A Novel Algorithm for Estimating Flow Length Distributions–LSM*

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Abstract. Traffic sampling technology has been widely deployed in front of many high-speed network applications to alleviate the great pressure on packet capturing.Increasingly passive traffic measurement employs sampling at the packet level. Packet sampling has become an attractive and scalable means to measure flow data on high-speed links. However, knowing the number and length of the original flows is necessary for some applications. This paper provides a novel algorithm, Least Square Method(LSM), that uses flow statistics formed from sampled packet stream to infer the absolute frequencies of lengths of flows in the unsampled stream. The theoretical analysis shows that the computational complexity of this method is well under control, and the experiment results demonstrate the inferred distributions are as accurate as EM algorithm.

1 Introduction

With the rapid increase of network link speed, packet sampling has become an attractive and scalable means to measure flow data. However, knowing the number and lengths of the unsampled flows remains useful for characterizing traffic and the resources required to accommodate its demands. Here are some applications: Resources Required for Collecting Flow Statistics: flow cache utilization and the bandwidth for processing and transmitting flow statistics are sensitive to the sampling rate, the number of flows, and flow lengths and duration; see [1,2]. Characterizing Source Traffic: the measured numbers of flows and the distribution of their lengths have been used to evaluate gains in deployment of web proxies [3], and to determine thresholds for setting up connections in flow-switched networks [4]. Sampling entails an inherent loss of information. We expect use statistic inference to recover information as much as possible.

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However, more detailed characteristics of the original traffic are not so easily estimated. Quantities of interest include the number of packets in the flow–we shall refer to this as the flow length–and the number of flows with fixed length.

1.1 Related Work

Kumar et al proposed a novel SCBF that performs per-flow counting without maintaining per-flow state in [5] and an algorithm for estimation of flow size distribution in [6]. Its disadvantage is that all packet must be processed due to not using sampling. Hohn and Veitch in [7] discussed the inaccuracy of estimating flow distribution from sampled traffic, when the sampling is performed at the packet level.

Although sampled traffic statistics are increasingly being used for network measurements, to our knowledge few studies have addressed the problem of estimating flow size distribution from the sampled packet stream. In [2], the authors studied the statistical properties of packet-level sampling using real-world Internet traffic traces. This is followed by [8] in which the flow distribution is inferred from the sampled statistics. After showing that the naive scaling of the flow distribution estimated from the sampled traffic is in general not accurate, the authors propose an EM algorithm to iteratively compute a more accurate estimation. Scaling method is simple, but it exploits the sampling properties of SYN flows to estimate TCP flow frequencies; EM algorithm does not rely on the properties of SYN flows and hence is not restricted to TCP traffic, but its versatility comes at the cost of computational complexity.

1.2 Some Elementary Concepts

This paper considers sampling some target proportion $p = 1/N$ of the packet stream. There are a number of different ways to implement this. Implementations include independent sampling of packets with probability $p = 1/N$, and periodic selection of every N^{th} packet from the full packet stream. In both cases we will call N the sampling period, i.e., the reciprocal of the average sampling rate. Although the length distributions by random and periodic sampling can be distinguished, the differences are, in fact, sufficiently small [8]. A flow is defined as a stream of packets subject to flow specification and timeout. When a packet arrives, the specific rules of flow specification determine which active flow this packet belongs to, or if no active flow is found that matches the description of this packet, a new flow is created. A TCP flow is a stream of TCP packets subject to timeout and having the same source and destination IP addresses, same source and destination port numbers. Similarly, a UDP flow is a stream of UDP packets associated with above specification. A general flow is a stream of packets subject to timeout and having the same source and destination IP addresses, same source and destination port numbers(not considering protocol). In this paper, we will use the term original flow to describe the above flow. A sampled flow is defined as a stream of packets that are sampled at probability $p = 1/N$ from an original flow.

