Rough Set Theory of Pattern Classification in the Brain

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Abstract. Humans effortlessly classify and recognize complex patterns even if their attributes are imprecise and often inconsistent. It is not clear how the brain processes uncertain visual information. We have recorded single cell responses to various visual stimuli in area V4 of the monkey's visual cortex. Different visual patterns are described by their attributes (condition attributes) and placed, together with the decision attributes, in a decision table. Decision attributes are divided into several classes determined by the strength of the neural responses. Small cell responses are classified as *class 0*, medium to strong responses are classified as *classes 1 to n-1* (*min*(*n*)=*3*), and the strongest cell responses are classified as class *n*. The higher the class of the decision attribute the more preferred is the stimulus. Therefore each cell divides stimuli into its own family of equivalent objects.

By comparing responses of different cells we have found related concept classes. However, many different cells show inconsistency between their decision rules, which may suggest that parallel different decision logics may be implemented in the brain.

Keywords: visual brain, imprecise computation, bottom-up, top-down processes, neuronal activity.

1 Introduction

We define after Pawlak [1] an information system as $S = (U, A)$, where U, A are nonempty finite sets called the *universe of objects* and the *set of attributes*, respectively. If $a \in A$ *and* $u \in U$, the value $a(u)$ is a unique element of V (where V is a value set). The *indiscernibility relation* of any subset B of A or $I(B)$, is defined [3] as follows: $(x, y) \in I(B)$ or $xI(B)y$ if and only if $a(x) = a(y)$ for every $a \in B$, where $a(x) \in V$. $I(B)$ is an equivalence relation, and $[u]_B$ is the equivalence class of u, or a *B-elementary granule*. The family of all equivalence classes of $I(B)$ will be denoted $U/I(B)$ or U/B. The block of the partition U/B containing u will be denoted by $B(u)$. The concept $X \subseteq U$ is *B-definable* if for each $u \in U$ either $[u]_B \subseteq X$ or $[u]_B \subseteq U \backslash X$. $\underline{B}X = \{u \in U: [u]_B \subseteq X\}$ is a lower approximation of X. The concept $X \subseteq U$ is *B indefinable* if there exists $u \in U$ such that $[u]_B \cap X \neq \phi\}$. $\overline{B}X = \{u \in U: [u]_B \cap X \neq \phi\}$ is an *upper approximation* of X. The set $BN_B(X) = \overline{B}X - \underline{B}X$ will be referred to as the *B-boundary region* of. X If the boundary region of X is the empty set

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than *X* is *exact* (*crisp*) with respect to *B*; otherwise if $BN_B(X) \neq \phi$ *X* is not *exact* (i.e., it is *rough*) with respect to *B*. We say that the *B*-lower approximation of a given set *A* is the set of union of all *B*-granules that are included in the set *A*, and the *B*-upper approximation of *A* is a set of the union of all B-granules that have nonempty intersection with *A*. We will distinguish in the information system two disjoint classes of attributes: condition and decision attributes. The system *S* will be called a decision table *S* = (*U, C, D*) where *C* and *D* are condition and decision attributes.

In this paper the universe *U* will be assumed to be all visual patterns that are characterized by their attributes *C*. The purpose of our research is to find how these objects are classified in the brain. Therefore we are looking to determine *D* on the basis of a single neuron recording from the visual area in the brain.

Imprecise reasoning is a characteristic of natural languages and is related to human decision-making effectiveness [2]. The brain, in contrast to the computer, is constantly integrating many asynchronous parallel streams of information [3], which help in its adaptation to the environment. Most of our knowledge about the function of the brain is based on electrophysiological recordings from single neurons. In this paper we will describe properties of cells from the visual area V4. This intermediate area of the ventral stream mediates shape perception, but different laboratories propose different often-contradictory hypotheses about properties of V4 cells. We propose the use of rough set theory (Pawlak, [1]) to classify concepts as related to different stimuli attributes. We will show several examples of our method.

