

# A Study of the Attenuation in the Properties of Haptic Devices at the Limit of the Workspace

Jose San Martin

Universidad Rey Juan Carlos, 28933 Mostoles, Spain  
jose.sanmartin@urjc.es

**Abstract.** In the context of the optimization in virtual reality systems involving a haptic device, this paper introduces a correction in the formula that defined the performance of the device near the boundary of its workspace. We introduce two corrections to an index based on the Manipulability which takes in account the frequency with which each zone of the application workspace is visited during the simulation process, in order to help the designer for obtaining the best positioning of the device respect to the virtual environment. We demonstrate the new formula studying three different tasks to be accomplished. Finally we look for this best positioning analyzing not only the displacement but the different orientations we can introduce in the virtual environment in order to take advantage of the best zones of the workspace in terms of Manipulability.

**Keywords:** Virtual reality, Haptic interface, Manipulability, Mechanical Performance, Optimal designing.

## 1 Introduction

The design of a virtual environment in occasions involves the integration of different mechanical devices. The system can include manipulators that, depending on the configuration can reach more or less easily the different points of the workspace. We defined Real Workspace (RW) as the volume corresponding to all points the end of a manipulator is capable of reaching (fig. 1). For each point there is calculated the value of the quality of the necessary configuration of the manipulator according to an index based on Manipulability criterion [1], [2]. This volume allows us to identify convenient zones to work [3], [4].

Studying this RW we have determined that as a representation of the performance of a mechanical device, near of the maximum range of the device, the algebraic values obtained in terms of Manipulability must be corrected. The paper presents an attenuation factor that solves this problem.

In previous works [5] we have studied the possibility of obtaining tools to help in the optimal designing of haptic devices. In order to accomplish this we studied the calculus of the best fitting of RW and the task to be realized, represented by a virtual environment.

A design criterion of a system that contains haptic devices is maximizing the efficiency of this relative positioning of the virtual environment inside of all the reachable space. Now we analyze for each relative positioning the possible orientation

of the virtual environment (in fact it will involve the physical situation of the device in the Real World). So with this additional study we can improve the performance of the optimal solution.

## 2 Manipulability

Manipulability of a device is its ability to move freely in all directions into the work-space [6]. The first formulation that allowed a mathematical simple quantification was brought up by Yoshikawa [7]. We use the formulation of Manipulability proposed by Cavusoglu et al. [8]:

$$\mu = \sigma_{\min}(J_u) / \sigma_{\max}(J_u). \tag{1}$$

Where:

$\sigma_{\min}$  and  $\sigma_{\max}$  are the minimum and maximum singular values of  $J_u$ , upper half of the manipulator Jacobian matrix.

In this terms the Manipulability index is a tool for evaluating the quality and the performance in the designing of a manipulator device [9], [10]. So the first step in the study of Manipulability is the analysis of the kinematics of a manipulator, in this case we have used the PHANToM OMNi of SensAble Technologies. From Jacobian we calculate using (1) the Manipulability measure for each point of the surrounding space the device can reach.

### 2.1 Manipulability Solid

We can extend the map 2D developed in figure 1 by analyzing the behavior of the device in its whole surrounding space. This RW defined in 3D is a volume of the space near of the OMNi that contains all points of the space that the End Effector can reach. If we assign to each of points the Manipulability measure calculated by (1), the resultant volume is the Manipulability Solid associated to the device.

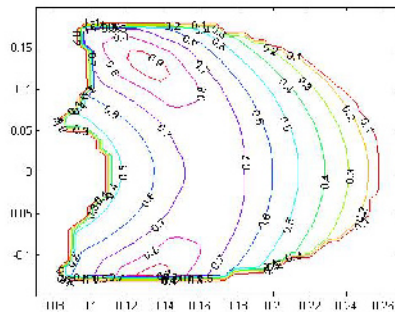
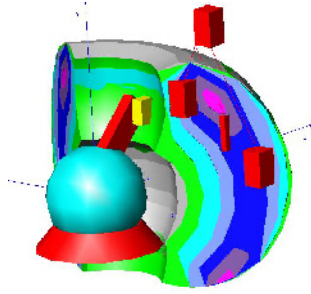


Fig. 1. OMNi device YZ Section from RW. Manipulability index values.