1.3 Contribution and Outline

This paper presents a novel algorithm for estimation of flow size distributions from sampled flow statistics. Our method is available not only to TCP flows but also to general flows. We complete this work using four approaches. The first formalizes the probability distribution of original flow length of a sampled flow length j. The second classifies two types of flows based on their probability that no packet is sampled. A flow is labeled as small (S) when it's probability that no packet is sampled is more than ε and as large (L) when it's probability that no packet is sampled is less than or equal to ε. The third gives a simple estimation method for large flows. The fourth uses Least Square Method to estimate the full distribution of small flows.

The rest of this paper is organized as follows. In Section 2 we analyze the probability models of the original flow length distributions of a sampled flow. In Section 3, we classify two types of flows: small flow and large flow. Then we present different estimation methods for small flows and large flows, respectively. In Section 4 we discuss the computational complexity of our method. Furthermore, we compare our method with EM algorithm in estimation accuracy and computational complexity. We conclude in Section 5.

2 Probability Distribution of Original Flow length

For a specific original flow F, let X_F denote the number of packets in F, Y_F denote the number of packets in the sampled flow from F. The conditional distribution of Y_F, given that $X_F = l$, follows a binomial distribution $Pr[Y_F = k|X_F = l] = B_p(l,k) = \binom{l}{k} p^k(1-p)^{l-k}$. By the conditional probability formula,

$$Pr[X_F = x|Y_F = y] = \frac{Pr[Y_F = y|X_F = x]Pr[X_F = x]}{Pr[Y_F = y]} \tag{1}$$

and by the complete probability formula,

$$Pr[Y_F = y] = \sum_{i=y}^{\infty} B_p(i,y)Pr[X_F = i] \tag{2}$$

we assume that original flow length has a uniform *a priori* distribution. Thus,

$$P[X_F = k] = P[X_F = k+1] \;\; for \;\; k = 1, 2, \cdots.$$

How to prescribe the default *a priori* distribution has always been a controversial issue in statistics. It is however a widely acceptable practice to use uniform as the default when there are no obviously better choices. Assuming uniform as the default is reasonable also for the following reason. We use Pareto distribution as the default to calculate the probabilities, but we find that they is very close to the probabilities calculated with uniform distribution. However, computational complexity of Pareto is not desirable. So we select uniform as the default.

For fixed k, we sum for all $B_p(l,k), l = k, k+1, \cdots$:

$$\sum_{l=k}^{\infty} B_p(l,k) = \sum_{l=k}^{\infty} \binom{l}{k} p^k (1-p)^{l-k} = p^k \sum_{l=0}^{\infty} \binom{l+k}{k} (1-p)^l$$

$$= p^k \sum_{l=0}^{\infty} \binom{l+k}{k} q^l = p^k (1-q)^{-k-1} = 1/p = N$$

Hence,

$$Pr[Y_F = y] = \sum_{i=k}^{\infty} B_p(l,k) Pr[X_F = i] = Pr[X_F = y] \sum_{i=k}^{\infty} B_p(l,k) = \frac{Pr[X_F = y]}{p}.$$

Therefore,

$$Pr[X_F = x | Y_F = y] = \frac{Pr[Y_F = y | X_F = x] Pr[X_F = x]}{Pr[Y_F = y]}$$

$$= \frac{Pr[Y_F = y | X_F = x] Pr[X_F = x]}{Pr[X_F = y]/p} = pB_p(x,y).$$

We obtain

Lemma 1. *The probability that a sampled flow of length k is sampled from an original flow of length l is*

$$Pr[X_F = l | Y_F = k] = \binom{l}{k} p^k (1-p)^{l-k}, l = k, k+1, \cdots. \quad (3)$$

Lemma 2. *The mean and variance of the above probability distribution are $E\xi = N(k+1) - 1$ and $D\xi = (N+1)N(k+1)$, respectively.*

Let $a_1 = \frac{B_p(l,k)}{B_p(l-1,k)} = 1 + \frac{kN+1-l}{(l-k-1)N}$. For $l \leq kN$, since $a_1 > 1$, hence $B_p(l,k)$ is increasing as l increases. For $l > kN+1$, since $a_1 < 1$, hence $B_p(l,k)$ is decreasing as l increases. At $l = kN+1$, $a_1 = 1$ means that $B_p(l,k)$ is maximized at $l = kN$ and $l = kN+1$. We have

Lemma 3. *The probability $Pr[X_F = l | Y_F = k]$ is maximized at $l = kN, kN+1$. It is increasing as l increases for $l < kN+1$ and decreasing as l increases for $l > kN+1$.*

In following section we will use this conditional probability to estimate large flow length distribution. To our knowledge no studies have addressed the problem of using conditional probability to estimate flow length distribution.