2 Method

Results of electrophysiological experiments are placed into the following decision table. Neurons are identified using numbers related to a collection of figures in [4]. Different measurements of the same cell are denoted by additional letters (*a, b, …*) and placed in the first column adjacent to the cell number. The next columns of the table describe stimulus attributes and their values. Stimulus attributes are as follows:

1. orientation in degrees appears in the column labeled *o*, and orientation bandwidth is labeled by *ob*.
2. spatial frequency is denoted as *sf*, and spatial frequency bandwidth is *sfb*
3. *x*-axis position is denoted by *xp* and the range of x-positions is *xpr*
4. *y*-axis position is denoted by *yp* and the range of y-positions is *ypr*
5. *x*-axis stimulus size is denoted by *xs*
6. *y*-axis stimulus size is denoted by *ys*
7. stimulus shape is denoted by *s*, with values of *s* are defined as follows: for grating *s=1*, for vertical bar *s= 2*, for horizontal bar *s= 3*, for disc *s= 4*, for annulus *s=5*.

Thus the full set of stimulus attributes is expressed as *B* = {*o, ob, sf, sfb, xp, xpr, yp, ypr, xs, ys, s*}. The cell's responses *r* are divided into several classes are placed in the last column of the table.

3 Results

We have analyzed the experimental data from several neurons recorded in the monkey's V4 [4]. Below we show a modified figure from the above work (Fig.1), along with the associated decision table (table 1).

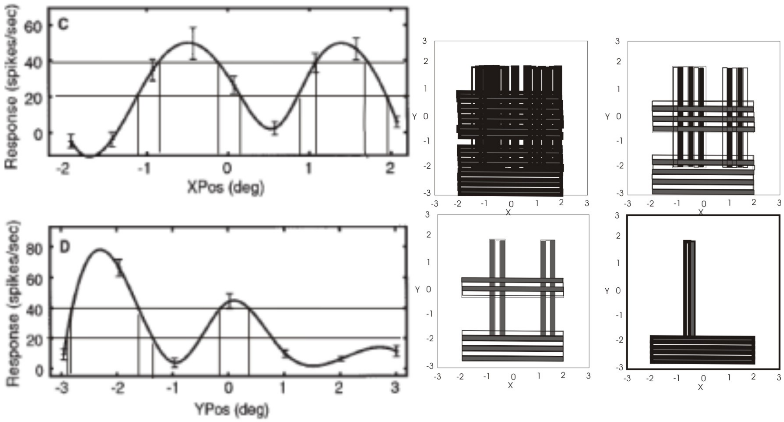

Fig. 1. Curves represent approximated responses of a cell from area V4 to vertical (C), and horizontal (D) bars. Bars change their position along the x-axis (Xpos) or along the y-axis (Ypos). Responses of the cell are measured in spikes/sec. Mean cell responses ± SE are marked in C and D plots. Cell responses are divided into 5 ranges (classes) but for simplicity only two horizontal lines are plotted. On the right are schematic representations of cell response on the basis of Table 1. Vertical and horizontal bars in certain x- and y-positions give significant responses: class 1 - upper left schematic, class 2 – upper right, class 3 - lower left, and class 4 – lower right schematic. These schematics represent decision rules for each response class.

On the basis of the decision table we have made a schematic of the optimal stimulus for this cell (Fig. 1, right side). Fig. 1 (left side) shows the cell's responses to the stimulus, which was a long narrow bar with vertical (Fig.1 C) or horizontal (Fig.1 D) orientation. The cell's responses are divided into strength classes (horizontal lines in plots of Fig. 1) with stimuli attributes placed in the decision table (Table 1). This table is converted into a schematic (right side of Fig. 1), which can be read as the decision rules related to four classes of cell responses. On the basis of this schematic the receptive field can be divided into smaller areas with different preferences, and these subfields can be stimulated independently as is shown in Fig. 2. Table 2 divides data from Fig. 2 and from Fig. 5 in [4] into decision classes, which determine equivalent classes of stimuli as shown on the schematic in lower part of Fig. 2.