**Fig. 2.** 3D Map of Manipulability. Detail of the positioning of an AW inside the RW.

This map constitutes one important feature of the OMNi that we can consider to be physically joined the device. We can realize this map as a zone of influence of the device in its environment [5].

Figure 2 shows a representation of the OMNi device together with its 3D map of Manipulability. We use RW in the optimization of the design of Virtual Reality (VR) systems that integrate haptic devices. From whole the RW we select a portion, the virtual environment which we called Application Workspace (AW), in red in fig. 2. We place AW inside RW taking advantage of best zones according to Manipulability index. In the figure we appreciate different options of positioning an AW.

### 3 Attenuation at the Limit of RW

A problem exists near the limit of the RW (final range of the manipulator) and therefore near singular configurations. We need to characterize the attenuation of the properties in the limit zone of RW in terms of Manipulability, redefining RW. At the bottom of figure 1, at the frontier of RW, the value of Manipulability turns suddenly to zero. That it is not a real effect. To solve this problem, we first proposed an attenuation factor of Manipulability index to apply to the points to less distance than 10mm of the border. In the present paper a new factor is proposed in a more complex way, according to physical phenomenon beside introducing another idea: factor must not depend only on the distance to the border, but also on other points that surround it, being singular or not, definitively, with the shape of the surface at the border.

The value of the attenuation factor is depending, in each cell, of two circumstances: the first one is the distance to the boundary in each one of the three principal directions XYZ. A cell can be near of the boundary only across the X direction or in all directions (as a geographical cape). So we must penalize each one contribution to a very low value of Manipulability. In this case the attenuation is proportional to the distance.

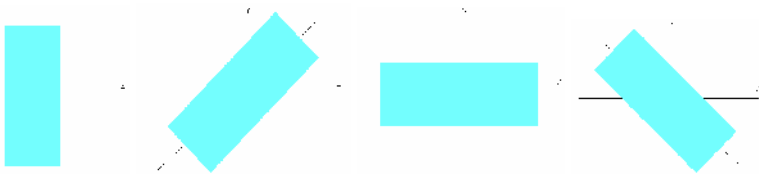
Assuming a discretization in cells of the RW, the second factor that contributes to the attenuation is the feature of the surrounding cells, that is, depending of the geometry of the RW, a cell can have or not, bad neighbors in terms of Manipulability. This “bad neighbors” affects to the performance of the cell. So for each cell, we must study the surrounding cube of cells (26 cells) in terms of distance to the XYZ borders. After

this we can recalculate the influence over the cell to study. Note that the new formulation will produce smoother surface of Manipulability than the algebraic one. Peaks are less high and valleys are less deep. We consider that these values are coherent with the conceptual definition of Manipulability.

## 4 Optimal Positioning

In addition we study the optimal positioning of a haptic device inside of a virtual environment. There are several scientific works aimed to develop methods of optimizing the use of manipulators [12]. Some of these works have used the measure of Manipulability as a criterion of optimization [13], [14], [15]. The design of a virtual reality system requires positioning the AW inside the solid of Manipulability. This intersection affect zones with different values of  $\mu$  (different colored volumes  $v_i$  in fig. 2). In order to solve this problem, we have developed an automatic searching process based on Simulated Annealing [16], [17]. Every step of the algorithm, the system searches randomly a solution near the current one, according to a probability that depends on the current temperature (similar to the metallurgy concept of Annealing). The algorithm allows calculating an optimal positioning of AW that produces a maximum value of the cost function.

Generalizing optimal design we study not only the position but also the orientation of a generic haptic manipulator arm. First we discretize in cells the RW and obtain 3 principal axes of the solid AW. For each positioning of the AW inside the RW, we define a set of rotations (fig. 3) for AW around principal axes. The study is divided in two parts: first one a discretization of the space is defined with a low precision, so the possible positions (and the orientations in each case) to study are much more limited. Both cases we use Simulated Annealing for searching optimal positioning and orientation. The second study has a better precision, but studying only zones we have obtained optimal values of Manipulability index in the previous study.



