Deriving Sparse Coefficients of Wavelet Pyramid Taking Clues from Hough Transform

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Abstract. Many applications like image compression requires sparse representation of image. To represent the image by sparse coefficients of transform is an old age technique. Still research is going on for better sparse representation of an image. A very recent technique is based on learning the basis for getting sparse coefficients. But learned basis are not guaranteed to span l_2 space, which is required for reconstruction. In this paper we are presenting a new technique to choose steerable basis of wavelet pyramid which gives sparse coefficients and better reconstruction. Here selection of steerable basis is based on clues from Hough transform.

1 Introduction

A sparse signal has a large portion of its energy contained in a small number of coefficients. One obvious reason why we desire to have sparse representations of signals is, more sparsely one can represent a signal, more efficiently one can compress the signal. Moreover, sparse representations allow us to do much more than just compression. For example, sparse representations allow us to violate the Nyquist sampling theorem without loss of fidelity [1]. Sparse representations leads to solutions to previously insolvable blind source separation problems [2]. It has applications including speech processing, biomedical signal processing, financial time-series analysis, and communications [3]. Fourier Transform,Discrete Cosine Transform,Wavelet Transform are a few examples of choosing suitable basis for sparse transformation. This motivates researchers to explore new technique for finding suitable basis that leads to maximum sparsity as well as information.

Recently, machine learning based technique for finding suitable sparse basis coefficients is gaining attention [4]. Here one takes Cauchy distribution of the coefficients as a prior, then learn image basis using MAP with minimum residual error as constraint. But learned basis through this mechanism do not guarantee about spanning of l_2 space. Note that, without spanning of l_2 space, the original input signal could not be reconstructed from the basis. Reconstruction drawback of learned basis motives the current research. We are proposing new basis which are tight frames and derived without any learning. In the proposal, steerable pyramid wavelet transform is used. It is noted that energy distribution among

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the basis is depending on features present in the image. One of these features is orientation of edges. Hough transform provides us inference about orientation of edges. Using this inference we choose suitable steerable basis such that image energy is mostly confined in one of the basis. This leads to coefficients of basis close to zero. The major contribution of the present submission includes the following,

1. A new set of steerable basis leads to sparse representation of the image.
2. The basis are constructed based on edge orientation present in the image and hence it is adaptive in the sense of image requirement.
3. The reconstruction of input image is guaranteed.
4. The basis selection is cost effective as it requires no learning.

Note that for rest of the manuscript the steerable basis when implemented in the images will be termed as steerable band pass filters.

The organization of the article is as follows. In Section 2, we have discussed learning based method [4]. Section 3 deals with the current proposal followed by implementation and results in Section 4.

2 Learning Based Method for Sparse Representation

The detail description of learning based sparse representation is available in [4]. Only the salient features of the same is discussed here to make the readers appreciate the current proposal which does not require any learning. With a particular choice of basis coefficients a, one can express image $I(\boldsymbol{x})$ with an assumption that the noise present is Guassian,white and additive.

$$I\left(\boldsymbol{x}\right) = \sum_{i} a_i \phi_i \left(\boldsymbol{x}\right) + v\left(\boldsymbol{x}\right) \tag{1}$$

Where $\boldsymbol{x}$ denotes a discrete spatial position, ϕ_i are basis functions, v is noise or uncertainty. Hence for given set of a_i's, we need to find ϕ_i so that the noise present is as less as possible. ϕ_i's are learned from the model given the data.

Image I as presented in equation (1) can be represented canonically as,

$$I = Ga + \nu \tag{2}$$

where vector a is coefficient for all scales, positions and levels, G is basis functions for coefficient vector a. Probability of generating Image I, given coefficients a is assumed to be Gaussian,

Under this set up, and assuming a Cauchy prior coefficients of a given image are determined by MAP estimate, The learning time is often high and this leads to a disadvantage of this mechanism . Another disadvantage may come from the fact that these basis are statistically independent, however not guaranteed to span l_2 which is required for reconstruction.

All learned basis are directional band pass filters which lead to a new direction. Taking clue from this, we propose a new approach to construct basis inferred from Hough Transform. This approach removes learning part of the basis.