Table 1. Decision table for the cell shown in Fig. 1. Attributes *ob, sf, sfb* were constant and are not presented in the table. Cell responses *r* below 10 spikes/s were defined *class 0*, above 10 spikes/s is defined as *class 1*, above 20 sp/s – *class 2*, above 40 sp/s - *class 3*, and above 50 sp/s – *class 4*.

Cell	o	xp	xpr	yp	ypr	xs	ys	s	r
12c	90	-0.6	1.4	0	0	0.4	4	2	1
12c1	90	-0.6	1.2	0	0	0.4	4	2	2
12c2	90	-0.6	0.6	0	0	0.4	4	2	3
12c3	90	1.35	1.3	0	0	0.4	4	2	1
12c4	90	1.3	1	0	0	0.4	4	2	2
12c5	90	1.3	0.5	0	0	0.4	4	2	3
12c6	90	-0.6	0	0	0	0.4	4	2	4
12d	0	0	0	-2	1.8	4	0.4	3	1
12d1	0	0	0	-2.2	1.6	4	0.4	3	2
12d2	0	0	0	-2.2	1.2	4	0.4	3	3
12d3	0	0	0	0.1	1.8	4	0.4	3	1
12d3	0	0	0	0.15	1.3	4	0.4	3	2
12d4	0	0	0	0.15	0.7	4	0.4	3	3
12d5	0	0	0	-2.2	0.9	4	0.4	3	4

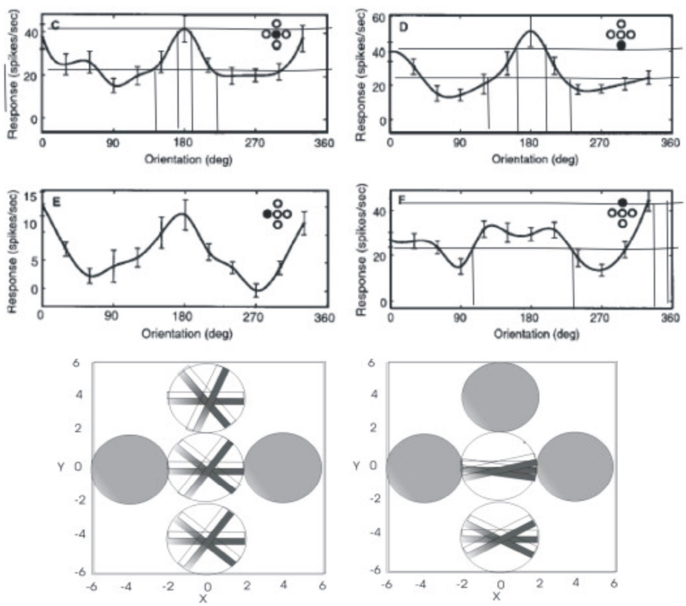

Fig. 2 Modified plots on the basis of [4] (upper plots), and their representation on the basis of table 2 (lower plots). C-F Curves represent responses to different orientations of one V4 cell when its subfields (their positions are shown in plots) are covered with 2 degree grating discs 2 degrees apart in a 6 degree receptive field. Lower plots: Gray circles indicate cell response below 10 spikes/s. Plots on the left are related to class 1, in the middle – class 2, and plots on the right are related to responses of class 3.

Table 2. Decision table for one cell shown in Fig. 2 (Figs. 3, 5 in [4]). Attributes xpr, ypr, s are constant and are not presented in the table. Cell # 3* from Fig. 3, cell# 5* from Fig. 5 cell #3_5* combined Figs. 3 and 5. Cell responses *r* below 10 spikes/s were defined as *class 0*, 10 - 20 spikes/s is defined as *class 1*, 20 - 40 sp/s – *class 2*, above 40 sp/s - *class 3*.