3 Estimation Method of Flow Length Distributions

3.1 Flow Classification: Large Flow and Small Flow

Let $g = \{g_j : j = 1, 2, \cdots, n\}$, where g_j is sampled flow frequencies of length j, be a set of sampled flow length frequencies, $f = \{f_i : i = 1, 2, \cdots, n, \cdots\}$ a set of original flow length frequencies to be estimated. Consider sampling the packets of an original flow of length Nj independently with probability $1/N$, the probability that no packet is sampled is $(1 - 1/N)^{Nj} = ((1 - 1/N)^N)^j$. $\{(1 - 1/N)^N\}$ is increasing in N and $\lim_{N\to\infty} (1 - 1/N)^N = 1/e < 0.37$. Thus for a given error ε,we require $(1-1/N)^{Nj} < (1/e)^j < \varepsilon$ and choose $j_{bord} \geq max(j(\varepsilon) = \lceil log(1/\varepsilon)\rceil, \alpha)$. For example, $j(0.01) = 5, j(0.001) = 7$. We classify two types of flows based on their probability that no packet is sampled. A flow is labeled as small (S) when it's probability that no packet is sampled is more than ε and as large (L) when it's probability that no packet is sampled is less than or equal to ε.

3.2 Estimation for Large Flows

For a sampled flow of length $j > j_{bord}$, by Lemma 3, the original flow length values of the $2N$ relatively large probabilities are $N(j - 1), \cdots, N(j + 1)$. We estimate the sampled flow is sampled from one of the $2N$ original flows. Then there are $\frac{g_j}{2N}$ sampled flows that are sampled from one of original flows of the above lengths in $g_j(j > j_{bord})$ sampled flows. Therefore, for all large flows of length $i > Nj_{bord}$, we have

$$f_i = \frac{1}{2N}(g_j + g_{j+1}), where\, j = \lfloor (i - 1)/N\rfloor. \tag{4}$$

From Equation (4) we can observe that the number of original flows of length i is calculated by using the numbers of the sampled flows of two different lengths. In fact, this is maximum likelihood estimation for large flows. Since scaling method only use a sample flow length, so this method is more precise. Furthermore, in order to improve accuracy, we can extend the original flow length interval so that a sampled flow can be sampled from one of the more than $2N$ original flows, e.g. $3N$, $4N$. In this case, in order to estimate the number of original flows of a fixed length we need involve the more sampled flows. Since the probability increases, so the estimation is more reliable.

3.3 Least Square Method for Small Flows

For all small flows of length $i \leq Nj_{bord}$, we estimate as follows:

$$g_j = \sum_{i=j}^{m} B_p(i, j) f_i, \qquad j = 1, \cdots, Nj_{bord} \tag{5}$$

where $m = \max\{i : f_i \neq 0\}$. For $i > Nj_{bord}$, substituting (4) into Equations (5):

$$\overline{g}_j = g_j - \sum_{i=Nj_{bord}+1}^{m} B_p(i,j) f_i = \sum_{i=j}^{Nj_{bord}} B_p(i,j) f_i, j = l, \cdots, Nj_{bord}. \quad (6)$$

For the above some $\overline{g}_j \leq 0$, we replace it with $\delta \overline{g}_{i-1}, 0 < \delta < 1$. For example, we may take $\delta = 0.94$. Hence, we may assume that all $\overline{g}_j > 0$. Since some coefficients of Equations (6) are zero or very small, solving the equations directly may follow a large deviation. Therefore, we use the heavy-tailed feather of flow to reduce the number of the indeterminates of Equations (6).

