

Introduction to Decidability of Higher-Order Matching

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Abstract. Higher-order unification is the problem given an equation $t = u$ containing free variables is there a solution substitution θ such that $t\theta$ and $u\theta$ have the same normal form? The terms t and u are from the simply typed lambda calculus and the same normal form is with respect to $\beta\eta$ -equivalence. Higher-order matching is the particular instance when the term u is closed; can t be pattern matched to u ? Although higher-order unification is undecidable, higher-order matching was conjectured to be decidable by Huet [2]. Decidability was shown in [7] via a game-theoretic analysis of β -reduction when component terms are in η -long normal form.

In the talk we outline the proof of decidability. Besides the use of games to understand β -reduction, we also emphasize how tree automata can recognize terms of simply typed lambda calculus as developed in [1, 3–6].

References

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