Cell	o	ob	sf	sfb	xp	yp	r
3_5c0	180	180	2.5	1.5	0	0	1
3c	172	105	2	0	0	0	2
3c1	10	140	2	0	0	0	2
3c2	180	20	2	0	0	0	3
3_5d0	180	180	2.5	1.5	0	0	1
3d	172	105	2	0	0	-2	2
3d1	5	100	2	0	0	-2	2
3d2	180	50	2	0	0	-2	3
3_5e	180	10	2	0	-2	0	1
3_5f0	180	180	2.5	1.5	0	2	1
3f	170	100	2	0	0	2	2
3f1	10	140	2	0	0	2	2
3f2	333	16	2	0	0	2	3
5a	180	0	2.3	2.6	0	-2	2
5b	180	0	2.5	3	0	2	2
5c	180	0	2.45	2.9	0	0	1
5c1	180	0	2.3	1.8	0	0	2

In order to find general decision rules (decision table reduct) we introduce a tolerance on a certain attribute values (discretization problem): all *ob* values with $0 < ob < 60$ we denote as ob_n (narrow orientation bandwidth), $ob > 100$ we denote as ob_w (wide orientation bandwidth), we write sfb_n if $0 < sfb < 1$ (small bandwidth), and sfb_w if $sfb > 1$. The **Decision rules** are as follows:

DR1: $ob_n \wedge xp_0 \longrightarrow r_3$, **DR2**: $ob_n \wedge xp_{-2} \longrightarrow r_1$,
DR3: $ob_w \wedge sfb_n \longrightarrow r_2$, **DR4:** $ob_w \wedge sfb_w \longrightarrow r_1$,
Notice that Figs 1 and 3 show possible configurations of the optimal stimulus. However, they do not take into account interactions between several stimuli, when more than one subfield is stimulated. In addition there are **Subfield Interaction Rules:**
SIR1: facilitation when stimulus consists of multiple bars with small distances (0.5-1 deg) between them, and inhibition when distance between bars is 1.5 -2 deg.
SIR2: inhibition when stimulus consists of multiple similar discs with distance between them ranging from 0 deg (touching) to 3 deg.
SIR3: Center-surround interaction, which is described below in detail.

The next part is related to the center-surround interaction **SIR3**. The decision table (Table 3) shows responses of 8 different cells stimulated with discs or annuli (Fig. 10 in [4]). In order to compare different cells, we have normalized their optimal orientation and denoted it as 1, and removed them from the table. We have introduced

Table 3. Decision table for eight cells comparing the center-surround interaction. All stimuli were concentric discs or annuli with xo – outer diameter, xi – inner diameter. All stimuli were localized around the middle of the receptive field, so that ob = xp = yp = xpr = ypr = 0 were fixed and we did not put them in the table. Cell responses *r* below 20 spikes/s were defined as *class 0*, 20 – 40 sp/s is defined as *class 1*, 40 – 100 sp/s – *class 2*, above 100 sp/s - *class 3*.

Cell	sf	sfb	xo	xi	s	r
101	0.5	0	7	0	4	0
101a	0.5	0	7	2	5	1
102	0.5	0	8	0	4	0
102a	0.5	0	8	3	5	0
103	0.5	0	6	0	4	0
103a	0.5	0	6	2	5	1
104	0.5	0	8	0	4	0
104a	0.5	0	8	3	5	2
105	0.5	0	7	0	4	0
105a	0.5	0	7	2	5	1
106	0.5	0	6	0	4	1
106a	0.5	0	6	3	5	2
107	0.5	0.25	6	0	4	2
107a	0.9	0.65	6	3	5	2
107b	3.8	0.2	6	3	5	2
107c	2.3	0.7	6	3	5	3
107d	2	0	6	2	5	2
107e	2	0	4	0	4	1
108	0.5	0	6	0	4	1
108a	1.95	0.65	4	0	4	2
108b	5.65	4.35	6	2	5	2
108c	0.65	0.6	6	2	5	3