3 Proposed Methodology Inferred from Hough Transform

Here we are proposing a new approach to get sparse distribution of coefficients by suitably selecting basis. We have used pyramid wavelet transform with basis as steerable functions. As discussed in [5], each steerable basis functions are rotated copy of each other. We can design basis of any orientation direction by interpolating these basis functions. Here we have used steerable filters or basis and wavelet pyramid structure .

General equation for pyramid structure as described in [6] is,

$$\hat{X}(\overrightarrow{w}) = \left\{ H_0(\overrightarrow{w})|^2 + |L_0(\overrightarrow{w})|^2 \left(|L_1(\overrightarrow{w})|^2 + \sum^{n} |B_k(\overrightarrow{w})|^2 \right) \right\} X(\overrightarrow{w}) + \text{aliasing terms} \quad (3)$$

Where X is input signal, $\hat{X}$ is output of the same when treated with filters, H_0 is first level high pass filter and L_0 is first level low pass filter. After that, low pass filter L_1 and steerable band pass filters $B_i's$ are repeated recursively on 2 by 2 decimation. The reconstruction is possible if the following constraints such as unitary response, recursion relation and aliasing cancellation. There is one more constraint that B_i's need to satisfy is angular constraint.

The spectrum of wavelet pyramid structure which is divided into three types output of High Pass,Low Pass and Band Pass filters. Note that the band pass filters are directional oriented and are called basis. These filters are steerable. The main emphasis of the current work to select proper band pass filter which generate coefficients as sparse as possible. The direction orientations are chosen from the Hough Space [7] generated out of Hough Transformation. Hough transform is a mapping of image into Hough space. As described in [7], Hough space is made up of two parameters, distance from origin (ρ) and and angle with x-axis (θ) (positive in anticlockwise direction). Any point in image plane (x, y) can be represented as following in the Hough Space ,

$$\rho = x\cos(\theta) + y\sin(\theta) \quad (4)$$

Note that infinite numbers of lines can pass through a single point of image plane. All lines passing through the point (x, y) should satisfy equation (4). Again for each point (x, y) in the image plane, a sinusoidal is generated in Hough space by varying ρ and θ. The lines passing through this point (x, y) in image plane increases co-occurrence of sinusoidal waves in Hough space. So, each line generates peak in hough space at a particular (ρ, θ). In other words each line in image can be represented as unique couple (ρ, θ) for $\theta\epsilon\,[0, \pi]$ and $\rho\epsilon R$ in Hough space. Finally, we conclude that hough transform is line to point conversion. Natural images have structures of edges and lines. If band pass filters (steerable basis) are getting tuned with lines, one can get sparse distribution for coefficients of band pass filters. We propose to tune orientation of steerable basis taking clue from Hough transform. Peaks in Hough space correspond to the lines or edges in image. Note that peaks at 0 and ±90 degrees are not considered as these angles do not bring any information. First, find the highest peak in a particular angle in Hough space. This angle indicates orientation direction of the most of the edges or the lines in the image. Hence this angle could be used to select the direction of steerable band pass filters. Finally, if required, from the Hough space some angles (few top peaks) that indicates orientation direction of edges present in the image are selected and used to choose the direction of steerable band pass filters.

In steerable pyramid structure, we choose first steerable basis in tune with the highest peak angle. This implies first steerable basis which is direction derivative function, has direction perpendicular to the orientation of highest peak angle in Hough space. First basis(band pass filter), thus contains maximum energy. So, other basis functions will have comparatively less energy leading toward sparse distribution of coefficients.

4 Exprimetal Results

Experiments are conducted for a set of natural images taken from [8]. Note that same set of images are used for learning the basis as presented in [4].

