alpha^2-\gamma^2 v_0 & \alpha^2\beta^2+\alpha^2 {v_0}^2+(\beta u_0-\gamma v_0)^2 \end{bmatrix} \tag{17}$$

It is solved linearly by ignoring the positive semi-definiteness requirement on the IAC. For the cases where the linear solution does not yield a positive semi-definite IAC, the closest positive semi-definite matrix is estimated as a post-processing step by dropping the negative eigenvalues.

We take into account the positive semi-definiteness of the IAC in the optimization algorithm and search for the optimal IAC in the feasible domain. To this end, we pose the minimization problem:

$$\begin{aligned} &\min_{\hat{\varpi}_0}\sum_i (L_1^2 - k({d'}_i^z\mathbf{z}_i-\mathbf{x}_0)^T\hat{\varpi}_0({d'}_i^z\mathbf{z}_i-\mathbf{x}_0))^2 \\ &subject \quad to \quad \hat{\varpi}_0 \succeq 0 \\ &\qquad\qquad\quad\;\; \hat{\varpi}_0 \in \aleph \end{aligned}, \tag{18}$$

where $\hat{\varpi}_0 \succeq 0$ describes the positive semi-definiteness, $\aleph$ is some initial convex region which can be given by estimated camera matrix $\bar{\mathbf{K}}$ with original linear 1D calibration. To improve the numerical conditioning of this method, the matrix

$$\mathbf{V} = \frac{1}{2}\begin{bmatrix} \sqrt{w^2+h^2} & 0 & w \\ 0 & \sqrt{w^2+h^2} & h \\ 0 & 0 & 2 \end{bmatrix} \tag{19}$$

is used to normalize $\hat{\mathbf{K}}$ by $\mathbf{K} = \mathbf{V}^{-1}\bar{\mathbf{K}}$, where w and h are respectively the width and height of images.

Assume that the domain $\aleph$ is given in the form of bounds $[l_{jk}, r_{jk}]$ on the five unknown symmetric entries $\hat{\varpi}_{0jk}$ of $\hat{\varpi}_0$. Then the above optimization problem can be stated as

$$\begin{array}{ll} \min\limits_{\hat{\varpi}_0} \sum\limits_i (L_1^2 - k(d'^z_i \mathbf{z}_i - \mathbf{x}_0)^T \hat{\varpi}_0 (d'^z_i \mathbf{z}_i - \mathbf{x}_0))^2 & \\ subject \quad to \quad \hat{\varpi}_0 \succeq 0 & . \\ \qquad\qquad l_{jk} \le \hat{\varpi}_{0jk} \le r_{jk} & \end{array} \tag{20}$$

Then introduce a new matrix $\nu = k\hat{\varpi}_0$. The objective function of the optimization problem (20) can be expressed as $\min\limits_{\hat{\varpi}_0} \sum\limits_i (L_1^2 - (d'^z_i \mathbf{z}_i - \mathbf{x}_0)^T \nu (d'^z_i \mathbf{z}_i - \mathbf{x}_0))^2$. With the bilinear equality constraint $(d'^z_i \mathbf{z}_i - \mathbf{x}_0)^T \nu (d'^z_i \mathbf{z}_i - \mathbf{x}_0) = L_1^2$, the bounds of variable of k can be given by simply inverting the bounds on $\hat{\varpi}_0$. The convex relaxation of the above optimization problem can be stated as

$$\begin{array}{ll} \min\limits_{\hat{\varpi}_0, k, \nu} \sum\limits_i (L_1^2 - (d'^z_i \mathbf{z}_i - \mathbf{x}_0)^T \nu (d'^z_i \mathbf{z}_i - \mathbf{x}_0))^2 & \\ subject \quad to \quad \nu_{jk} \le k_u \hat{\varpi}_{jk} + l_{jk} k - k_u l_{jk} & \\ \qquad\qquad \nu_{jk} \le k_l \hat{\varpi}_{jk} + u_{jk} k - k_l u_{jk} & \\ \qquad\qquad \nu_{jk} \ge k_l \hat{\varpi}_{jk} + l_{jk} k - k_l l_{jk} & . \\ \qquad\qquad \nu_{jk} \ge k_u \hat{\varpi}_{jk} + u_{jk} k - k_u u_{jk} & \\ \qquad\qquad \hat{\varpi}_0 \succeq 0 & \\ \qquad\qquad l_{jk} \le \hat{\varpi}_{0jk} \le r_{jk} & \\ \qquad\qquad k_l \le k \le k_r & \end{array} \tag{21}$$

The objective function of the optimization problem (21) is convex quadratic. The constraint set includes linear inequalities and a positive semi-definiteness constraint. Such problem can be efficiently solved using interior point methods which can obtain their global optimum and have a number of software packages for implementation. In our implementation, SeDuMi [19] is used.