**Fig. 3.** Different orientations of the AW to study (0-45-90-135 grades) in each positioning

### 4.1 Frequency Map

Studying the problem of the optimal positioning we have seen that our application has zones of AW which are more used than others. These are, for instance in a simulation, the specific areas where the intervention is effectuated. It is desirable that the haptic device provides its best performance in the most visited zones.

So we propose, for the cases in which the use of the workspace is not homogeneous, to perform an additional analysis taking in account this heterogeneity. It involves

the study of the End Effector movement across the AW during the simulation process. As a result of the trace of the navigation across the virtual environment, we can obtain data referred to the frequency each single cell of the AW is visited ( $f_{ijk}$ ). With this value we can create a map of frequency for each task the device has to accomplish.

The optimal positioning is characterized by Useful Manipulability  $\hat{\mu}_v$ , considering a tri-dimensional grid of cells (i, j, k) of RW:

$$\hat{\mu}_v = \sum_{ijk} \mu_{vijk} \cdot f_{ijk} \tag{2}$$

Where:

- $\mu_{vijk}$  volumetric average manipulability of a cell. We need to study the distribution of Manipulability inside of the cell and calculate an average value.
- $f_{ijk}$  frequency of visits sampled during a task in each cell.

Note that if the size of cells (i,j,k) is small enough we can consider that Manipulability is constant and we can use the more simple formula:

$$\hat{\mu}_v = \sum_{ijk} \mu_{ijk} \cdot f_{ijk} \tag{3}$$

Where:

- $\mu_{ijk}$  is the Manipulability of a cell calculated by (1).

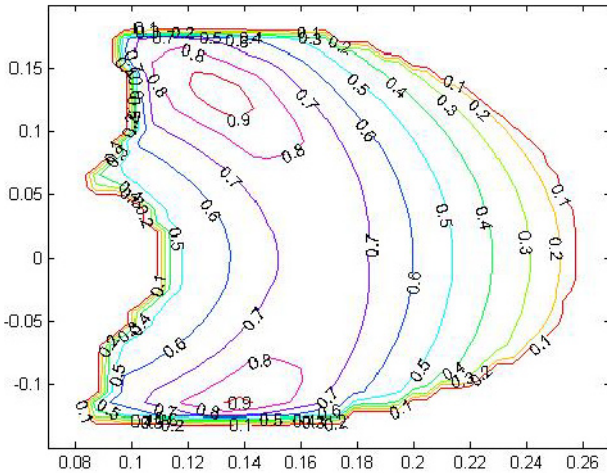
With this criterion the best positioning of the OMNi in terms of frequency of use will produce maximum value of  $\hat{\mu}_v$ , which is the cost function in the Simulated Annealing process.

## 5 Results

The solution of the first proposed problem, the attenuation of the RW-Manipulability map near the frontier is formulated. Note that figure 4 represents only the part of the plane X=0 that corresponds with the real workspace. So we can see the attenuation factor at frontier zone (portion of the volume of thickness 10mm) in this figure. Contrasting fig. 4 and previous one without attenuation, figure 1, results that the Manipulability in the boundary limits falls smoothly and now the maximum, that it was at the border line, is clearly displaced inside the workspace.

### 5.1 Positioning and Orientation of the Virtual Environment

We use defined measures for Optimal designing to calculate the grade of suitability of a manipulator in a real application. As an example we use a previously developed simulation that consists of the positioning-orientation of an OMNi device used as a component of the mechanical platform of a Minimal Invasive Surgery Trainer [12]. The virtual environment is the internal cavity of a human shoulder (Figure 5).



**Fig. 4.** OMNi device YZ Section from RW. Manipulability index including frontier-attenuation effect.

In order to introduce the study of the orientation of a generic haptic manipulator arm in a positioning, and taking in account the computational cost, the study is divided in two resolution parts. Both cases we use Simulated Annealing for searching optimal positioning and orientation.