Figure 2 shows trained basis as described in Section 2, These are learned for image shown in Figure 1. Starting with real random basis, tunning has been carried out through training process (Section 2) for one and half hour in a pentium-IV,1.7 GHz CPU machine. Note that tunning of basis may improve if

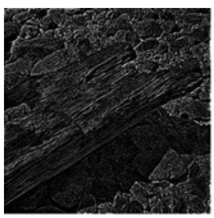

Fig. 1. Original Natural Image

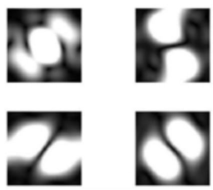

Fig. 2. Trained basis filters starting from real random basis. Training is carried out for one and half hour.

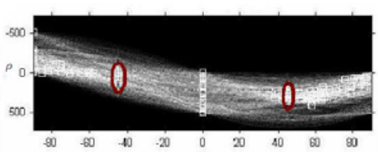

Fig. 3. Hough transform of the image shown in Figure 1. The circle indicates the maxima at Hough space.

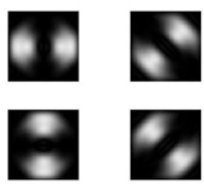

Fig. 4. Directional band pass filter whose orientation direction is inferred from Hough transform (Figure 3).

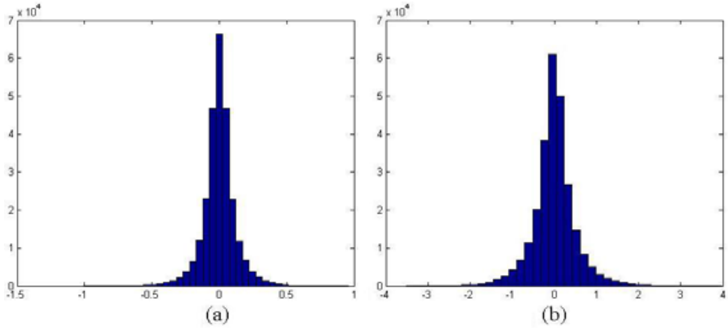

Fig. 5. Histogram of one level coefficients: (a) tunned band pass filters, (b) trained filters inferred from Hough transform

Table 1. Comparison of Filter Coefficients Distribution

	Trained Filter	Inferred Band Pass Filter
Mean	2.4835e-006	-1.3159e-018
Variance	0.2568	0.0147

Table 2. Comparison of quality measures for reconstruction with steerable basis (Inferred Band Pass Filter) and trained basis (filter)

	MSE	SNR	PSNR
Inferred Band Pass Filter	0.0546	2.5706	23.6916
Trained Filter	0.2895	-4.6715	16.4495

one extends the time of learning. However we felt that would be too costly as far as time is concern. So, we stop after one and half hour of learning. Trained basis (Figure2) are like directional derivatives where directions are perpendicular to orientation corresponding to peak angles of hough space as shown in Figure 3. This justifies our claim that orientation direction of steerable basis could be inferred from Hough Transform. At the same time these basis will generate sparse distribution of coefficients. Proposed directional band pass filters obtain from Hough are shown in Figure 4. The sparsity of proposed basis filters is compared with that of trained basis filters. The histogram of one level coefficients of the proposed band pass filter inferred from Hough transform is presented in Figure 5 (a) , where as same for trained basis is presented in Figure 5 (b) . The sparsity of the former seems to be efficient for compression. The mean and variance values of both histograms are presented in Table 1. Variance of coefficients of the proposed filters is significantly less that of trained filters. In Table 2, we have compared Quality Measures of reconstructed images with steerable basis and learned basis. This shows steerable basis has good reconstruction capability with sparse coefficients.

5 Conclusion

This article poses a new approach to make wavelet pyramid steerable basis coefficients as sparse as possible. The orientation directions required to select steerable band pass filters have been identified from the Hough transform and thus required no learning, yet it is appeared to be efficient for sparsity and reconstruction. Unlike same basis applied to a set of images, the basis proposed here seems to be tailor made for a given image. Though the current sparse basis that is learned [4], yet it is worth mentioning that current work of finding orientation direction for band pass filters has been assured by the learning based technique. Instead of learning the orientation directions, the same is inferred from Hough space generated out of Hough transform. Reconstruction power that is guaranteed by steerable pyramid structure, accuracy in sparsity and the cost effectiveness are major contribution of the present work.

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