Once $\hat{\varpi}_0$ is obtained, the normalized camera matrix $\hat{\mathbf{K}}$ can be computed by equation (17). Finally the camera matrix can be obtained by de-normalization with $\mathbf{K} = \mathbf{V}\hat{\mathbf{K}}$.

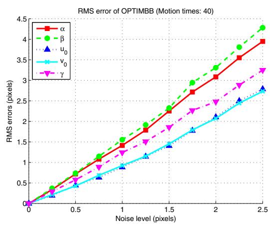
4 Experiments

To validate the proposed algorithm, computer simulated data and real image data experiments have been performed. Some results are reported in this section.

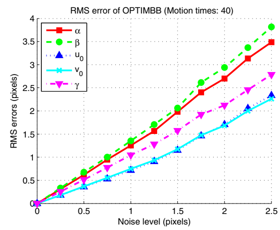
4.1 Simulated Data

In this experiment, the intrinsic parameters of the simulated camera are $\alpha = 500$, $\beta = 500$, $u_0 = 512$, $v_0 = 512$ and $\gamma = 0.0$. The distance between two markers **A** and **C** on the 1D object is $L_1 = 20$, the distance between two markers **B** and **C** is $L_2 = 10$. The initial positions of three markers are $\mathbf{A} = (0, 0, 25)^T$, $\mathbf{B} = (5\sqrt{2}, 0, 25 + 5\sqrt{2})^T$, $\mathbf{C} = (10\sqrt{2}, 0, 25 + 10\sqrt{2})^T$ respectively. Rotate the 1D object N times with randomly selected angle and axis around fixed point

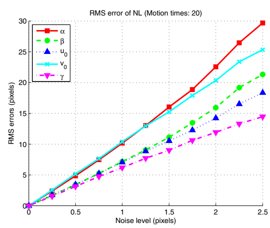
A, then $2N + 3$ space points can be obtained which include three points on the initial position. Assumed that the camera coordinate system coincides with the world coordinate system, image points of these markers can be obtained by equation (1).



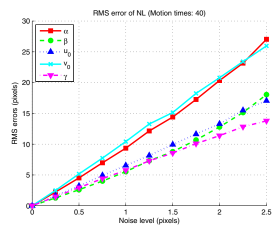
(a) OPTIMBB with 20 rotations



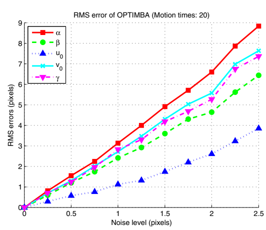
(b) OPTIMBB with 40 rotations



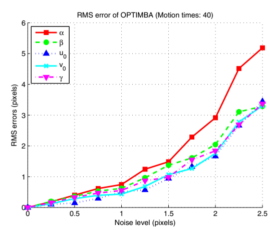
(c) NL with 20 rotations



(d) NL with 40 rotations



(e) OPTIMBA with 20 rotations



(f) OPTIMBA with 40 rotations

Fig. 3. Experimental results of the proposed optimal, NL and conventional bundle adjustment 1D calibration with 20 and 40 rotations of 1D object

Add Gaussian noise with mean 0 and standard derivations σ to image points. The noise level σ is varied from 0 to 2.5 pixels with a step of 0.25 pixels. Here we use three methods, the proposed optimal algorithm based on branch and bound framework (OPTIMBB), the normalized linear algorithm (NL) and the conventional optimal algorithm based on bundle adjustment (OPTIMBA), to calibrate the camera. In order to provide more statistically meaningful results, for each noise level we perform 1000 trials. The RMS errors of estimated inner parameters relative to the ground truth are computed to evaluate the performance. Results are shown in Figure 3.

We can see in Figure 3, errors of the proposed OPTIMBB algorithm are less than that of the normalized linear algorithm and the conventional OPTIMBA algorithm. Errors of the proposed OPTIMBB algorithm increase almost linearly with the noise level while the conventional OPTIMBA algorithm does not linearly increase with noise level due to trapping into local minima.

4.2 Real Images Data

For the experiment with real data, distances between two adjacent markers on the 1D object are 15cm. Rotate the 1D object 15 times around one fixed marker. Then use measurements of makers' image to calibrate the camera with the OPTIMBB, NL and OPTIMBA 1D calibration. We also perform 2D calibration and take its results as the ground truth. Calibration results are shown in Table 1. We can see that the performance of the proposed OPTIMBB algorithm is better than the other algorithm.

Table 1. Experimental results with real image data

	α	β	γ	u_0	v_0
OPTIMBB	2097	2080	-0.4	764	525
NL	1923	1938	-1.8	806	552
OPTIMBA	2034	1988	1.9	762	537
2D	2075	2078	0	757	514

5 Conclusions

In this paper, we have investigated to improve the accuracy of 1D calibration. A global optimal algorithm based on branch and bound framework is proposed. By constructing tight convex relaxations of the objective functions and minimizing in a branch and bound optimization framework, the optimal solution, which can provide ε-optimality, is obtained. Experiments prove the validity of the proposed method.

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