In the first study we have used a resolution of 8mm of the cell side, with only 50000 cells to study. Using the above defined AW we calculate the best only-positioning of the AW inside the RW using (2). We obtain a value of cost function:

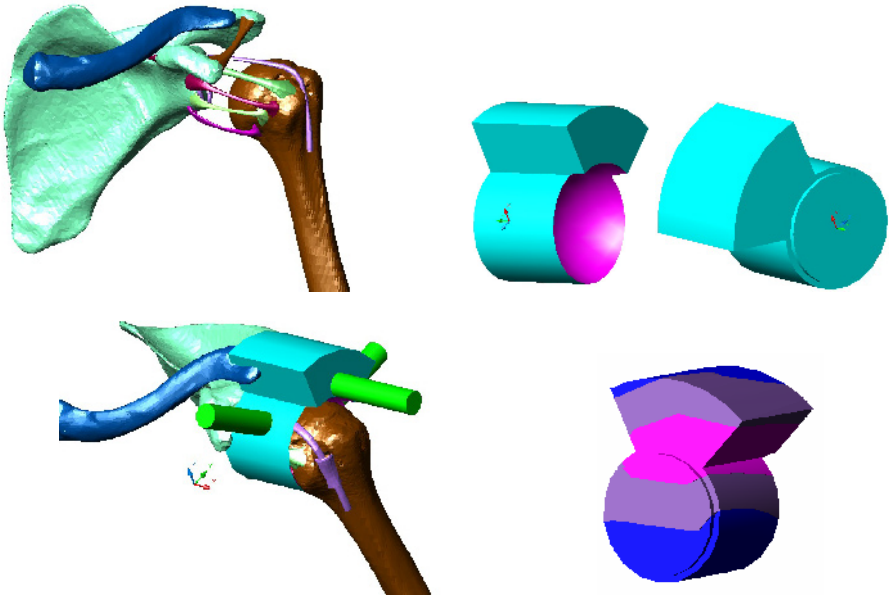
$$\hat{\mu}_v = 0.8521 \text{ as optimal solution.} \tag{4}$$

Now we remake the problem, with the same resolution, but including, for each possible positioning, 4 possible orientations, according to 0 grades (only positioning), 45, 90 and 135 grades of inclination (in fact other inclinations to consider are symmetrical to these). We study this cases for each axis XYZ. As a result we obtain:

$$\hat{\mu}_v = 0.8725 \text{ as optimal solution.} \tag{5}$$

So there is an option of the positioning-orientation (in the optimal orientation was 45 grades) that improves the only positioning study. The improvement is a very low percentage of 2.34%.

The second study has a better precision, cells of 2mm-side. In this case we must study near 2 million cells. The computational cost of this analysis is high so we



**Fig. 5.** Study of  $\mu_v$  in a real implementation. 5-1 Anatomical model. 5-2 Two views of AW. 5-3 Shoulder view plus AW. Entry portals in green. 5-4 AW intersection with RW as the example indicated in figure 2.

decide to simplify the study: we consider only zones we have obtained good values of Manipulability index in the low resolution study. Again we introduce the three possible orientations in each XYZ axis. As a result of using (3) we obtain:

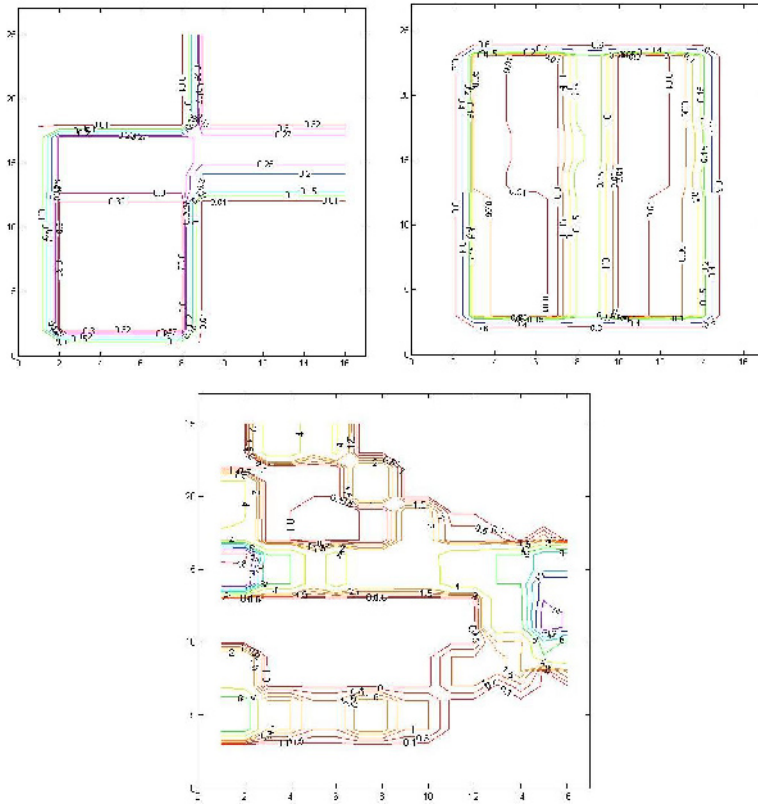
$$\hat{\mu}_v = 0.8838 \text{ as optimal solution.} \quad (6)$$

In this case the improvement is only of 1.28%.

## 5.2 Cases of Study: Frequency Map

In order to study the effect of realizing different tasks for the same haptic device in the same virtual environment, we select three tasks: task 1 is a simulation of a minimal invasive surgery, a shoulder arthroscopy (fig. 6-1) where most part of the work is most divided in two different zones (subacromial and glenohumeral). According to the trace of the movements of a surgeon in an intervention we have create the Frequency map of the figure. Note that in task 1 the navigation is located in two well-defined sectors.

The second task is a simulation of the movement of a probe-camera in the interior of a boiler in a operation searching for fissures. In this case most part of the navigation is in the frontier zone although all the movements pass through the center zone (fig. 6-2). The third task is a fictional task of the working of a machine tool. This task involves changing the tool, welding, coupling of parts, etc. Estimating the navigation across the virtual environment we can define its frequency map (fig. 6-3).



**Fig. 6.** Frequency map of tasks 1, 2 and 3 in the same Virtual Environment

So depending on the task, the optimal designing process defined previously concludes that haptic device must be placed in a different position of CG and orientation regard to principal axes in degrees:

Task 1.  $\hat{\mu}_v = 0.8042$ , AW at position XYZ (0.05, 0.13, 0.13).

Task 2.  $\hat{\mu}_v = 0.7911$ , AW at position XYZ (0.01, 0.13, 0.12).

Task 3.  $\hat{\mu}_v = 0.7717$ , AW at position XYZ (-0.02, 0.13, 0.115).

## 6 Conclusions

The definition of an attenuation factor solves the problem in the boundary of the current algebraic formulation and puts in hands of mechanical engineers a new tool to measure the performance of a manipulator taking in account the actual characteristics of the mechanical implementation.

In the generalization of the process of obtaining the optimal design the high resolution study indicate that the improvement of the global value of Manipulability index is



small from preliminary study. We have defined an error value in percent when using the first test instead of the second. Our conclusion is that in most part of applications exists only a minimum error, and a search of optimal positioning with high resolution but without orientation study can be an acceptable search. We recommend the low resolution positioning-orientation option in cases where the RW is much bigger than virtual environment-AW.

Finally, we have demonstrated that the most important factor in order to select a haptic device to cope with a task is the kind of task to be done. It is obvious that the size of the AW defines a size of device in terms of a necessary RW, but through three examples we have seen that a same device in a same virtual environment has very different performance.