For $i, l \in [8, Nj_{bord}], i > l$, we have:

$$f_i = (l/i)^k f_l \quad (7)$$

Substituting Equation (7) into Equations (6):

$$\overline{g}_j = \sum_{i=j}^{l} B_p(i,j) f_i + \sum_{i=l+1}^{Nj_{bord}} B_p(i,j)(l/i)^k f_l, \qquad j = 1, \cdots, Nj_{bord}. \quad (8)$$

Now, we let the values of k increase from 0.5 to 5.0 by increment 0.1. Applying each concrete k of the above values to compute Equations (8), we obtain:

$$y_j^{(k)} = \sum_{i=1}^{l} x_{ji}^{(k)} f_i \qquad j = 1, \cdots, Nj_{bord}. \quad (9)$$

There are l indeterminates and Nj_{bord} equations in Equations (9) where $Nj_{bord} > l$. According to [9], we know least square estimate is unbias and its variance is least. Hence, we use least square method to solve Equations (9) and get the solutions $f_i^{(k)}, \ i = 1, \cdots, l$. If each $f_i^{(k)} > 0, i = 1, \cdots, l$, then let $m_k = \sum_{i=1}^{Nj_{bord}} (y_j^{(k)} - \sum_{i=1}^{l} x_{ji}^{(k)} f_i^{(k)})^2$. We find the value of k such that m_k is minimized in all positive solutions. Denoting the found value as $\overline{k}$, we substitute the corresponding $f_l^{(\overline{k})}$ into Equation (7). Finally, we obtain $f_i^{(\overline{k})}, i = 1, \cdots, Nj_{bord}$. We write our estimation of original small flows as $f_i^{(\overline{k})}, i = 1, \cdots, Nj_{bord}$.

4 Evaluations and Comparison

Computational complexity. Let i_{max} denote the maximum original flow length. The computation for binomial coefficients of Equations (5) is $O(Nj_{bord} i_{max})$. The computation for Least Square Method needs little time. We compare the computational complexity of our method against the best known EM algorithm in [8] for estimating flow distribution from sampled traffic. In [8] for all ϕ_i completing an EM iteration is $O(i_{max}^2 j_{size})$,where j_{size} denote the number of non-zero sampled flow length frequencies g_j. We collect all IP packet heads during a period of 300 minutes at Jiangsu provincial network border of China Education and Research Network (CERNET) (1Gbps) to do offline experiment. For

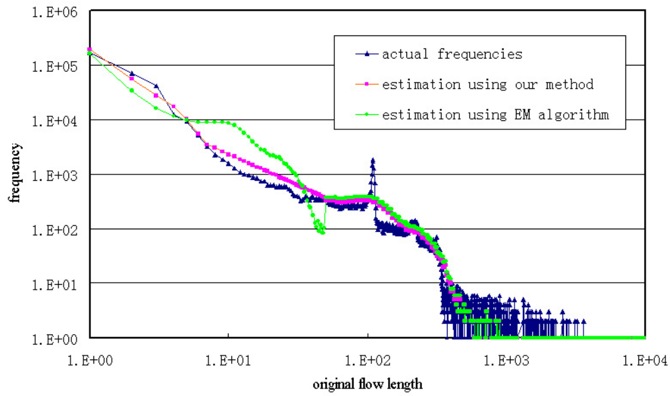

Fig. 1. Comparison of our method and EM algorithm at sampling period $N = 10$ for Jiangsu trace

Table 1. WMRD of our method and EM algorithm

trace	Sampling period	WMRD of our method	WMRD of EM algorithm
Abilence III	10	18%	29%
	30	23%	29%
	100	31%	34%
Jiangsu	10	15%	18%
	30	21%	19%
	100	34%	38%
Abilence I	10	13%	15%
	30	22%	23%
	100	31%	35%

IP header data during a period of 1 minute, sampling packets with $p = 1/10$, $i_{max} = 2000$, $j_{size} = 400$,in our method let $\varepsilon = 0.01$, then $j_{bord} = 5$, thus $(Nj_{bord})i_{max} = 2000 * 50 = 0.1 * 100^3$. However, $i_{max}^2 j_{size} = 2000 * 2000 * 400 = 1600 * 100^3$ in EM algorithm of [8].