a tolerance on values of *sf*. We have denoted as sf_{low} (low spatial frequency) all $sf < 1$, as sf_m for $1.7 < sf < 3.5$, and sf_h for $4 < sf$. We have calculated if the stimulus contains sf_{low}, sf_m or sf_h by taking from the table $sf \pm sfb$ and skipping *sfb*. For example, in the case of 108b the stimulus has *sf*: 8.6 $\pm$7.3 c/deg, which means that sf_m or sf_h are values of the stimulus attributes. We can also skip *s*, which is determined by values of *xo* and *xi*.

Stimuli used in these experiments can be placed in the following categories:

$Y_o = |sf_{low}\, xo_7\, xi_0| = \{101, 105\}$; $Y_1 = |sf_{low}\, xo_7\, xi_2| = \{101a, 105a\}$; $Y_2 = |sf_{low}\, xo_8\, xi_0| = \{102, 104\}$; $Y_3 = |sf_{low}\, xo_8\, xi_3| = \{102a, 104a\}$; $Y_4 = |sf_{low}\, xo_6\, xi_0| = \{103, 106, 107, 108, 20a\}$; $Y_5 = |sf_{low}\, xo_6\, xi_2| = \{103a, 106a, 107a, 108b, 20b\}$; $Y_6 = |sf_{low}\, xo_8\, xi_0| = \{104, 108a\}$; $Y_7 = |sf_{low}\, xo_8\, xi_3| = \{104a, 108a\}$; $Y_6 = |sf_{low}\, xo_4\, xi_0| = \{107e, 108a\}$.

These are equivalence classes for stimulus attributes, which means that in each class they are indiscernible *IND*(*B*). We have normalized orientation bandwidth to 0 in {20a, 20b} and spatial frequency bandwidth to 0 in cases {107, 107a, 108a, 108b}.

There are four classes of responses, denoted as r_0, r_1, r_2 , r_3. Therefore the expert's knowledge involves the following four concepts:

$| r_o | = \{101, 102, 102a, 103, 104, 105\}$, $| r_1 | = \{101a, 103a, 105a, 106, 107b, 108\}$
$| r_2 | = \{104a, 106a, 107, 107a, 107b, 107d, 108a, 108b\}$, $| r_3 | = \{107c, 108c\}$, which are denoted as X_o, X_1, X_2 , X_3.

We want to find out whether equivalence classes of the relation $IND\{r\}$ form the union of some equivalence relation $IND(B)$, or whether $B \Rightarrow \{r\}$.

We will calculate the lower and upper approximation [1] of the basic concepts in terms of stimulus basic categories:

$\underline{B} X_o = Y_o = \{101, 105\}$, $\overline{B} X_o = Y_o \cup Y_2 \cup Y_3 \cup Y_4 = \{101, 105, 102, 104, 102a, 104a, 103, 106, 107, 108\}$,

$\underline{B} X_1 = Y_1 \cup Y_5 = \{101a, 105a, 103a\}$, $\overline{B} X_1 = Y_1 \cup Y_5 \cup Y_6 \cup Y_4 = \{101a, 105a, 103a, 106, 108, 107e\}$,

$\underline{B} X_2 = 0$, $\overline{B} X_2 = Y_3 \cup Y_4 \cup Y_5 \cup Y_6 = \{102a, 104a, 103a, 107a, 108b, 106a, 20b, 103, 107, 106, 108, 20a, 107b, 108a\}$

Concept 0 and concept 1 are roughly *B-definable*, which means that only with some approximation can we say that stimulus Y_0 did not evoke a response (concept 0) in cells 101, 105. Other stimuli Y_2, Y_3 evoked no response (*concept 0*) or weak (*concept 1*) or strong (*concept 2*) response. This is similar for concept 1. However, concept 2 is internally *B-undefinable*. Stimulus attributes related to this concept should give us information about cell characteristics, but data from table 3 cannot do it. We can find quality [1] of our experiments by comparing properly classified stimuli $POS_B(r)=\{101, 101a, 105, 105a\}$ to all stimuli and responses:

$$\gamma\{r\} = |\{101,101a,105,105a\}| / |\{101,101a, \ldots,20a,20b\}| = 0.2.$$