## Acknowledgements

I would like to thank the anonymous reviewers for their helpful comments and to Gracian Trivino and the rest of the GMRV group at Universidad Rey Juan Carlos. This work was funded in part by the URJC - Comunidad de Madrid project CCG08-URJC/DPI-3647 and by the Spanish Ministry of Education and Science (grant TIN2007-67188).

## References

1. Yoshikawa, T.: Foundations of Robotics: Analysis and Control. MIT Press, Cambridge (1990)
2. Yoshikawa, T.: Manipulability of Robotic Mechanisms. The International Journal of Robotics research (1985)
3. San Martin, J., Trivino, G.: Measurement of Suitability of a Haptic Device in a Virtual Reality System. In: Proc. 2nd International Conference on Virtual Reality HCII 2007 (July 2007)
4. Pham, H.H., Chen, I.-M.: Optimal Synthesis for Workspace and Manipulability of Parallel Flexure Mechanism. In: Proceeding of the 11th World Congress in Mechanism and Machine Science, Tianjin, China, August 18-21 (2003)
5. San Martin, J., Trivino, G.: Mechanical Design of a Minimal Invasive Surgery Trainer Using the Manipulability as Measure of Optimization. In: IEEE International Conference on Mechatronics ICM 2007, Kumamoto, Japan (May 2007)
6. Murray, R.M., Li, Z., Sastry, S.S.: A mathematical introduction to robotic manipulation. CRC Press, Inc., Boca Raton (1994)
7. Yoshikawa, T.: Manipulability and redundancy control of robotic mechanisms. In: Proceedings of IEEE International Conference on Robotics and Automation, March 1985, vol. 2, pp. 1004-1009 (1985)
8. Cavusoglu, M.C., Feygin, D., Tendick, F.: A Critical Study of the Mechanical and Electrical Properties of the PHANToM Haptic Interface and Improvements for High Performance Control. Teleoperators and Virtual Environments 11(6), 555-568 (2002)
9. Yamamoto, Y., Yun, X.: Unified analysis on mobility and manipulability of mobilemanipulators. In: Proceedings. 1999 IEEE International Conference on Robotics and Automation, Detroit, vol. 2, pp. 1200-1206 (1999)

10. Yokokohji, Y., Yoshikawa, T.: Guide of master arms considering operator dynamics. *Journal of dynamic systems, measurement, and control* 115(2A), 253–260 (1993)
11. Sobh, T.M., Toundykov, D.Y.: Optimizing the tasks at hand (robotic manipulators). *Robotics & Automation Magazine* 11(2), 78–85 (2004)
12. Alqasemi, R.M., McCaffrey, E.J., Edwards, K.D., Dubey, R.V.: Analysis, evaluation and development of wheelchair-mounted robotic arms. In: 9th International Conference on Rehabilitation Robotics, ICORR 2005, June 28–July 1, 2005, pp. 469–472 (2005)
13. Guilamo, L., Kuffner, J., Nishiwaki, K., Kagami, S.: Manipulability optimization for trajectory generation. In: *Proceedings 2006 IEEE International Conference on Robotics and Automation, ICRA 2006*, May 15–19, 2006, pp. 2017–2022 (2006)
14. Masuda, T., Fujiwara, M., Kato, N., Arai, T.: Mechanism Configuration Evaluation of a Linear-Actuated Parallel Mechanism Using Manipulability. In: *Proceedings of the 2002 IEEE International Conference on Robotics & Automation*, Washington, DC (May 2002)
15. Bayle, B., Fourquet, J.-Y., Renaud, M.: Manipulability of Wheeled Mobile Manipulators: Application to Motion Generation. *The International Journal of Robotics Research* 22(7-8), 565–581 (2003)
16. Kirkpatrick, S., Gelatt Jr., C.D., Vecchi, M.P.: Optimization by Simulated Annealing. *Science* (220), 671–680 (May 13, 1983)
17. Aragon, C.R., Johnson, D.S., McGeoch, L.A., Shevon, C.: Optimization by Simulated Annealing: An Experimental Evaluation; Part II, Graph Coloring and Number Partitioning. *Operations Research* 39(3), 378–406 (1991)