Game and Market Equilibria: Computation, Approximation, and Smoothed Analysis

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I will present some recent advances in algorithmic game theory especially about Nash equilibria. As you may have already known, the notion of Nash equilibria has captured the imagination of much of the computer science theory community, both for its many applications in the growing domain of online interactions and for its deep and fundamental mathematical structures. As the complexity and scale of typical Internet applications increase, the problem of efficiently analyzing their game-theoretic properties becomes more pointed.

In particular, I will cover the recent results in settling several open questions about Nash equilibria. After a quick review the result of Chen and Deng that

> BIMATRIX, the problem of finding a Nash equilibrium in a two-person game, is a complete problem in the complexity class **PPAD** (Polynomial Parity Argument, Directed version) introduced by Papadimitriou in 1991,

I will focus on the approximation and smoothed complexity of equilibrium computation and prove the following two theorems:

- BIMATRIX does not have a fully polynomial-time approximation scheme, unless every problem in **PPAD** is solvable in polynomial time.
- The smoothed complexity of the classic Lemke-Howson algorithm, and in fact, of any algorithm for BIMATRIX is not polynomial, unless every problem in **PPAD** is solvable in randomized polynomial time.

Our results demonstrate that, even in this simplest form of non-cooperative games, equilibrium computation and approximation are polynomial-time equivalent to fixed-point computation. If time permits, I will also cover the extensions of these results to other equilibrium problems such as in trading and market economies.

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