Estimation accuracy: We adopt Weighted Mean Relative Difference (WMRD) as our evaluation metric. Suppose the number of original flows of length i is n_i and our estimation of this number is $\hat{n}_i$. The value of WMRD is given by: WMRD= $\frac{\sum_i |n_i - \hat{n}_i|}{\sum_i (\frac{n_i + \hat{n}_i}{2})}$.

We use three traces in our comparison experiments. The first trace is the first publicly available 10 Gigabit Internet backbone packet header trace from NLANR: Abilence III data set [10]. In our experiments , we used a minute of traffic from the trace. The second trace, which contains packets during a 5-minute period , was collected at Jiangsu provincial network border of China Education and Research Network (CERNET) on April 17, 2004. The backbone capacity is 1000Mbps; mean traffic per day is 587 Mbps. We call this trace as Jiangsu trace. The third trace, which contains packets during a 10 minute period, was obtained from NLANR: Abilence I [11]. Figure 1 compares the two estimators of Jiangsu trace derived by our method and EM algorithm of [8] at sampling period $N = 10$. Observe that they are so close. Table 1 shows the estimation accuracy of our algorithm is close enough to that of EM algorithm. In most cases, our algorithm is much more accurate.

5 Conclusions

Estimating the distribution of flow length is important in a number of network applications. In this paper we present a novel method for estimation of flow length distributions from sampled flow statistics. The main advantage is that it could significantly reduce the computational complexity. The theoretical analysis shows that the computational complexity of our method is well under control. The experimental results demonstrate that our method achieves an accurate estimation for flow distribution.

References

1. Duffield, N.G., Lund, C., Thorup, M.: Charging from sampled network usage. In: ACM SIGCOMM Internet Measurement Workshop, November 2001, pp. 245–256 (2001)
2. Duffield, N.G., Lund, C., Thorup, M.: Properties and Prediction of Flow Statistics from Sampled Packet Streams. In: ACM SIGCOMM Internet Measurement Workshop, November 2002, pp. 159–171 (2002)
3. Feldmann, A., Caceres, R., Douglis, F., Glass, G., Rabinovich, M.: Performance of Web Proxy Caching in Heterogeneous Bandwidth Environments. IEEE INFOCOM 99, 107–116 (1999)
4. Feldmann, A., Rexford, J., Caceres, R.: Efficient Policies for Carrying Web Traffic over Flow-Switched Networks. IEEE/ACM Transactions on Networking 6, 673–685 (1998)
5. Kumar, A., Xu, J., Li, L., Wang, J.: Space Code Bloom Filter for Efficient Traffic Flow Measurement. In: IEEE INFOCOM 2004, pp. 1762–1773 (2004)
6. Kumar, A., Sung, M., Xu, J.(Jim.), Wang, J.: Data streaming algorithms for efficient and accurate estimation of flow size distribution. In: ACM Sigmetrics 2004, pp. 177–188 (2004)
7. Hohn, N., Veitch, D.: Inverting Sampled Traffic. In: Internet Measurement Conference October 27-29, 2003, Miami Beach, Florida, USA (2003)
8. Duffield, N.G., Lund, C., Thorup, M.: Estimating Flow Distributions from Sampled Flow Statistics. IEEE/ACM Transation on Networking 13, 933–945 (2005)
9. Xuan, L.: Applied Statistics, pp. 80–89. Tsinghua University Press, Beijing (1999)
10. NLANR: Abilene-III data set, `hppt://pma.nlanr.net/Special/ipls3.html`
11. NLANR:Abilene-I data set, `http://pma.nlanr.net/Traces/long/bell1.html`