We can also ask what percentage of cells we have fully classified. We obtain consistent responses from 2 of 9 cells, which means that $\gamma\{cells\} = 0.22$. This is related to the fact that for some cells we have tested more than two stimuli. What is also important from an electrophysiological point of view is there are negative cases. There are many negative instances for the concept 0, which means that in many cases this brain area responds to our stimuli; however it seems that our concepts are still only roughly defined. We have the following decision rules:

DR5: $x_{o7}\, x_{i2}\, s_5 \longrightarrow r_1$; **DR6**: $x_{o7}\, x_{i0}\, s_4 \longrightarrow r_0$, **DR7**: $x_{o8}\, x_{i0}\, s_4 \longrightarrow r_0$.

They can be interpreted as the statement that a large annulus (s5) evokes a weak response, but a large disc (s4) evokes no response. However, for certain stimuli there is inconsistency in responses of different cells (Table 3): 103: $x_{o6}\, x_{i0}\, s_4 \longrightarrow r_0$, 106: $x_{o6}\, x_{i0}\, s_4 \longrightarrow r_1$.

4 Discussion

The purpose of our study has been to determine how different categories of stimuli and particular concepts are related to the responses of a single cell. We test our theory

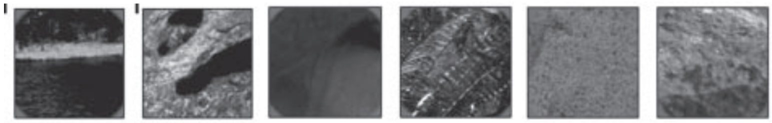

Fig. 3. In their paper David et al. [5] stimulated V4 neurons (medium size of their receptive fields was 10.2 deg) with natural images. Several examples of their images are shown above. We have divided responses of cells into three concept categories. The two images on the left represent cells, which give strong responses, related to our expertise *concept 2*. The two images in the middle evoke medium strength responses and they are related to *concept 1*. The two images on the right gave very weak responses; they are related to *concept 0*.

on a set of data from David et al. [5], shown in Fig. 3. We assume that the stimulus configuration in the first image on the left is similar to that proposed in Fig. 2; therefore it should give a strong response. The second image from the left can be divided into central and surround parts. The stimulus in the central disc is similar to that from Fig. 2 (*DR1*). Stimuli on the upper and right parts of the surround have a common orientation and a larger orientation bandwidth ob_w in comparison with the center (Fig. 2). These differences make for weak interactions between discs as in *SIR2* or between center-surround as in *SIR3*. This means that these images will be related to *concept 2*. Two middle images show smaller differences between their center and surround. Assuming that the center and surround are tuned to a feature of the object in the images, we believe that these images would also give significant responses. However, in the left image in the middle part of Fig. 3, stimuli in the surround consist of many orientations (ob_w) and many spatial frequencies (sfb_w); therefore medium class response is expected *DR4* (*concept 1*). The right middle image shows an interesting stimulus but also with a wide range of orientations and spatial frequencies *DR4*. There are small but significant differences between center and surround parts of the image. Similar rules as to the previous image can be applied. In consequence brain responses to both images are related to *concept 1*. In the two images on the right there is no significant difference between stimulus in the center and the surround. Therefore the response will be similar to that obtained when a single disc covers the whole receptive field: *DR6, DR7*. In most cells such a stimulus is classified as *concept 0*.

In summary, we have showed that using rough set theory we can divide stimulus attributes in relationships to neuronal responses into different concepts. Even if most of our concepts were very rough, they determine rules on whose basis we can predict neural responses to new, natural images.

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